Variable-Sample Methods for Stochastic Optimization

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In this article we discuss the application of a certain class of Monte Carlo methods to stochastic optimization problems. Particularly, we study variable-sample techniques, in which the objective function is replaced, at each iteration, by a sample average approximation. We first provide general results on the schedule of sample sizes, under which variable-sample methods yield consistent estimators as well as bounds on the estimation error. Because the convergence analysis is performed pathwisely, we are able to obtain our results in a flexible setting, which requires mild assumptions on the distributions and which includes the possibility of using different sampling distributions along the algorithm. We illustrate these ideas by studying a modification of the well-known pure random search method, adapting it to the variable-sample scheme, and show conditions for convergence of the algorithm. Implementation issues are discussed and numerical results are presented to illustrate the ideas.

Categories and Subject Descriptors: G.1.6 [Numerical Analysis]: Optimization—global optimization; G.3 [Probability and Statistics]: Probabilistic algorithms (including Monte Carlo); I.6.1 [Simulation and Modeling]: Simulation Theory

General Terms: Algorithms, Design, Theory

Additional Key Words and Phrases: Monte Carlo methods, pathwise bounds, random search, stochastic optimization

1. INTRODUCTION

In the past few years, a great deal of attention has been devoted to theoretical and practical aspects of combining simulation and optimization techniques to solve practical problems. This is required in problems where the objective function cannot be evaluated exactly but rather must be estimated by simulation. Indeed, many simulation packages have now optimization procedures implemented in the software. Conversely, there has been considerable amount of research on optimization methods that incorporate sampling in order to deal with uncertainty factors.

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