Quasi-Monte Carlo Applications

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Outline

• Other applications
• Results
• Option Model
• Error results
• Basic analysis

In Option Pricing

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In Option Pricing

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Motivation

- Error result for standard Monte-Carlo
- Validity of pseudo-random generators
- Really random? Collinear patterns possible
Error Results for Quasi-Monte Carlo

• Result for Halton sequence
• General result - Irrationals
Expected Error Analysis

- Expected error:
- Mean zero/FORM of multi-dimensional Wiener Process
- Function distribution model
- Wozniakowski, Traub

Expected Error Analysis
Implications

- **Form of functions**
  - Bounded variation
  - Unit hypercube
  - "Randomly" generated

- **Validity**
  - In applications, unlikely to hold

- **BUT** - can you use a specific function form?
Option Models

- "Derivative" securities
  - Call: Buy a share at a given price at specific time
    » If by a specific time - American (European)
  - Put: Sell a share at a given price at specific time
  - Straddle: Buy or sell

- Why?
  - Reduce risk (hedge)
  - Speculate
  - Arbitrage

- Original analysis - L. Bachelier (1900 - Brownian motion)
Valuing an Option

- (European) Call Option on Share assuming:
  - Buy at $K$ at time $T$; Current time: $t$; Share price: $S_t$
  - Volatility: $\sigma$; Risk-free rate: $r_f$; No fees; Price follows Ito process

Valuing option (Black/Scholes):
- Assume risk neutral world (annual return=$r_f$ independent of risk)
- Find future expected value and discount back by $r_f$

Share Price, $S_T$
Strike, $K$

\[
\text{Call value at } t = C_t = e^{-r_f(T-t)} \left[ \mathbb{E} \left( (S_T-K)^+ \right) \right]
\]

- Volatility: $\sigma$; Risk-free rate: $r_f$; No fees; Price follows Ito process
- Buy at $K$ at time $T$; Current time: $t$; Share price: $S_t$

(European) Call Option on Share assuming:
Black-Scholes Difficulty

- American options
  - Decision at all t - exercise or not?
  - Analysis difficult
  - Sample paths (Monte Carlo)

Alternative?

American options
Monte Carlo Method for Option Valuation

• General form

• Procedure

• Typical dimensions
  – 30-180 stocks
  – 365+ for mortgages (each holder has option to exercise)
Methods Considered

- Pseudo-Schrage
- Quasi-Irrationals
  - Halton
  - Sobol
  - Faure
  - Irrationals
  - Schrage
  - Sobol
Examples

- Exercise (strike) price - 30
- Current price -
- Volatility -
- Expiration date -

Examples
Sample times

- Sobol
- Faure
- Halton
- Irrationals
- Pseudo
Stopping Rule Results
Other applications
Conclusions