Discrete and Continuous Models in Stochastic Scheduling

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OUTLINE

• Motivation - Short and Long Term Framework
  • Long-Term: Capacity Decisions: Flexibility
  • Short-Term: Production scheduling
  • Different role of risk
  • Results on cycles and matching up
  • Types of uncertainty
  • General approach toward risk
  • Option uses
  • Problems of uncertainty

• Summary
• Computation
Long Term Horizon Decisions (Years)

- Strategies
- Overall Capacity
- Sources of Uncertainty
- Product Mix
- Market
- Competitors

Short to Medium Term Decisions (< Year)

- Actual Production
- Daily to Monthly Mix
- Variable Productive Capacity
CAPACITY DECISIONS

- What to produce?
- Where to produce? (When?)
- How much to produce?
- Where to produce? (When?)
- What to produce?

EXAMPLE: Models 1, 2, 3; Plants A, B

Should B also build 2?
MEASURING VALUE

• SUPPOSE RISK NEUTRAL: (expected cost)

RESULT: Does not correspond to decision maker preference

• CONSEQUENCE: For financial objectives

• RESOLUTION: Use economic/financial theories:
  – Efficient Market Theory
  – Capital Asset Pricing Model

CONSEQUENCE: For financial objectives

• RESOLUTION: Know how to assess based on risk
• **RISK/RETURN TRADEOFF:**

**NEED: Symmetric Risk**

- All investments on security market line
- Firms need not diversify
- Investors can diversify

**BASICS OF CAPM**
IMPLICATIONS FOR CAPACITY

DECISIONS

VALIDITY OF SYMMETRY:
• Unlikely:
  » Constrained resources
  » Correlations among demands

ALTERNATIVES?
• Option Theory
  » Allows for non-symmetric risk
  » Explicitly considers constraints
  » Sell at a given price

 Likely:
• Validity of Symmetry:
USE OF OPTIONS

- Capacity limits cut off potential revenue like selling option to competitor
- Values asymmetric risk
- Competitor revenue like selling option to competitor limits cut off potential

Steps with capacity evaluation:
- Adjust revenue to risk-free equivalent
- As in Black-Scholes model
- Can evaluate as if risk neutral
- Assumption: risk-free hedge

Results from finance:
- Discount at riskless rate
EVALUATING THE OPTION

- CANNOT USE EXPECTATIONS (SINGLE FORECASTS) ALONE BECAUSE OF:
  - Correlated Demand
    - Models 1, 2, 3 similar
  - Capacity Limit - cuts off revenue growth
    - => Asymmetric payoff
RESULTS OF OPTION-STOCHASTIC PROGRAMMING

- Gives value measure
- Incorporates uncertainty and any available information
- Can be used for varying model lifetimes/production periods
- Integrates capacity decisions across firm (not just within 1 plant)
- Can use for utilization/lost sales/what-if analyses
- Model
GENERALIZATIONS FOR OTHER LONG-TERM DECISION

START: Eliminate constraints on production
- Demand uncertainty remains - assume that is symmetric
- Can value unconstrained revenue with market rate, \( r \)

IMPLICATIONS OF RISK NEUTRAL HEDGE:

Can model as if investors are risk neutral

\[ \frac{1}{1+r_t} c_t x_t \]

\[ \frac{1}{(1+r)^t} c_t (1+r_f)^t x_t \]

Future value: this new quantity is constrained

\( \Rightarrow \) value grows at risk-free rate, \( r_f \)
CONSTRAINT MODIFICATION

FORMER CONSTRAINTS: $A_t x_t \leq b_t$

NOW: $A_t x_t \frac{(1+r)^t}{(1+r')^t} \leq b_t$

\[
\begin{align*}
x_t &\leq b_t \\
\frac{(1+r)^t}{(1+r')^t} &\leq b_t
\end{align*}
\]
To compensate for lower risk with constraints,

\[
\begin{align*}
\max_{\mathbf{c}} \quad \mathbf{q} \geq \mathbf{A} \mathbf{x} \\
\text{s.t.} \quad \mathbf{c}^\top \mathbf{1} = 1 \\
\mathbf{c} \geq 0
\end{align*}
\]

EQUIVALENT TO:

\[
\begin{align*}
\max_{\mathbf{c}} \quad \mathbf{q} \geq \mathbf{A} \mathbf{x} \\
\text{s.t.} \quad \mathbf{c}^\top \mathbf{1} = 1 \\
\mathbf{c} \geq 0
\end{align*}
\]

Want to find (present value):

NEW PERIOD 1 PROBLEM
EXTREME CASES

1. All slack constraints:

\[
\frac{1}{(1+r)^t} \max \left[ \frac{c^T x}{b} \right] = \frac{c^T x}{b} - \frac{(1+r)^t}{(1+r)^t} \]

i.e. same as if unconstrained - risky rate

\[
\frac{1}{(1+r)^t} \max \left[ \frac{c^T x}{b} \right] \]

i.e. same as if determinisitic - riskfree rate

NO SLACK:

\[
\frac{1}{(1+r)^t} \max \left[ \frac{c^T x}{b} \right] = \frac{c^T x}{b} - \frac{(1+r)^t}{(1+r)^t} \]

i.e. same as if unconstrained - risky rate

\[
\frac{1}{(1+r)^t} \max \left[ \frac{c^T x}{b} \right] \]

i.e. same as if determinisitic - riskfree rate
OVERALL RESULTS - LONG-TERM

• CAN ADAPT OBJECTIVE TO RISK
• USE RATE FROM FIRM AS WHOLE
• ADJUST ALL CONSTRAINTS ON REVENUE
  - ASSUMES INVEST LIKE WHOLE FIRM
  - SYMMETRIC RISK
• CAN ADAPT OBJECTIVE TO RISK

ATTITUDE TOWARD INVESTMENT

END RESULT SHOULD REFLECT INVESTOR GENERATORS BY RATE RATIOS

TERM

OVERALL RESULTS - LONG-TERM
SHORT-TERM UNCERTAINTIES

- EFFECTIVE CAPACITY LIMITED BY:
  - Uncertain Yields - Quality Loss
  - Machine Breakdowns
  - Uncertain Yields - Quality Loss
  - Variable Production Rates
  - Unforeseen Orders
  - Lack of Material/Supplies
  - Logistical Problems
  - Basic Optimization Problem
  - Must Define Objectives
  - Look at Structure

GENERAL FRAMEWORK
Formulation:

\[
\text{MIN } E \left[ \sum_{t=1}^{T} f_t(x_t, x_{t+1}) \right]
\]

s.t.

\[
x_t \in X_t \quad \text{nonanticipative}
\]

\[
\mathbb{P} \left( \forall (x_t, x_{t+1}) \in \mathcal{P}: t \leq T \quad (1) \right) \geq 0
\]

DEFINITIONS:

- \( x_t \): aggregate production of all components
- \( f_t \): defines transition - only if resources available and includes subtraction of demand

GENERAL MULTISTAGE MODEL
DYNAMIC PROGRAMMING

VIEW

STAGES: \( t = 1, \ldots, T \)

STATES: \( x_t \rightarrow B(x_t) \text{ (or other transformation)} \)

VALUE FUNCTION:

\[
Y_t(x_t) = E[y_t(x_t, x_{t+1})] \quad \text{s.t. } x_{t+1} \in X_{t+1}(x_t) \quad \text{where } X_{t+1}(x_t) \quad \text{is the random element and}
\]

ASSUMPTIONS:

- CONVEXITY
- EARLY AND Lateness PENALTIES

\[
E[y_t(x_t, x_{t+1})] = \min_x \left( f_t(x_t, x_{t+1}, x_t) + Y_{t+1}(x_{t+1}) \right)
\]
• LEADS TO MATCH-UP FRAMEWORK
  - Return to optimal cycle
  - From other disruptions:
    - Turnpike: (Birge/Dempster)
      - May indicate Kanban/ConWIP type optimality
      - Optimal if stationary or cyclic distributions
    - Cyclic schedules:
      - Derive supporting prices
      - Can define optimality conditions
      - Optimality:

RESULTS
PRODUCTION SCHEDULING
MATCH-UP BASICS

• METHOD: (Bean, Birge, Mittenthal, Noon)

• START: FIND a PRE-SCHEDULE (CYCLIC):
  FROM FORECASTS/NORMAL RANDOMNESS

• MATCH-UP PROCESS:
  WHEN DISRUPTIONS OCCUR, RECOGNIZE THEM
  TO DEVELOP RESPONSE, CONSTRUCT A PLAN TO
  MATCH UP WITH THE PRE-SCHEDULE IN THE FUTURE
  OVERALL PATTERN REPRESENTS SETTING GOALS
  AND REACTING

• MAY ALSO USE TO IMPROVE IN SHORT RUN

MATCH-UP BASICS
MATCH-UP PROBLEM

• GOAL: FIND A PERIOD OVER WHICH TO CHANGE SCHEDULE

MATCH-UP

MATCH-UP

HORIZON

HORIZON

TIME

TIME

DISRUPTION

DISRUPTION

MACHINE

MACHINE

A

B

C

DEFINE HORIZON

DEFINE SCENARIOS

DEFINE PATTERNS
HORIZON DEFINITION

ISSUES:
- LONG ENOUGH TO:
  » SMOOTH OUT RESPONSE
  » MAINTAIN LONG-TERM GOALS
  » MAKE ECONOMIC CHOICE
- SHORT ENOUGH TO:
  » NOT Undo OPTIMALITY IN PRE-SCHEDULE
  » COMPARE MANY ALTERNATIVES
  » ALLOW RAPID RESPONSE

RESOLUTION - DAILY FOR SHORT-TERM

ISSUES:
- LONG ENOUGH TO:
  » SMOOTH OUT RESPONSE

ISSUES:
- SHORT ENOUGH TO:
  » ALLOW RAPID RESPONSE
  » COMPARE MANY ALTERNATIVES
  » NOT Undo OPTIMALITY IN PRE-SCHEDULE
SCENARIO DEFINITION

• ISSUES:
  – NEED TO CAPTURE POSSIBLE FUTURE OUTCOMES
  – MUST MODEL INFINITE NUMBERS OF POSSIBILITIES
  – DIFFICULTIES PROCESSING INTERRUPTIONS
  – LIMITED KNOWLEDGE BASES EXISTING
  – INFINITE NUMBERS OF POSSIBILITIES

• APPROACH
  – START WITH INITIAL KNOWLEDGE
  – USE ALL INFORMATION TO ACHIEVE BEST MATCH
  – MUST MODEL INFINITE NUMBERS OF POSSIBILITIES
SOLUTION APPROACH

FORMULATION: BASIC MODELS

• FORMULATION
  • ASSUME HORIZON AND SCENARIOS
  • PRIME WITH SOLUTION RESPONSES FROM KNOWLEDGE CACHE

• SCENARIOS: \( S \)

• PROBABILITIES: \( p(\omega) \)

• GROUPS: \( S_1, ..., S_t \) at \( t \)

• KNOWLEDGE CACHE

• PRIME WITH SOLUTION RESPONSES FROM ASSUME HORIZON AND SCENARIOS

COUPLED MULTISTAGE STOCHASTIC NONLINEAR PROGRAM

\[
\begin{align*}
\min_{S_1, ..., S_t} & \quad \sum_{k \in K} \sum_{i \in I} \sum_{t \in T} \sum_{\omega \in \Omega} p(\omega) \left( x(i,t,s_\omega) - x(i,t,\omega) \right) \in \Omega, \forall i, t, s_\omega \in S \in \Omega \\
\text{s.t.} & \quad \sum_{k \in K} x(i,t,s_\omega) \leq d(t), \forall t = 1, ..., T \\
& \quad x(i,t,s_\omega) \geq 0, \forall i, t, s_\omega \\
& \quad \text{Nonanticipativity constraint}
\end{align*}
\]

RELAX NONANTICIPATORY CONSTRAINTS AND PLACE IN CONSTRAINT WITH MULTIPLIER UPDATED ON EACH ITERATION.

LAGRANGIAN APPROACH: THIS SAYS DECISION CANNOT DEPEND ON FUTURE.
STANDARD APPROACHES

• PARTITIONING
• BASIS FACTORIZATION
• INTERIOR POINT FACTORIZATION
• BASIS FACTORIZATION PARTITIONING
• MONTE CARLO APPROACHES
• DECOMPOSITION

- REGULARIZED (RUSZCZYNSKI)
- DANTZIG-WOLFE (PRIMAL VERSION)
- BENDER’S, L-SHAPED (VAN SLYKE - WETS0
• Production Uncertainty in Long and Short Terms

• Capacity Example - Long Term

• Use of Option for Risk

• Cyclic Optimality

• Short Term Scheduling - Short Term

• Approximation

• Decomposition

• Computation

• Match-Up

Summary