Systemic Risk Components as Deposit Insurance Premia

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PRELIMINARY

Abstract

In light of current events, there have been several proposals to establish a theory of financial system risk management analogous to portfolio risk management. One important aspect of portfolio risk management is risk attribution, the process of decomposing a portfolio risk measure into components that are attributed to individual assets or activities. This paper considers the total premium required to insure all deposits in the banking system as the systemic risk measure. The component of this risk measure attributable to a bank could serve as the bank's deposit insurance premium. The richer structure of a banking system, compared to a portfolio, makes the theory of systemic risk components more complicated than the theory of portfolio risk components. This paper proposes a scheme for systemic risk attribution that could be used in setting deposit insurance premia.

1 Introduction

This paper studies systemic risk in losses to a deposit insurer, such as the Federal Deposit Insurance Corporation (FDIC) in the United States, and the implications for deposit insurance premia. *Systemic risk* involves risk that arises because of the structure of the banking system and connections between banks. Systemic risk is not the same thing as *systematic risk*, which is risk explained by factors that influence the economy as a whole. Systematic risk in deposit insurance losses has to do with the impact on the loss distribution of dependence among banks' assets (Kuritzkes et al., 2005; Lehar, 2005). Systemic risk includes systematic

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risk and also *contagion*, the transmission of losses from one bank to another. Contagion can take several forms. One is contagion through asset markets: when a distressed bank is compelled to sell assets into an illiquid market, the resulting drop in asset prices can cause losses to other banks holding similar assets (Brunnermeier & Pedersen, 2009; Cifuentes et al., 2005; Diamond & Rajan, 2005; Krishnamurthy, 2009; Shin, 2008). In this paper, we instead focus on counterparty contagion, that is, contagion via direct bilateral links among banks: loss transmission occurs when a bank is unable to make promised payments on securities that it issued and are held by other banks. This enables us to make use of the Eisenberg & Noe (2001) model of contagion via direct bilateral links.

Why include systemic risk when modeling deposit insurance? One advantage is improved estimation of the loss distribution. Information is available or potentially available to regulators about the network of obligations among banks that is created by issuing and trading equity, debt, and derivative securities. This information is vital to proper assessment of deposit insurance losses in extreme scenarios. Studying the correlations among banks' asset helps most to assess the correlation among losses in ordinary times and at banks without significant exposure to derivative securities. It does not seem well-suited to the task of assessing deposit insurance losses in extreme scenarios in which contagious defaults take place and banks suffer losses that seem almost impossibly large, relative to asset volatilities measured in ordinary times. This can happen in part because of derivative securities whose values are nonlinear in underlying security prices. (As an analogy, it is hard to manage the risk of a portfolio containing derivative securities without actually looking at the structural relationship between the derivative securities and underlying securities. For example, estimating the correlation between the market values of a stock and a put option on that stock when it is out of the money is not adequate for assessing the tail of the loss distribution of a portfolio that is long the stock and short the put option.) This is one reason that it is worth including systemic risk when modeling deposit insurance.

This paper analyzes the significance of systemic risk for deposit insurance losses and investigates new schemes for setting deposit insurance premia, using a basic model of systemic risk. We consider a simplified model of deposit insurance which does not incorporate realistic features regarding when and how risk is measured. Thus, the results may not lead immediately to a practical, politically implementable new scheme for setting deposit insurance premia. The broad goal is to yield some insight into how systemic risk affects the deposit insurer and encourage discussion of the incentive effects that deposit insurance premium schemes have on banks' risk-sharing behavior.

The point is to do for systemic risk what fair-market methods of setting deposit insurance premia have done for systematic risk (Duffie et al., 2003; Pennachi, 2006). *Actuarially fair* methods set a bank's deposit insurance premium equal to the expected loss in insuring that bank's depositors. *Fair-market* methods do the same thing, but they compute the expectation under a different probability measure. Instead of a probability measure that describes the real world, they use a *pricing measure*. When the expectation is taken under the pricing measure, an asset's expected discounted future value equals the asset's price. This reflects systematic risk and investors' preferences. Because bank failure tends to occur in bad economic scenarios, in which there is high marginal utility of wealth, fair-market deposit insurance premia tend to be higher than actuarially fair premia. The premium for a bank with high exposure to systematic risk is higher than the premium for a bank with low exposure to systematic risk. Pennachi (2006) finds that in the absence of fair market deposit insurance premia, banks have an incentive to amass a systematically risky asset portfolio. This is bad for the deposit insurer because it produces worse losses in the worst scenarios, increasing the variability of the insurer's loss distribution. It also concentrates these losses in bad economic scenarios, when it would be especially painful for the government to pay out large sums on deposit insurance.

Likewise, it is bad for the deposit insurer when banks engage in systemically risky behavior. Consider banks' decisions about their financial leverage and the proportion of their assets held in cash. Having more equity or more cash makes a bank less individually risky to the deposit insurer, by reducing insured losses. It also makes the bank less systemically risky: the safer the bank, the less likely that it will default on its obligations to other banks and thus cause insured losses there. Depending on one's viewpoint, contagious default is a negative externality, or risk-sharing among banks that prevents deposit insurer losses is a positive externality. Setting a bank's deposit insurance premium equal to the expected insured loss at that bank makes this effect an externality: the bank has no incentive to take into account the consequences of its decisions elsewhere in the system.[1] This is so whether the expectation is taken in the actuarially fair or fair-market sense. This paper proposes a deposit insurance premium scheme that takes into account the effect that a bank has on expected insured losses at all banks.

A simple model of the banking system that incorporates systemic risk appears in Section 2. The systemic risk measure is the *total deposit insurance premium*, which is the expectation of total insured losses throughout the system. (The results apply whether the expectation is taken in the actuarially fair or fair-market sense.) The analogy between risk management of the banking system and portfolio risk management suggests that a systemic risk measure can be decomposed into a sum of risk components, each attributable to one of the system's components. A portfolio's components are assets or activities such as trading strategies or business divisions of a firm. We take the components of the banking system to be banks, so as to decompose the deposit insurance premium for the system into components charged to each bank. A brief review of the theory of risk components occupies Section 3. New results on systemic risk components of the total deposit insurance premium appear in Section 4. Because of the links between banks in the banking system, a feature that a portfolio does not have, the theory of systemic risk components is more complicated than that of portfolio risk components. Thus, we arrive at risk components based on deposit insurance

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[1] One might wonder whether a sufficient incentive arises from a deposit insurance premium based on a expected insured losses at a bank because it could affect prices that banks negotiate for trades. Would it not be possible that, when a trade would reduce the insured losses at one bank, both parties would have an incentive to make the trade because the bank whose premium would drop could offer a favorable price to its counterparty to get the deal done? The model studied here indicates that this is not sufficient, because there can be trades between two banks that reduce the insured losses at a third bank.

losses that are markedly different from those of Lehar (2005, § 5.3). Because he was studying only systematic risk and not systemic risk, he was able to regard deposit insurance losses as losses on a portfolio of put options and apply the theory of portfolio risk components. An implication of Section 4 is that there is no simple, unique decomposition of systemic risk involving contagion, as there is for portfolio risk. Section 5 elaborates one systemic risk decomposition, and Section 6 shows why expected insured losses at each bank do not seem to correspond to risk components of expected total insured losses. Section 7 discusses future research directions.

2 A Model of the Banking System

Consider a one-period model of the banking system. At the beginning of the period, each bank invests in some assets, such as cash, riskless securities, and risky loans to businesses. The assets are funded by equity and insured deposits. (It would be straightforward to include uninsured bank debt along with insured deposits among the liabilities.) Each bank also trades contingent claims with other banks at the beginning of the period. That is, it issues securities such as stocks, bonds, and derivative securities, and sells them to other banks. To simplify the exposition, assume that the contingent claims have zero value and zero impact on the balance sheet at the beginning of the period. The sole purpose of this discussion of the beginning of the period is to motivate the following description of what happens at the end of the period, which is the object of study.

At the end of the period, all the assets mature, and bank i is due to receive p_i in interest and principal. However, some of its borrowers may default, so bank i only receives $\tilde{P}_i \leq p_i$, including recovery on defaulted loans. (Capital letters represent random variables and tildes indicate random payments that may be less than promised payments.) Bank i owes its depositors d_i. The random matrix X specifies the net amounts that banks owe each other to settle contingent claims: X_{ij} is the amount that bank i owes bank j, after netting.[2] (Any diagonal element $X_{ii} = 0$.) In total, bank i is obligated to pay $O_i = d_i + X_{i\cdot}1$.

Here 1 represents a column vector of the appropriate length whose elements are all one, so that $X_{i\cdot}1 = \sum_j X_{ij}$. Matrix products have the following interpretation in this model. Let y be a row vector and z be a column vector whose elements are all zero or one. They represent sets of banks. The vector y is associated with the set $\{i : y_i = 1\}$, e.g. if $y = [0\ 1\ 1]$ then products involving y include payments from banks 2 and 3. The product $yXz = \sum_i \sum_j y_i X_{ij} z_j$ is the sum of all promised payments from banks in the set $\{i : y_i = 1\}$ to banks in the set $\{j : z_j = 1\}$. Define e_i to be the column vector whose elements are all zero except that the ith element is one. Then $X_{i\cdot}1 = e_i^\top X1 = \sum_j X_{ij}$ is the sum of all promised payments from bank i to any bank.

Banks may not be able to meet their obligations, in which case they default. We assume

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[2] Netting means that X is nonnegative and at most one of any pair X_{ij} and X_{ji} can be nonzero. It is also possible not to use netting as well as to have different levels of seniority among the securities a bank issues (Eisenberg & Noe, 2001).

that deposits have absolute priority over contingent claims, and all banks have equal priority with each other in settlement of contingent claims after netting. The matrix $\tilde{X}$ represents the payments that banks actually make to each other. Eisenberg & Noe (2001) provide a method for computing these payments. The computation is not extremely simple because the payments made into bank i are $\tilde{I}_i = \tilde{P}_i + 1^\top \tilde{X}_{\cdot i}$, and then bank i is then able to pay no more than $\tilde{I}_i$, which depends on what other banks were able to pay. The situation is complicated because cycles may exist, e.g. bank 1 must pay bank 2, which must pay bank 3, which must pay bank 1; it is somewhat involved but not difficult to determine how a default by one of the banks in the cycle affects defaults by the others. Thus we can compute $\tilde{X}$, $\tilde{I}$, and the vector $\tilde{O}$ of total payments that each bank actual makes to depositors plus other banks. Bank i actually pays $\tilde{O}_i = \min\{O_i, \tilde{I}_i\}$; to its depositors, bank i actually pays

$$\tilde{D}_i = \min\{d_i, \tilde{I}_i\}. \tag{1}$$

Given the scenario, there are three sets of banks:

1. The set $\mathcal{H}$ of *surviving banks*, which do not default: $\tilde{O}_i = O_i$ if $i \in \mathcal{H}$.
2. The set $\mathcal{G}$ of *defaulting banks*, which default on their obligations to other banks, but are able to satisfy their depositors: for $i \in \mathcal{G}$, $\tilde{D}_i = d_i$ but $\tilde{X}_{i\cdot} < X_{i\cdot}$.
3. The set $\mathcal{F}$ of *failed banks*, at which there are insured losses: for $i \in \mathcal{F}$, $\tilde{D}_i < d_i$.

We also define indicator functions: $F_i = 1_{\{i \in \mathcal{F}\}}$, i.e. $F = 1$ if $i \in \mathcal{F}$ and $F = 0$ otherwise, $G_i = 1_{\{i \in \mathcal{G}\}}$, and $H_i = 1_{\{i \in \mathcal{H}\}}$. The insured loss at bank i is

$$L_i = d_i - \tilde{D}_i = (d_i - \tilde{I}_i)F_i = (d_i - \tilde{P}_i - 1^\top \tilde{X}_{\cdot i})F_i.$$

Because of equal priority among banks with claims on a defaulting bank $i \in \mathcal{G}$, the fraction $\tilde{X}_{ij}/\tilde{X}_{i\cdot}1$ of bank i's total payments to other banks that goes to bank j is $\Pi_{ij} = X_{ij}/X_{i\cdot}1$. Then

$$\tilde{X}_{i\cdot} = \begin{cases} 0 & \text{if } i \in \mathcal{F} \\ (\tilde{I}_i - d_i)\Pi_{i\cdot} & \text{if } i \in \mathcal{G} \\ X_{i\cdot} & \text{if } i \in \mathcal{H} \end{cases}. \tag{2}$$

A specification of the banking system is $(d, \tilde{P}, X)$: in this model, all other relevant quantities can be computed from these. The total insured losses across the whole banking system are $L^\top 1 = L^\top F$. Our systemic risk measure is the total deposit insurance premium $\mathrm{E}\left[L^\top F\right]$, the expected total losses, either under the real-world probability measure so that the aggregate premium is actuarially fair or under a pricing measure such that it is a fair market premium.

3 Background on Risk Components

Let ρ be a portfolio risk measure: where Y is the random future value of a portfolio, $\rho(Y)$ is its risk. (Examples of ρ include expected loss, standard deviation, value at risk, and expected shortfall.) The portfolio value can be written as $Y = \sum_i \theta_i A_i$ where θ_i is the number of shares of asset i and A_i is the random future value of a share of asset i. It is helpful to construct a risk function r defined by

$$r(\theta) = r\left(\sum_i \theta_i A_i\right). \tag{3}$$

The theory of risk components asks how the portfolio risk measure can be decomposed as $\rho(Y) = \sum_i \rho_i(Y)$, where $\rho_i(Y)$ is the risk attributed to the investment in asset i, in a way that satisfies certain axioms. Different authors adopt different axioms, yet the consensus of the literature (Denault, 2001; Kalkbrener, 2005; Tasche, 2004) is that the only desirable risk decomposition is the *Euler allocation* or *gradient allocation*

$$\rho_i(Y) = \theta_i \frac{\partial r}{\partial \theta_i}(\theta).$$

The meaning of the Euler allocation is seen from the interpretation of the partial derivative: the risk component allocated to the position in asset i is proportional to the rate at which the risk measure increases as the position in asset i increases.

A prime matter of concern in the theory of risk components is to establish that the risk components satisfy $\rho(Y) = \sum_i \rho_i(Y)$, known as *full allocation*. In the present context, this means that the deposit insurance premia set as risk components sum up to the total expected losses, which are an appropriate total deposit insurance premium. For a discussion of when the Euler allocation is a full allocation, see Buch & Dorfleitner (2008) and Fischer (2003). Roughly speaking, the Euler allocation is a full allocation at θ if r is suitably differentiable at θ and positively homogeneous, i.e. for a scalar λ and any θ', $r(\lambda\theta') = \lambda r(\theta')$. A bank's insured loss $L_i = (d_i - \tilde{I}_i)^+$ involves a non-differentiable function: there are scenarios in which a bank is on the cusp of defaulting on its obligation to its depositors, so that perturbing the system in some directions increases the insured losses, while perturbing it in other directions leaves them at zero. However, the systemic risk measure is an expectation of insured losses across scenarios, which has a smoothing effect. It seems that in typical applications, r would be differentiable at most points, to speak loosely.

4 Components of Aggregate Deposit Insurance Premia

We now apply the Euler allocation principle to the total deposit insurance premium: $\rho = \mathrm{E}\left[L^\top F\right]$. The result is a decomposition of the total deposit insurance premium into premia for each bank, reflecting the way in which a marginal increase in the size of bank i contributes to the total deposit insurance premium. But what does it mean to consider "a marginal increase in the size of bank i," especially when there are connections among banks? How can the aggregate deposit insurance premium be written as a function of the banks' sizes in the manner of Equation (3)?

There were obvious answers to such questions for portfolios, but more than one answer is possible for banking systems. Everything under study in the banking system, including the total deposit insurance premium ρ, can be computed from $(d, \tilde{P}, X)$. This systemic risk measure ρ is positively homogeneous in $(d, \tilde{P}, X)$. If we multiply $(d, \tilde{P}, X)$ by λ, we get the new system $(\lambda d, \lambda\tilde{P}, \lambda X)$. Given the same scenario, the same banks fail in the old and new system, but the losses in the new system are λ times as large. If $(d, \tilde{P}, X)$ can be viewed as functions of the banks' sizes as measured by their initial assets $\bar{a}$, then ρ can be viewed as a function $r(\bar{a})$, and the Euler allocation used to compute individual banks' risk components, for use as deposit insurance premia. It is plausible to regard the amount d_i that bank i owes to depositors and its income $\tilde{P}_i$ as fixed multiples of its initial assets $\bar{a}_i$. That is, as bank i grows, it keeps the same proportions among its liabilities and among its assets. The difficulty is in modeling the effect of changing bank i's size on the contingent claims traded among banks, which determine X.

Because of our interest in positive homogeneity as a condition for the Euler allocation to satisfy full allocation, we consider what happens when we multiply bank i by λ, that is, when we shift attention to the system that arises when the vector of initial assets is $[\bar{a}_1, \ldots, \bar{a}_{i-1}, \lambda\bar{a}_i, \bar{a}_{i+1}, \ldots, \bar{a}_n]^\top$. As discussed, this means multiplying bank i's deposits d_i and income from loans $\tilde{P}_i$ by λ, and we now consider what effect we should imagine it having on X. We consider several possible schemes. Recall that $X_{i\cdot}$ is the vector of payments that bank i makes to other banks, while $X_{\cdot i}$ is the vector of payments that bank i receives from other banks. Possible schemes include:

1. When bank i is multiplied by λ, X stays the same.
2. When bank i is multiplied by λ, $X_{i\cdot}$ and $X_{\cdot i}$ are both multiplied by λ.
3. When bank i is multiplied by λ, $X_{i\cdot}$ is multiplied by λ while $X_{\cdot i}$ stays the same.
4. When bank i is multiplied by λ, $X_{i\cdot}$ stays the same while $X_{\cdot i}$ is multiplied by λ.
5. When bank i is multiplied by λ, $X_{i\cdot}$ and $X_{\cdot i}$ are both multiplied by $\sqrt{\lambda}$.

Their interpretations are not obvious: why should increasing the assets of bank i be coupled with an increase in the net payments it makes to other banks, but not on the net payments it receives (Scheme 3)? Or vice versa (Scheme 4)? These schemes seem to be at odds with the way contingent claims with zero initial value are actually traded: the possibility of making greater payments should be counterbalanced by the possibility of receiving greater payments. Scheme 5 involves growth in both payments made and received by bank i, but the square root seems rather artificial. There is a fundamental difficulty here in that any change to X as a result in the change of the size of bank i involves a change that affects another bank j whose size did not change. The merits and interpretations of these schemes will be seen after investigating the resulting risk components.

We have already supposed that multiplying the system by λ, that is, taking $\lambda \bar{a}$ to be the vector of initial assets, involves multiplying d and $\tilde{P}$ by λ. Furthermore, ρ is positively homogeneous in $(d, \tilde{P}, X)$. Therefore, what is required for r to be positively homogeneous is that multiplying the system by λ should multiply X by λ. Thus, Schemes 1 and 2 fail to yield positive homogeneity, so we abandon them. Under Scheme 1, multiplying the system by λ does not change X. Under Scheme 2, multiplying the system by λ multiplies X by λ^2. The other schemes all yield positive homogeneity of r.

Given any such scheme, the Euler allocation says that the risk component, or deposit insurance premium levied on bank k, is

$$\bar{a}_k \frac{\partial}{\partial \bar{a}_k} \mathrm{E}\left[L^\top F\right] = \mathrm{E}\left[\bar{a}_k \frac{\partial}{\partial \bar{a}_k} L^\top F\right] \tag{4}$$

under suitable regularity conditions. The sensitivity of the total insured losses $L^\top F$ to the size of bank k is

$$\frac{\partial}{\partial \bar{a}_k} L^\top F = \frac{\partial}{\partial \bar{a}_k} (d - \tilde{D})^\top F = \sum_j \left(\frac{\partial d_j}{\partial \bar{a}_k} - \frac{\partial \tilde{D}_j}{\partial \bar{a}_k} \right) F_j,$$

where the derivative exists. Throughout the paper, we ignore those points where a marginal change to the size of bank k alters the sets $\mathcal{F}$ of failed banks, $\mathcal{G}$ of defaulting banks, and $\mathcal{H}$ of surviving banks. Given our assumption that a bank's deposits and income from loans scale with its initial assets,

$$\frac{\partial d_j}{\partial \bar{a}_k} = 1_{\{j=k\}} \frac{d_k}{\bar{a}_k} \quad \text{and} \quad \frac{\partial \tilde{D}_j}{\partial \bar{a}_k} = F_j \frac{\partial}{\partial \bar{a}_k} (\tilde{P}_j + 1^\top \tilde{X}_{\cdot j}) = \left(1^\top \partial_k \tilde{X}_{\cdot j} + 1_{\{j=k\}} \frac{\tilde{P}_k}{\bar{a}_k} \right) F_j$$

where the notation $\partial_k \tilde{X}$ represents a matrix whose (i,j)th element is $\partial \tilde{X}_{ij} / \partial \bar{a}_k$. Thus the deposit insurance premium levied on bank k in Equation (4) is

$$\mathrm{E}\left[(d_k - \tilde{P}_k) F_k - \bar{a}_k 1^\top (\partial_k \tilde{X}) F \right]. \tag{5}$$

The first term, $(d_k - \tilde{P}_k) F_k$, is the insured loss at bank k in the absence of risk transfer among banks, except that F_k indicates whether bank k fails in the presence of risk transfer. That is, this first term looks at the insured loss that would have occurred, without risk transfer, at bank k in scenarios where it actually fails even though there is risk transfer. The second term, $\bar{a}_k 1^\top (\partial_k \tilde{X}) F = \bar{a}_k \partial_k \left(1^\top \tilde{X} F \right)$, leads to a reduction in bank k's deposit insurance premium that arises because risk transfer among banks mitigates insured losses because banks that have not failed make payments to failed banks.[3] The expression $1^\top \tilde{X} F$ is the total amount actually paid by any bank to failed banks.

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[3] In the one-period model studied here, a system without risk transfer among banks always entails greater insured losses than a system without risk transfer, because depositors have priority over contingent claims. This is a limitation of the model studied here, but the model could be enriched.

5 Scheme 3: Increasing Obligations

According to Scheme 3, $\partial_k X_{ij} = 1_{\{i=k\}} X_{kj}/\bar{a}_k$, that is, changing the size of bank k scales all the payments that it has promised to make. This can affect the payments that bank k actually makes and the payments that other banks make. We need to compute $1^\top(\partial_k \tilde{X})F$, which appears in Equation (5). The effect of changing the size of bank k depends on which banks have failed or defaulted. We continue to ignore points of non-differentiability.

From Equation (2) we see that changing the size of bank k only affects its own payments and those made by defaulted (but not failed) banks in $\mathcal{G}$. If $i \in \mathcal{F}$, then $\partial_k \tilde{X}_{i\cdot} = 0$, because a failed bank i makes no payments to other banks. If $i \in \mathcal{H}$ and $i \neq k$, then $\partial_k \tilde{X}_{i\cdot} = 0$, because a surviving bank makes its promised payments: $\tilde{X}_{i\cdot} = X_{i\cdot}$, which does not depend on the size of bank k. Therefore $1^\top(\partial_k \tilde{X})F = \partial_k \left(1^\top \tilde{X} F\right)$ consists of two effects: an effect on payments made directly by bank k to failed banks in $\mathcal{F}$, and an effect on payments that bank k makes to defaulted banks in $\mathcal{G}$ and indirectly reach failed banks.

We can envision the banking system as a network in which money flows according to $\tilde{X}$. Increasing the size of bank k increases flows on some of the links in the network. Failed and surviving banks, in $\mathcal{F}$ and $\mathcal{H}$, act as sinks, while extra money entering a defaulted bank, in $\mathcal{G}$, flows on to other banks. A failed bank in $\mathcal{F}$ absorbs extra money by giving it to its depositors, a surviving bank in $\mathcal{H}$ absorbs extra money into its equity, and a defaulted bank in $\mathcal{G}$ sends the extra money to other banks, to whom it has not fulfilled its obligations. It may be complicated to analyze $\partial_k \tilde{X}$ because of cycles in the network of payments: for example, if $k \in \mathcal{G}$, increasing the size of bank k can cause extra money to flow through other banks in $\mathcal{G}$ and back to bank k, whence it flows out to other banks again. It is not obvious how to determine the ultimate destination of extra money arriving at a bank merely by inspecting $\tilde{X}$ and $\mathcal{G}$. Staum (2009) develops a version of the Eisenberg & Noe (2001) algorithm which computes $\tilde{X}$ and also a matrix Π^*, where Π^*_{ij} is the amount of extra money ultimately absorbed by bank j when an extra dollar is injected at bank i. (It has the property that $\Pi^*_{ij} = 0$ if $i \in \mathcal{F}$ or $\mathcal{H}$, or if $j \in \mathcal{G}$.) Then we can write a formula for total insured losses that has no explicit dependence on $\tilde{X}$:

$$1^\top \tilde{X} F = H^\top X F + H^\top X \Pi^* F + (\tilde{P} - d)^\top \Pi^* F. \tag{6}$$

The three terms all represent amounts of money arriving at failed banks, respectively: directly from surviving banks, indirectly from surviving banks via defaulting banks, and from the funds available at defaulting banks for transfer to other banks.

A key to working out the solution in Scheme 3 is that changing the size of bank k has no impact on the fraction Π of payments that any bank sends to each other bank, or the ultimate destination Π^* of money injected at any bank. In this scheme, the only variables on the right side of Equation (6) that change with a marginal change in the size of bank k are X and $\tilde{P} - d$. Specifically,

$$\partial_k X_{ij} = 1_{\{i=k\}} X_{kj}/\bar{a}_k \quad \text{and} \quad \partial_k(\tilde{P}_i - d_i) = 1_{\{i=k\}}(\tilde{P}_k - d_k)/\bar{a}_k.$$

Therefore

$$\bar{a}_k \partial_k \left(1^\top \tilde{X} F \right) = H_k X_{k\cdot} F + H_k X_{k\cdot} \Pi^* F + G_k(\tilde{P}_k - d_k) \Pi^*_{k\cdot} F.$$

Under Scheme 3, the risk component in Equation (5) is

$$\mathrm{E}\left[F_k(\tilde{P}_k - d_k) - G_k(\tilde{P}_k - d_k)\Pi^*_{k\cdot} F - H_k X_{k\cdot}(I + \Pi^*)F \right]. \tag{7}$$

After Equation (5), we saw that everything except the first term, $F_k(\tilde{P}_k - d_k)$, leads to a reduction in bank k's deposit insurance premium due to mitigation of total insured losses by risk transfer among banks. We see that Equation (7) gives bank k credit for the help it provides to other banks, mitigating losses in insuring their depositors. This provides an incentive for the bank to have higher proportions of equity and cash, increasing the probability that it is able to live up to its obligations to other banks and thus prevent insured losses there.

6 The Usual Premium Scheme

It is usual to think of a bank's deposit insurance premium as a premium required to insure its own depositors. This might be the expected insured losses at bank k,

$$\mathrm{E}\left[L_k\right] = \mathrm{E}\left[d_k - \tilde{D}_k \right] = \mathrm{E}\left[(d_k - \tilde{P}_k - 1^\top \tilde{X}_{\cdot k}) F_k \right]. \tag{8}$$

Comparing this to Equation (5), we see that using expected insured losses at bank k as its deposit insurance premium gives bank k credit for all the help it receives in those scenarios when it fails, causing insured losses. Consequently, due to the full allocation property, banks can get no credit for the help they provide in mitigating the losses that arise from insuring depositors at other banks: the credit for loss mitigation is entirely given to the banks that receive it. As discussed in the introduction, this may not be desirable.

We next consider how using expected insured losses at a bank as its deposit insurance premium might fit into the new framework under study. To fit it into this framework requires finding a scheme for the dependence of X on $\bar{a}$ that makes Equations (5) and (8) equal. They are equal if $\partial_k \tilde{X}_{ij} = 1_{\{j=k\}} \tilde{X}_{ij} / \bar{a}_k$. This seems similar to Scheme 4, but it is not the same: Scheme 4 says $\partial_k X_{ij} = 1_{\{j=k\}} X_{ij} / \bar{a}_k$ if $k = j$. I conjecture that there is no way for $\partial_k \tilde{X}_{ij} = 1_{\{j=k\}} \tilde{X}_{ij} / \bar{a}_k$ to hold generally, within the model under study, using the approach of Eisenberg & Noe (2001) for dealing with defaults on obligations among banks. The basic reason relates to the payment $\tilde{X}_{ik}$ from a defaulting bank $i \in \mathcal{G}$ to bank k. If $X_{\cdot k}$ is multiplied by λ, this changes $\tilde{X}_{ik}/\tilde{X}_{i\cdot}1$, the share of bank i's payments going to k, from $\Pi_{ik} = X_{ik}/X_{i\cdot}1$ to $\lambda X_{ik}/(\lambda X_{ik} + \sum_{j \neq k} X_{ij})$, but that is not, in general, the same as multiplying by λ the amount $\tilde{X}_{ik}$ actually paid. Bank i's total payment $\tilde{X}_{i\cdot}1$ to other banks does not increase unless bank i gets more funds, which does not happen as a result of increasing the promised payments $X_{\cdot k}$ to bank k. It seems that the only way to ensure, in general, that $\tilde{X}_{ik}$ is multiplied by λ is to multiply everything in the system by λ. This would prevent r from being positively homogeneous, and it does not make Equations (5) and (8) equal.

7 Research Directions

This preliminary version of the paper gives an analysis of deposit insurance premia based on one sensitivity analysis scheme for generating systemic risk components, and shows that it corresponds to giving credit to a bank for the beneficial effects of its trades with other banks in mitigating losses in insuring their depositors. It remains to investigate different schemes (such as Schemes 4 and 5 in Section 4) for decomposing systemic risk and thus setting deposit insurance premia. To compare and evaluate these schemes, it would be desirable to investigate explicitly the resulting incentive effects, in the manner that Pennachi (2006) treated fair-market vs. actuarially fair deposit insurance: how might banks' behavior change if their deposit insurance premia were set according to these schemes? The sensitivity analysis done here could also be carried out for richer models of contagion, such as a multi-period model of counterparty contagion (see footnote 3), or a model that includes contagion through asset prices (see Section 1).

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