Diversification: How Much is Enough?

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The rule of thumb that nearly all of the diversifiable risk is eliminated in a portfolio of 10 securities dates back to the classic paper of Evans and Archer [2]. The authors discuss the mathematical relationship between portfolio size and risk, and rightly conclude that there is a phenomenon of diminishing returns: each additional security added to the portfolio produces an ever smaller decrease in risk. Both Evans and Archer and their later critics, such as Statman [5], conclude that investors should perform a marginal cost/benefit analysis in order to determine an appropriate number of securities to hold. In practice, there is no widely used method for computing this number, but Statman shows that many investors' policies appear to be suboptimal.

For instance, one may address the question of how many funds belong in a fund of funds. Has the fund of funds industry attained an acceptable level of diversification? Data from TASS Management, a London-based information services firm monitoring commodity trading advisors (CTAs) and hedge funds, provides an industry average. Funds of funds contain a mean of fewer than five funds, and a median of four. Of course, TASS's database is not a random sample of the universe of funds, and the low 24% response rate to this question raises some doubts as to the accuracy of this number. Nonetheless, in the absence of obvious biases leading to underresponse by funds with large numbers of submanagers, it seems safe to claim that the average is not much more than five funds.

How good is the amount of diversification provided by five funds? Evans and Archer answer such questions using a mathematical model of random stock-picking. This allows them to assume that each security has the same expected return and the same correlation with all other securities. From this assumption, statistical theory yields a simple relationship between portfolio size and variance:

$$\sigma_n^2 = \sigma_M^2 + (\sigma_1^2 - \sigma_M^2) / n$$

where σ_1^2 is the expected variance of a single randomly selected security, σ_M^2 is the variance of the market portfolio of securities, and σ_n^2 is the expected variance of a randomly selected portfolio of n securities. This relationship is illustrated graphically in the appended figure.

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The above formula shows how the expected variance of a portfolio of n randomly selected securities depends on only two empirically estimated quantities, σ_1 and σ_M. In all cases, a portfolio of size 5 eliminates 80% (1 – 1/5) of the diversifiable variance, while a portfolio of size 20 eliminates 95% (1 – 1/20) of the diversifiable variance. However, σ_1 and σ_M must be estimated for each class of securities. The greater the ratio of the diversifiable risk ($\sigma_1^2 - \sigma_M^2$) to nondiversifiable risk σ_M^2, the greater the benefits of diversification.

A comparison of several asset classes illustrates this principle. The data for CTAs and hedge funds comes from the TASS database, years 1989-1996, and incorporates a correction for survivorship bias discussed in Park [4]. The data for NYSE stocks comes from Elton and Gruber ([1], p. 61). The following table shows that increasing the size of the portfolio from 5 to 20 decreases the expected portfolio standard deviation by differing proportions for CTAs, hedge funds, and NYSE stocks. The greatest improvement is for stocks, and the least for CTAs. All show a substantial reduction in risk ($1 - \sigma_{20}/\sigma_5$).

Asset Class	Nondiversifiable Std. Deviation	Diversifiable Std. Deviation	σ_1	σ_5	σ_{20}	Reduction In Risk
CTAs	3.57	3.34	6.91	4.44	3.81	14.33%
Hedge Funds	2.05	2.42	4.47	2.71	2.24	17.59%
NYSE Stocks	2.66	4.17	6.83	3.87	3.01	22.31%

Statman suggests a clever method for assigning a value to this reduction in risk, which may be adapted to the present problem. Suppose that an average fund of funds manager has investments in five randomly selected hedge funds, and feels that he has achieved an acceptable level of risk at 2.71% monthly standard deviation. As an average fund of hedge funds manager, he has accumulated historical returns over 1989-1996 of 12.83%. How much better could he have done by investing in 20 randomly selected hedge funds while leveraging his investment up to 2.71% standard deviation? Increasing standard deviation from 2.24% to 2.71% allows the manager a leverage factor of 1.21 (2.71/2.24). This results in annual returns of 15.76%, or a gain of 2.93% per annum over his actual strategy. The following table summarizes these results for CTAs, hedge funds, and stocks:[3]

Asset Class	Annual Return	Leverage	Interest Rate	Leveraged Return	Gain In Return
CTAs	12.81	1.17	0%	15.09	2.28
Hedge Funds	12.83	1.21	0%	15.76	2.93
Stocks (Lend)	17.20	1.29	5%	20.70	3.50
Stocks (Borrow)	17.20	1.29	7%	20.12	2.93

In the case of stock investors who lend or borrow, Statman presents this method with the profits from leverage reduced by the risk-free or call-money rates, respectively.

[3]Volatility data comes from Elton and Gruber [1] for NYSE stocks as before, while returns data comes from Ibbotson [3] for common stocks 1989-1996.

This reduces the leveraged stock return from what would have been 22.61% without interest costs. His paper further makes an attempt to estimate the costs of diversification from the fees charged by S&P500 index funds. Both of these difficulties are greatly reduced for a fund of CTAs or hedge funds. CTAs commonly provide investors with leverage up to a certain level, or trade entirely "notional" amounts of money for their clients. Also, fees in these industries are typically percentages of money managed or profits generated. Splitting money among several funds incurs no greater charge than concentrating it in one fund. The only barrier to diversification is the ability to make the minimum investment in each fund.

On the other hand, some might argue that another barrier to diversification is the ability to pick a sufficiently large number of good funds. Of course, the same argument could be made about picking stocks. Nonetheless, no mutual fund manager would adopt a policy of buying only the five stocks he thought would do best. Despite the greater costs of diversification in the mutual fund industry, due to transaction costs and turnover among stocks, mutual funds have gone much further in pursuit of diversification. With hundreds of CTAs and hedge funds in existence, it is hard for a fund of funds manager to claim that he can only find five good ones. Since the penalty for inadequate diversification is in effect foregone profits, it seems highly desirable that funds of funds should learn to emulate the more mature mutual fund industry and embrace diversification more fully.

References

[1] Elton, E.J., and M.J. Gruber. *Modern Portfolio Theory and Investment Analysis*, 5th ed. New York: John Wiley & Sons (1995).

[2] Evans, J.L., and S.H. Archer. "Diversification and the Reduction of Dispersion: An Empirical Analysis." *Journal of Finance*, 23 (Dec. 1968), 761-767.

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[5] Statman, M. "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis*, 22 (Sep. 1987), 353-363.