Efficient and Robust Design for Transshipment Networks

Transshipment, the sharing of inventory among parties at the same echelon level, can be used to reduce costs in a supply chain. The effectiveness of transshipment is in part determined by the configuration of the transshipment network. We introduce chain configurations in transshipment settings and show their superiority over grouping configurations that are suggested in the literature. Further, we extend our research to general configurations and develop properties of low cost and robust transshipment networks. In addition, we provide managerial insights regarding preferred configurations based on problem parameters.

1. Introduction

According to the 15th Annual State of Logistics Report [22], logistic costs in the United States are rising, from $910 billion in 2002 to $936 billion in 2003. Inventory costs account for a third of this total. The report identifies "the ability to respond faster to changing customer needs" and "the flexibility to adjust manufacturing and delivery cycles" as keys to success in this competitive environment. Uncertainty in customer demand and operating costs can lead to major supply chain inefficiencies, causing lost revenue, poor customer service, high inventory levels and unrealized profits.

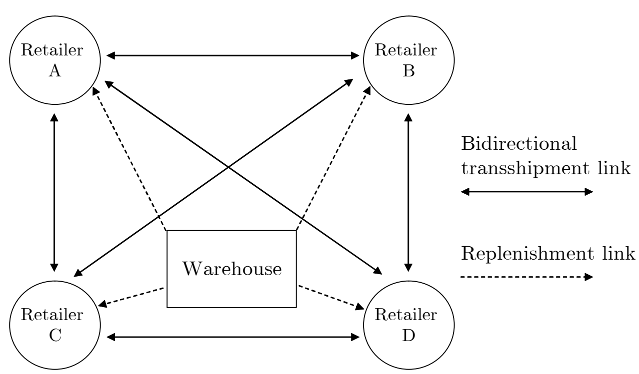


Figure 1: Supply Chain With Transshipments

Inventory transshipment is a promising strategy to provide operational flexibility to mitigate the effect of demand uncertainty. Transshipment is the sharing of inventory among locations at the same echelon level of a supply chain. In Figure 1, four retailers are supplied from one warehouse. Rather than relying solely on their own inventory or costly emergency replenishment from the warehouse, retailers can collaborate to address demand uncertainty.

Transshipment achieves the benefits of risk pooling to meet uncertain demand while maintaining low inventory levels at individual locations. The use of transshipment has been facilitated by advances in information technology and information sharing. Companies using transshipments include Footlocker, Macy's, and a group of chip manufacturers (NEC, Toshiba) sharing a common supplier, ASML.

Recent papers in transshipment models have considered restrictions on transshipment network connectivity since it is not always possible to transship directly among all locations; see [18] and [6]. Establishing a link between locations requires investments in bidirectional communication channels that enable information sharing, physical distribution systems and financial and administrative arrangements. Alternatively, transshipment networks without direct links between all locations tend to consolidate transshipment flows on a few routes, which can reduce the demand for communication channels and transportation (vehicles and drivers) and lower the overall complexity of a system. This is important in the case of outsourced transportation between locations that is negotiated in advance or in the case of multiple products which share common transshipment methods.

This paper considers the efficiency and robustness of a range of transshipment network configurations. *Efficient* networks achieve low expected inventory and transshipment costs. *Robust* networks maintain efficiency even with changes to cost or demand parameters. We present analytical results to show that the chain configuration is more efficient than the group configuration that appears in the literature. Using a numerical study, we analyze the characteristics of configurations under a range of cost and demand parameters, and provide insights on the parameters that affect the configuration choice. Section 2 reviews related literature. Section 3 describes operational and strategic transshipment problems. Section 4 presents comparisons of basic configurations. Section 5 discusses research extensions.

2. Literature review

The transshipment literature has focused on operational decisions for a fixed network design, such as the transshipment amount between locations and the order amount at each location. Most work considers two locations ([16], [18] and [19]), or locations that are identical in cost parameters ([13] and [18]). [14] and [7] consider locations that are non-identical in demand and cost parameters. [17] and [19] allow replenishment lead times larger than one; in the others, transshipment lead times are negligible and replenishment lead times are one period.

The literature on transshipment network design is more recent. In addition to the complete pooling network in which any location may transship to any other location, [18] and [6] consider group configurations, in which locations are divided into groups and transshipment is allowed only within groups. It is shown that while group configurations cannot achieve all of the savings of complete pooling, the savings are considerable, and the number of links is smaller. [7] compare five transshipment configurations, differing in the number and cost of links. The paper quantifies the value of transshipments, but does not analyze network configurations. [21] consider a transshipment network with one supplier and three retailers with six network design options which they refer to as operational flexibility levels. They find the retailers' optimal order quantities for any given flexibility level with the newsvendor network model of [20] and analyze the interaction between optimizing order quantities and increasing operational flexibility. In this paper we consider configurations with one supplier and N retailers, determining optimal order quantities with the method of [7].

While the transshipment literature considers only a limited number of network configurations, other network configurations have been considered in the manufacturing and service operations literature. The chain configuration has been shown to be an efficient structure in supply chains and labor cross-training. [12] and [3] study the chain structure in supply chains. They show that by assigning the capacity of factories to products according to a chain structure in which each factory only produces two types of products, as in Figure 2, one can achieve most of the potential benefit of complete pooling in which all factories are able to produce all products. [8], [15], [4], [9], [10], [11] and [2] highlight the properties of the chain structure in different production and repair/maintenance environments.

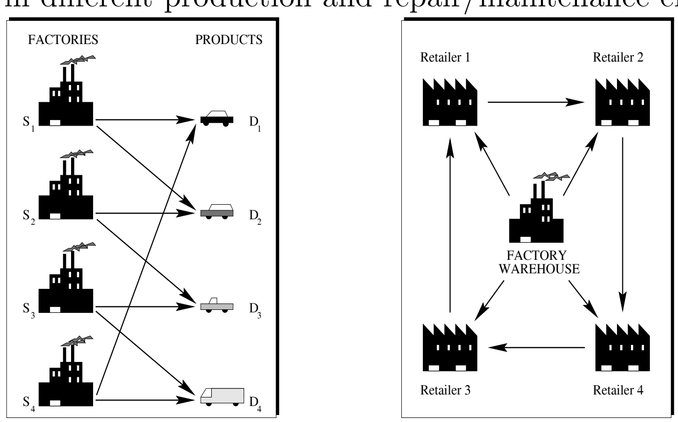


Figure 2: (left) Chaining in a manufacturing setting; (right) Chaining in a transshipment setting

Our research addresses the gap between the transshipment literature and network design by studying a wide range of network configurations, including the chain configuration. Like [1], we investigate the chain analytically.

3. Transshipment problems

Section 3.1 reviews the operational transshipment problem from the literature and Section 3.2 introduces the strategic transshipment network design problem studied in this paper.

3.1 Existing work on operational transshipment problems

The objective of the operational transshipment problem is to minimize inventory and transshipment costs per period for a given network design with known demand and cost parameters. Given N retailers, facing independent, stationary demand, events occur as follows:

1. Replenishment from the warehouse arrives from orders made in the previous period; backlogged demand is satisfied. The inventory level is equal to the base-stock level.
2. Demand is observed.
3. Transshipment decisions are made and occur immediately. Transshipment costs are incurred.
4. Demand is satisfied or backlogged. Holding and shortage costs are incurred.
5. Inventory level is updated.
6. Replenishment orders are made.

The replenishment lead time from the warehouse is one time period, and it is assumed that the warehouse has sufficient capacity to respond to all orders. Transshipments serve as a quicker source of supply if demand exceeds available inventory. Retailers follow a base-stock policy which [7] prove minimizes costs in a transshipment setting. At each location, base-stock level and periodic transshipment decisions are made to minimize the total long-run expected costs, which is the sum of inventory holding, shortage and transshipment costs of all locations. With a base-stock policy, the system regenerates itself every period; minimizing the expected cost for one period is equivalent to minimizing the long-run expected costs.

In the literature, locations are often assumed to be identical, as in this paper. Identical locations incur the same costs and observe demand from an identical distribution, as in cases of retailers serving near-homogenous populations over moderately sized geographic regions. Further, analysis of nonidentical locations is case-dependent and does not provide general insights, see Section 5 . Since variable replenishment costs are the same across locations and unmet demands are backlogged and eventually satisfied, these costs are ignored. Similar to [16], [18], [7] and other works, fixed costs of orders and transshipments are assumed to be incurred every period or negligible when compared with other costs.

The operational transshipment problem uses the following parameters:

$\mathcal{N}$	set of retail locations (also called "nodes"), $i \in \{1..N\}$
$\mathcal{K}$	set of directed transshipment links $(i,j) \in \mathcal{K}$, defined by configuration, $\mathcal{K} \subseteq (\mathcal{N} \times \mathcal{N})$
d_i	observed demand at location $i \in \mathcal{N}$ per period
c_t	cost of transshipping one unit along one link
c_s	cost of one unit of shortage for one period
c_h	cost of holding one unit in inventory for one period.

We define decision variables S_i and X_{ij}, and auxiliary variables I_i^+ and I_i^- as follows:

S_i	base-stock level at location $i \in \mathcal{N}$
X_{ij}	number of items transshipped on link $(i,j) \in \mathcal{K}$
I_i^+	net surplus at end of time period (after transshipment) at location $i \in \mathcal{N}$
I_i^-	net shortage at end of time period (after transshipment) at location $i \in \mathcal{N}$.

Given a base-stock level vector $\mathbf{S}$, the following linear program presented by [7] is solved to determine transshipment flows for each period of observed demand.

$$\min z(\mathcal{K}, \mathbf{S}) = c_t \sum_{(i,j)\in\mathcal{K}} X_{ij} + c_s \sum_{i\in\mathcal{N}} I_i^- + c_h \sum_{i\in\mathcal{N}} I_i^+ \tag{1a}$$

subject to

$$\sum_{j:(i,j)\in\mathcal{K}} X_{ij} - \sum_{j:(j,i)\in\mathcal{K}} X_{ji} + I_i^+ - I_i^- = S_i - d_i \qquad \forall i \in \mathcal{N} \tag{1b}$$

$$\sum_{j:(i,j)\in\mathcal{K}} X_{ij} \leq S_i \qquad \forall i \in \mathcal{N} \tag{1c}$$

$$X_{ij} \geq 0 \qquad \forall (i,j) \in \mathcal{K} \tag{1d}$$

$$I_i^+, I_i^- \geq 0 \qquad \forall i \in \mathcal{N} \tag{1e}$$

The objective function (1a) minimizes $z(\mathcal{K}, \mathbf{S})$, the sum of transshipment costs, shortage costs and holding costs, given $\mathcal{K}$, $\mathbf{S}$ and the demand realization. Constraints (1b) state that demand must be satisfied by inventory and/or transshipments, or backlogged. Constraints (1c) state that retailers cannot transship more than the base-stock level. Node $i \in \mathcal{N}$ may both receive units from location j_1 and transship units to location j_2 for some j_1, j_2 such that $(j_1, i) \in \mathcal{K}$ and $(i, j_2) \in \mathcal{K}$, up to its base-stock level; i.e., if j_1 has a surplus and j_2 has a shortage. Constraints (1d) and (1e) are non-negativity constraints. We use the infinitesimal perturbation analysis (IPA) procedure from [7] to find optimal base-stock levels and the optimal expected cost for a given configuration.

The objective (1a) is linear in holding, shortage and transshipment costs. A transshipment involving one or more links deducts one unit of surplus at a node to fulfill one unit of shortage at another. This action incurs transshipment costs to avoid one unit of holding cost and one unit of shortage cost. We define a *shift* to be a transshipment along a single link. The number of *profitable shifts* is the number of links along which units are transferred to meet shortage and still reduce costs. The number of profitable shifts, J, is the floor of the ratio between the sum of holding and shortage costs and transshipment cost; $J = \lfloor \frac{c_h + c_s}{c_t} \rfloor$. Our modeling allows for multiple shifts, yet unless the transshipment cost is low and demand uncertainty is high, results indicate that more than two shifts are rare. In such cases, configurations which depend on multiple shifts to balance shortages and surpluses are less efficient than those in which locations are directly linked to one node that acts as a warehouse.

Formulation (1) with $c_s = 1$ and $c_h = c_t = 0$ is similar to the manufacturing model in [12]. Retailers satisfy demand at other retailers by shifting capacity (i.e., inventory) along multiple links with no penalty. Transshipment and inventory costs in the transshipment setting limit this flexibility. Capacity in manufacturing models in [12] is independent of network design; in transshipment, retailers adjust base-stock levels to minimize system costs.

3.2 The strategic transshipment network design problem

The strategic transshipment network design problem determines the optimal network configuration (i.e., the link set $\mathcal{K}$) given a limit, P, on the number of transshipment links. To compare the efficiency of transshipment networks, we introduce $Z(\mathcal{K}, \mathbf{S})$ to denote the optimal *expected* cost (over demand realizations) of a network with transshipment link set $\mathcal{K}$ and base-stock level vector $\mathbf{S}$. The transshipment network design problem is formulated as:

$$\min Z(\mathcal{K}, \mathbf{S}) \tag{2a}$$

subject to

$$|\mathcal{K}| \leq P \tag{2b}$$

$$\mathcal{K} \subseteq (\mathcal{N} \times \mathcal{N}) \tag{2c}$$

The objective function (2a) minimizes the expected cost. Constraint (2b) limits the size of the link set, and constraint (2c) defines the possible choices of links between nodes. Efficient networks are those that have low values of $Z(\mathcal{K}, \mathbf{S})$ when compared to other networks. Robust networks are those that, after being selected over other networks under one set of cost and demand parameters, still perform better given deviations from the initial parameters.

4. Network configurations

Figure 3 shows the group configurations from the literature (3(a) and 3(b)) and chain configurations (3(c) and 3(d)). Let group(l) denote a group configuration of M groups of l nodes and $l(l-1)$ directed links each, for any positive integer M such that $lM = N$. Group(2) in Figure 3(a) and the unidirectional chain in Figure 3(c) have 6 links while group(3) in Figure 3(b) and the bidirectional chain in Figure 3(d) have 12 links or 6 bidirectional links.

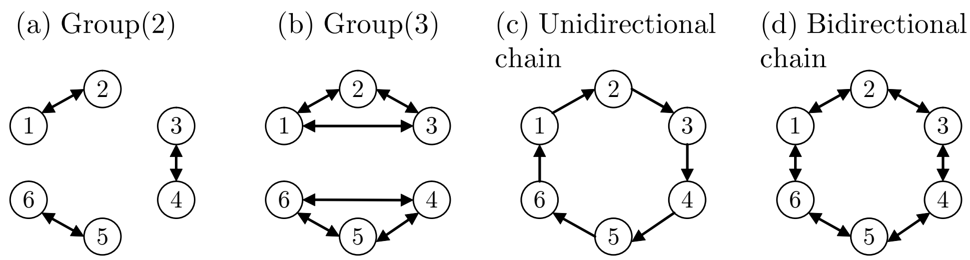


Figure 3: Basic network configurations with $N = 6$ locations

Section 4.1 presents analytical comparisons of chain and group configurations, and Section 4.2 presents numerical comparisons of the chain configuration with a range of configurations.

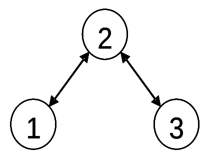
4.1 Analytical results for chain and group configurations

We show that the group configuration incurs higher expected costs than the chain configuration under the identical location assumption. The unidirectional chain outperforms group(2), both with $N = 2M$ directed links, and the bidirectional chain outperforms group(3), both with $N = 3M$ bidirectional links. Proofs for the lemmas (needed to prove the theorems) and Theorems 1 and 3 are provided in the online appendix.

Lemma 1 *Given a network with identical nodes, a vector of base-stock levels,* $\mathbf{S}$, *cost parameters that allow for only one profitable shift, and a fixed configuration, then: minimizing the expected number of units in inventory, minimizing the expected number of units in shortage or maximizing the number of units transshipped will yield the minimum total expected cost.*

Lemma 2 *Given an identical node network with nodes facing independent demand, linked for transshipments either as a chain or in a group(l) configuration, the optimal base-stock levels are identical across locations in a configuration.*

Lemma 3 *Consider three identical nodes linked in the following manner:*



where the nodes face independent demand, their base-stock levels are identical, and where cost parameters allow for only one profitable shift. The expected ending inventory/shortage after transshipments at nodes 1 and 3 are positively correlated.

Theorem 1 *Given an identical node network with $N = 2M$ locations ($M = 1, 2, \cdots$), the optimal expected cost in a unidirectional chain configuration is lower than or equal to the optimal expected cost in a group(2) configuration.*

Theorem 2 states that the bidirectional chain outperforms group(3) in expectation for networks with six nodes. Theorem 3 extends the result to networks with multiples of three nodes.

Theorem 2 *Given a network with six identical locations facing independent demand, the optimal expected cost in a bidirectional chain configuration is lower than or equal to the optimal expected cost in a group(3) configuration.*

Proof: From Lemma 2, the optimal base-stock levels at all locations within each configuration are identical. We prove that for the same base-stock level at each location, $\mathbf{S}$, the bidirectional chain outperforms group(3) for one profitable shift. With more than one profitable shift, the chain configuration can only improve while the group(3) configuration sees no benefit. Recall that $\mathcal{K}$ is the set of transshipment links in formulation (1). Let $Z(\mathcal{K}_c)$ and $Z(\mathcal{K}_g)$ denote the optimal expected inventory, shortage and transshipment costs for a network with chain links, $\mathcal{K}_c$, and group(3) links, $\mathcal{K}_g$, respectively. The argument $\mathbf{S}$ is omitted from $Z(\mathcal{K}, \mathbf{S})$ since the base-stock levels are the same. We prove that $Z(\mathcal{K}_c) \leq Z(\mathcal{K}_g)$, according to the outline in Figure 4.

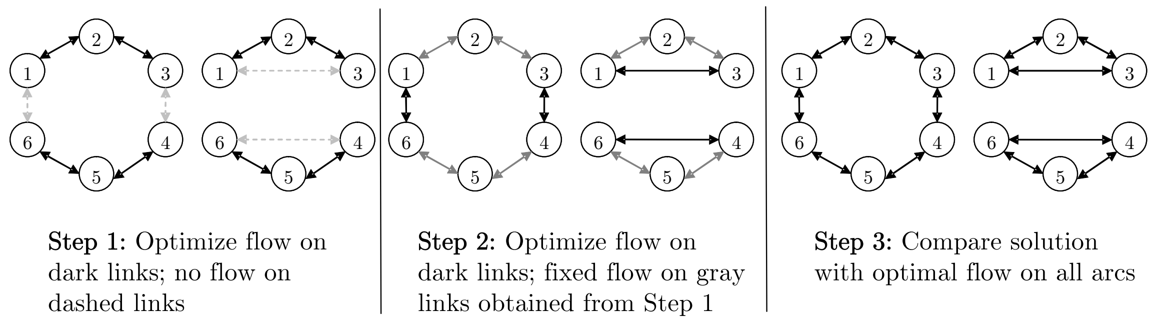


Figure 4: Outline of proof of Theorem 2

Step 1: Remove two links from each network, such that the same links remain in both networks. Let $\mathcal{K}' = \{(1,2),(2,3),(4,5),(5,6)\}$ be the set of remaining links. Given a demand instance, the transshipment quantities are found with formulation (1). The solutions for both networks are the same since the link set is $\mathcal{K}'$ for both. The optimal flows on these networks, assuming no flow on the removed links, is denoted $\mathbf{X}'$ with cost $Z(\mathcal{K}')$.

Step 2: Add back the removed links. The resulting link sets are $\mathcal{K}_c = \{(1,2),(2,3),(3,4),(4,5),(5,6),(6,1)\}$ and $\mathcal{K}_g = \{(1,2),(2,3),(3,1),(4,5),(5,6),(6,4)\}$, for chain and group(3), respectively. Fix the flows on the links in $\mathcal{K}'$ at the levels of $\mathbf{X}'$, allow flows on the reintroduced links to lower costs. Denote the costs after Step 2 as $Z(\mathcal{K}_c|\mathbf{X}')$ and $Z(\mathcal{K}_g|\mathbf{X}')$.

From Lemma 3, the expected net inventory at the corner nodes (i.e., nodes 1 and 3, and nodes 4 and 6) are positively correlated. The expected net inventory at the endpoints of connecting links in the chain network (i.e., nodes 3 and 4, and nodes 1 and 6) are independent. There will be less transshipment flow on the new links in the group(3) network than on the new links in the chain network. Since only one shift is allowed, maximizing transshipments will minimize cost, from Lemma 1. Hence, $Z(\mathcal{K}_c|\mathbf{X}') \leq Z(\mathcal{K}_g|\mathbf{X}')$.

Step 3: Attempt to improve the solution of each configuration by changing the link flows. We show that the solution of group(3) cannot be improved while the solution to the chain may be improved. Consider six exhaustive cases for nodes 1, 2, and 3 in group(3).

1. *All nodes are short.* No transshipments are possible.
2. *No nodes are short.* No transshipments are needed.
3. *Only one corner node (e.g. node 1) is short.* In Step 1, the middle node ships to node 1. If node 1 is still short, then node 3 ships in Step 2. Since holding and transshipment costs are identical at each node, redistributing flows cannot lower the total cost.

4. *Only the middle node is short.* Corner nodes do not ship to each other; the flows after Step 2 are optimal.
5. *Corner node (e.g. node 1) is not short and other nodes are short.* In Step 1, node 1 ships to the middle node, and, if possible, ships to node 3 in Step 2. Again since shortage and transshipment costs are identical, the flows after Step 2 are optimal.
6. *Middle node is not short and all other nodes are short.* Since both corner nodes are short, the flows after Step 2 are optimal.

In all six cases, changing transshipment flows among nodes 1, 2 and 3 after Step 2 cannot lower the total cost. The same holds for nodes 4, 5, and 6, thus $Z(\mathcal{K}_g|\mathbf{X}') = Z(\mathcal{K}_g)$.

For the chain, we show by example that $Z(\mathcal{K}_c) \leq Z(\mathcal{K}_c|\mathbf{X}')$. Consider inventory before transshipment= (-10, 5,-5, 0, 0, 10). An optimal solution in Step 1 is $x_{21} = 5$ and 0 otherwise. In Step 2, $x_{61} = 5$ and 0 otherwise. The total inventory of this solution is 5, the shortage is 5, and the number of transshipments is 10. However, the optimal solution, $x_{23} = 5$ and $x_{61} = 10$, results in no inventory or shortage with 15 transshipments. From Steps 2 and 3, $Z(\mathcal{K}_c) \leq Z(\mathcal{K}_c|\mathbf{X}') \leq Z(\mathcal{K}_g|\mathbf{X}') = Z(\mathcal{K}_g)$. As a result, $Z(\mathcal{K}_c) \leq Z(\mathcal{K}_g)$ for one profitable shift. With more than one profitable shift, the chain configuration benefits while the group configuration does not. Therefore, $Z(\mathcal{K}_c) \leq Z(\mathcal{K}_g)$.□

Theorem 3 *Given a network with $N = 3M$ identical locations ($M = 1, 2, \cdots$) facing independent demand, the optimal expected cost in a bidirectional chain configuration is lower than or equal to the optimal expected cost in a group(3) configuration.*

4.2 Numerical results for chain and general configurations

We extend the previous results by comparing the bidirectional chain configuration with configurations with the same number of bidirectional links, using network configurations with N nodes and N transshipment links, where $N = 6, 12$, and 18. The numerical tests incorporate a range of parametric values. We fix the sum of holding and shortage costs at 13 with three cases for holding and shortage costs, denoted by $\tau = (c_h, c_s) \in \{(2, 11), (4, 9), (6, 7)\}$, and consider six transshipment costs: $c_t = 2, 4, 6, 8, 10, 12$, resulting in four profitable shifts: $J = 1, 2, 3, 6$. Demand at each node follows a Gamma distribution with mean of 100 and five coefficients of variation: $\gamma = 0.25, 0.5, 1, 1.5, 2$. For each scenario, we perform pairwise comparisons of the chain with the other configurations.

Consider a pairwise comparison of the chain configuration with configuration f. The optimal expected cost for each configuration is obtained with the IPA method using its optimal base-stock level vector. Let $Z(\mathcal{K}_c|\mathbf{S}_c)$ be the optimal expected cost of the chain and $Z(\mathcal{K}_f|\mathbf{S}_f)$ be the optimal expected cost of configuration f. Let $\Lambda(\gamma, c_t, \tau)$ denote the percent of pairwise comparisons in which the chain has a lower expected cost than the other configuration with scenario values γ, c_t, and τ. Let $\Delta(\gamma, c_t, \tau)$ denote the deviation of the expected cost of the chain from the expected cost of the lower cost configuration in pairwise comparisons in which the chain is not the lower cost configuration:

$$\Delta(\gamma, c_t, \tau) = \frac{Z(\mathcal{K}_c|\mathbf{S}_c) - Z(\mathcal{K}_f|\mathbf{S}_f)}{Z(\mathcal{K}_f|\mathbf{S}_f)} \times 100\%.$$

Let $\bar{\Delta}(\gamma, c_t, \tau)$ be the average deviation of all pairwise comparisons in which the chain does not yield the lower cost and $\Delta^*(\gamma, c_t, \tau)$ be the maximum deviation of these comparisons.

We compare the relative magnitude of $\Delta(\gamma, c_t, \tau)$ with $\Theta(\gamma, c_t, \tau)$, the difference between the highest and lowest optimal expected costs for all configurations for scenario values γ, c_t, and τ. We denote the set of all configurations, including the chain, as F, and calculate $\Theta(\gamma, c_t, \tau)$ as follows:

$$\Theta(\gamma, c_t, \tau) = \left(\frac{\max_{f \in F}\{Z(\mathcal{K}_f|\mathbf{S}_f)\} - \min_{f \in F}\{Z(\mathcal{K}_f|\mathbf{S}_f)\}}{\min_{f \in F}\{Z(\mathcal{K}_f|\mathbf{S}_f)\}} \right) \times 100\%.$$

For networks with $N = 6$, there are 20 possible configurations (other than chain), shown in the Appendix. Table 1 presents the results as a function of coefficient of variation, the factor that was observed to have the most significant impact on the results. The table presents the number of total comparisons(#) at each value of γ and the value of the efficiency metrics for a fixed γ: $\Lambda(\gamma, \cdot, \cdot)$, $\bar{\Delta}(\gamma, \cdot, \cdot)$, $\Delta^*(\gamma, \cdot, \cdot)$, and $\Theta(\gamma, \cdot, \cdot)$. We use Λ, $\bar{\Delta}$, Δ^*, and Θ when the values of γ, c_t, and τ are constant, or when these values are unambiguous.

γ	#	$\Lambda(\gamma, \cdot, \cdot)$	$\bar{\Delta}(\gamma, \cdot, \cdot)$	$\Delta^*(\gamma, \cdot, \cdot)$	$\Theta(\gamma, \cdot, \cdot)$
0.25	360	100%	-	-	38.2%
0.5	360	100%	-	-	38.3%
1	360	100%	-	-	35.3%
1.5	360	98.3%	1.2%	2.6%	27.2%
2	360	90.0%	1.6%	7.5%	20.1%
Total	1800	97.7%	1.4 %	7.5%	38.3%

Table 1: Efficiency of the chain network for $N = 6$ as a function of γ

The chain is more efficient in 97.7% of all comparisons. The chain is the most efficient configuration for values of $\gamma \leq 1$, implying that chain remains efficient regardless of the cost parameters for low to moderate levels of demand uncertainty. The chain is still highly efficient at $\gamma = 1.5$ (high value of Λ and low values of $\bar{\Delta}$ and Δ^*). At $\gamma = 2$, other configurations may be more efficient than the chain, although the chain is more efficient in 90% of the comparisons. The maximum deviation between the chain and more efficient configurations is 7.5%, which is small relative to the maximum cost spread of 20%. Values of Θ decrease as γ increases, suggesting that when demand is highly uncertain, the configuration is less important since the high costs to accommodate this uncertainty are unavoidable.

In Tables 2 and 3, we explore the efficiency of the chain with respect to cost parameters for γ=2 (i.e., the fifth row of Table 1). Table 2 presents results as a function of holding and shortage costs, and Table 3 presents results as a function of transshipment costs.

c_h	c_s	#	$\Lambda(2,\cdot,\tau)$	$\bar{\Delta}(2,\cdot,\tau)$	$\Delta^*(2,\cdot,\tau)$	$\Theta(2,\cdot,\tau)$
2	11	120	95.0%	1.2%	4.8%	20.1%
4	9	120	90.0%	1.9%	7.5%	13.9%
6	7	120	85.0%	1.5%	7.0%	8.7%
Total		360	90.0%	1.6%	7.5%	20.1%

Table 2: Efficiency of the chain network for $N = 6$ for γ= 2 as a function of c_h and c_s

Table 2 suggests that while the efficiency of the chain configuration in terms of Λ decreases with higher holding costs and lower shortage costs, the deviation between the chain and other configurations does not change significantly. The maximum deviation among all configurations decreases with higher holding costs and lower shortage costs.

c_t	#	$\Lambda(2,c_t,\cdot)$	$\bar{\Delta}(2,c_t,\cdot)$	$\Delta^*(2,c_t,\cdot)$	$\Theta(2,c_t,\cdot)$
12	60	100%	-	-	0.8%
10	60	100%	-	-	2.4%
8	60	100%	-	-	4.4%
6	60	98.3%	0.01%	0.01%	7.2 %
4	60	78.3%	0.7%	2.1%	12.6 %
2	60	63.3%	2.3%	7.5%	20.1 %
Total	360	90.00%	1.6%	7.5%	20.1%

Table 3: Efficiency of the chain network for $N = 6$ and γ= 2 as a function of c_t

Table 3 suggests that the cases in which the chain is less efficient correspond to scenarios with high demand variability and low transshipment costs (i.e., $\gamma = 2$ and $c_t = 2$ and 4). When demand variability is high and transshipment costs are low relative to holding and shortage costs, configurations that centralize inventory to maximize pooling are desirable.

In these cases, the most efficient configuration is Configuration 4 in the Appendix, which we refer to as the *star* network (an extra link exists compared to a typical star since all configurations must include 6 links). The star network stores additional inventory at one centralized node and transships items to other nodes as needed. This results in a lower cost than the chain since centralized inventory pooling reduces the relatively high holding costs with highly variable demand. We make the following observations.

Observation 1 *When demand uncertainty is low or transshipment costs are high relative to holding and shortage costs, multiple shifts are not significantly beneficial in all configurations. Under these conditions, balanced base-stock levels among all locations are desirable and the chain is likely to be the most efficient configuration for N=6.*

Observation 2 *When demand uncertainty is high and transshipment costs are low relative to holding and shortage costs, configurations that utilize multiple shifts are not the most efficient. Rather, these conditions promote centralized risk pooling and the star configuration is likely to be the most efficient configuration for N=6.*

We confirm Observations 1 and 2 in networks with $N = 12$ and 18. Since the number of possible unique configurations for 12 and 18 location/link scenarios are much greater, we consider 25 randomly generated unique networks in the pairwise comparisons with the chain and star configurations. The networks and result tables are presented in the Online Appendix. The chain is more efficient in over 97% of the comparisons with the randomly generated configurations. For $\gamma \leq 1$, the chain is most efficient when compared to the 25 randomly generated configurations. When the chain is inferior, the average value of $\bar{\Delta}$ is less than 2% with a maximum deviation below 7%. When compared to the star configuration, the chain is more efficient in all cases with $\gamma \leq 0.5$. Consistent with Observation 2, the star performs significantly better than the chain when $\gamma \geq 1.5$ and c_t is small relative to $c_s + c_h$. In pairwise comparisons of the star with the randomly generated networks at $\gamma \geq 1.5$ and $c_t \leq 4$, the star configuration is the most efficient. In conclusion there are many scenarios in which the chain is the most efficient configuration; however, in cases in which centralized inventory is desirable (high demand uncertainty and low transshipment cost relative to shortage and holding costs), the star configuration is more efficient.

Based on the above analysis, the chain is a robust, efficient configuration. We present the three key desirable properties for an efficient and robust transshipment network. While we have identified other desirable properties, they are all related to those mentioned below.

- *Appropriate pool size.* To achieve the benefits of risk pooling, nodes should have access to the inventory of other nodes. The number of nodes that should be accessible (i.e., the pool size) is impacted by demand variation. As variation increases, the pool size should increase.
- *Short Transshipment Paths.* As expected, reducing the path length (number of shifts) between locations in a transshipment pool can lower transshipment costs. Note that only paths of length less than the number of profitable shifts are included in the pool.
- *Balanced Node Degree.* The degree of a node is equal the number of incident links. Networks with balanced node degrees have well-distributed transshipment links throughout the network in identical networks. Balancing links among all nodes leads to low inventory levels at all nodes.

5. Discussion

Our work is a first step in transshipment network design research. Several of the initial assumptions may be relaxed in future research. In particular, it is important to investigate the transshipment network design problem in which the unit transshipment cost between different locations depends on the specific pair of locations (non-homogeneous transshipment costs). Such transshipment cost structure can represent the distance between the locations, or other location-pair characteristics.

Unfortunately, the above extension complicates the analysis quite considerably. A chain configuration with non-homogeneous transshipment costs must be further defined by the sequence in which the locations are ordered within the chain, and there are (n-1)! possible sequences. Each sequence is characterized by different total transshipment costs along the chain, and finding the sequence with minimal total transshipment costs is equivalent to the well known TSP problem. However, ordering the locations according to the optimal TSP sequence does not necessarily minimize total inventory and transshipment costs, and therefore, may not provide the best chain sequence. The locations will differ in their accessibility due to different (transshipment) costs of connected links and varying base stock levels.

With the grouping configuration and non-homogeneous transshipment costs, similar and additional issues arise. While equal group sizes are preferable for the case of homogeneous costs, this may not necessarily be the case with non-homogeneous costs. In the latter case, the group size may be significantly affected by the closeness of a group of locations to other locations, as expressed in terms of the transshipment costs. Thus, the group size is yet another factor to be determined.

It appears that with non-homogeneous transshipment costs the best configuration depends heavily on the specific cost parameters. Thus we believe that our analytical results cannot be extended to this more general case. Another important and interesting case is when the demand parameters differ among locations. This again will lead to unequal base stock levels affecting the required or available stock going into or out of each location, thus complicating again the desired configuration problem. Future research can build on the framework and basic fundamental analysis in this paper to study particular cases of these more general settings.

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Appendix

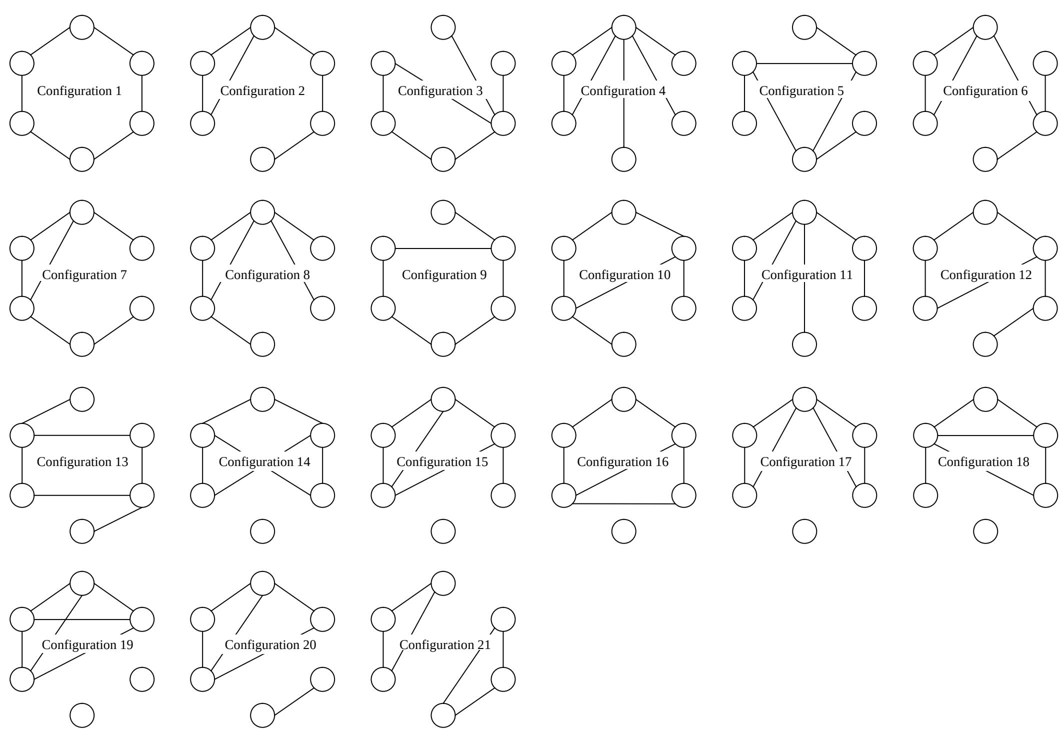

Figure A.1: All 6 node - 6 link network configurations