New Perspectives on Mixed-Integer Convex Optimization with Applications in Statistical Learning

Simge Küçükyavuz

NORTHWESTERN UNIVERSITY
Joint work with...

Alper Atamtürk
Berkeley

Salar Fattahi
Michigan

Andrés Gómez
USC

Peijing Liu
USC

Linchuan Wei
Northwestern

Supported by NSF grants 2007814 and 2006762 (with A. Gómez)
Time travel

Before we talk about Mixed-Integer Convex Quadratic Programs, let’s do an experiment to see how far we’ve come in Mixed-Integer Linear Programming (MILP)
Impact of cutting planes in Mixed-Integer Linear Programming (MILP) software

Without cuts

Explored 1933736 nodes (3989094 simplex iterations) in 38.70 seconds
Thread count was 8 (of 8 available processors)

With cuts

Cutting planes:
- Gomory: 4
- Implied bound: 22
- MIR: 19
- Flow cover: 35
- Flow path: 18
- Relax-and-lift: 2

Explored 1 nodes (426 simplex iterations) in 0.10 seconds
Thread count was 8 (of 8 available processors)

Extended formulation

Explored 0 nodes (238 simplex iterations) in 0.03 seconds
Thread count was 8 (of 8 available processors)
What's the Secret Sauce?

Polyhedral Theory for MILP

- Original formulation
- Stronger formulation
- Ideal formulation (convex hull, facets)

- **Structured** cutting planes (Cover, flow cover, flow path, etc.)
- General-purpose cutting planes (Gomory, MIR, disjunctive, etc.)
- Presolve, heuristics, branching, ...

MIQP with indicators

\[
\begin{align*}
\min_{x,z} & \quad a^T x + b^T z + \frac{1}{2} x^T Q x \\
\text{s.t.} & \quad x_j(1 - z_j) = 0, \quad j \in [n] := \{1, \ldots, n\} \\
& \quad x \in \mathbb{R}^n, \ z \in \mathbb{Z} \subseteq \{0, 1\}^n
\end{align*}
\]

or equivalently, in its epigraph form,

\[
\begin{align*}
\min_{x,z,t} & \quad a^T x + b^T z + \frac{1}{2} t \\
\text{s.t.} & \quad t \geq x^T Q x, \ x \circ (1 - z) = 0, \ x \in \mathbb{R}^n, \ z \in \mathbb{Z}
\end{align*}
\]

Alternative formulation of non-convex complementarity constraint

\[-Mz \leq x \leq Mz \quad \text{(Big-M constraint)}\]

Weak continuous relaxation
Motivating Example: Best Subset Selection

Given model matrix $A_{m \times n}$ and response vector $y \in \mathbb{R}^m$

$$
\min_{x: \|x\|_0 \leq k} \|y - Ax\|_2^2,
$$

where $\|x\|_0 = \sum_{i=1}^n 1_{\{x_i \neq 0\}}$ is the "\(l_0\) norm," $k \in \mathbb{Z}$ is a given cardinality.

Here, $Z = \{z \in \{0, 1\}^n : \sum_{i=1}^n z_i \leq k\}$, $Q = A^T A$, $a = -y^T A$

NP-hard. (Chen et al, 2017)
Other Applications

- Structured regression (e.g., Bertsimas et al, 2021; Hazimeh and Mazumder, 2020)

- Probabilistic graphical models (e.g., Küçükyavuz et al., 2020)

- Portfolio optimization (e.g., Bienstock, 1996)

- Power systems (e.g., Bacci et al., 2019)

- Machine scheduling (e.g., Aktürk et al., 2009)
Special Cases: $n = 1$ (or $Q$ is diagonal)

$$R \equiv \{(z, x, t) \mid t \geq Q_{11}x^2, \ x \circ (1 - z) = 0, \ z \in \{0, 1\}\}$$

$$\text{cl conv}(R) \equiv \{(z, x, t) \mid t \geq Q_{11}\frac{x^2}{z}, \ z \in [0, 1]\} \text{ (Big-M free, SOCP)}$$

Convention: $\frac{0}{0} = 0$.

Perspective reformulation: Ceria and Soares (1999), Frangioni and Gentile (2006), Aktürk et al. (2008), Günlük and Linderoth (2010)...

Why perspective? For convex function $f : \mathbb{R} \to \mathbb{R}$ with $f(0) = 0$ its perspective function $\phi(x, z) = zf\left(\frac{x}{z}\right) : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ is also convex.
Solution approaches leveraging perspective formulation

1. Find “good” diagonal matrix $D$, $D_{ii} \geq 0$ such that $Q - D \succeq 0$
   - Using minimum eigenvalue of $Q$ (Frangioni, 2006)
   - Using SDP heuristics (Frangioni, 2007)
   - Using ridge regularization (Bertsimas and Van Parys, 2020)
   - Maximizing relaxation quality (Zheng et al., 2014; Dong et al., 2015)

2. Use branch-and-bound based on the perspective reformulation

$$
\min_{x,z} \quad a^T x + b^T z + \frac{1}{2} x^T (Q - D) x + \frac{1}{2} \sum_{i=1}^{n} \frac{D_{ii} x_i^2}{z_i}
$$

s.t. $-Mz \leq x \leq Mz$ \hspace{1cm} \text{(Big-M constraint)}

$x \in \mathbb{R}^n$, $z \in Z \subseteq \{0, 1\}^n$
Research Questions

- Can we exploit matrix and constraint structure to obtain stronger relaxations? (Part 1)

- What does strong mean for MIQP? Can we leverage polyhedral theory for MIQP? (Part 2)
Special Cases: Rank-one convex function $f$

$$X = \{(z, x, t) \in \{0, 1\}^n \times \mathbb{R}^{n+1} | t \geq f(q^T x), x \circ (1 - z) = 0, z \in Z\}$$

Quadratic: $f(q^T x) = (q^T x)^2$ for a given vector $q \in \mathbb{R}^n$, i.e., $Q = qq^T \succeq 0$.

Theorem (Wei, Gómez, Küçükyavuz, 2022)

If $f$ is convex, $f(0) = 0$, and $Z$ is 'connected', then

$$\text{cl conv}(X) = \left\{(z, x, t) | z \in \text{conv}(Z), t \geq f(q^T x), t \geq (\pi^T z)f\left(\frac{q^T x}{\pi^T z}\right), \forall \pi \in F\right\},$$

where $F$ is a family of strong separating inequalities for

$$\text{conv}(Z\backslash\{0\}) = \text{conv}(Z) \cap \{z \in \mathbb{R}^n : \pi^T z \geq 1, \forall \pi \in F\}$$

- New perspectives
  Subsumes all related convexifications to date; first convexification for a logistic loss function.

Special Case: $f(q^T x) = (q^T x)^2, Z = \{0, 1\}^n$

$$R \equiv \{(z, x, t) \in \{0, 1\}^n \times \mathbb{R}^{n+1} : t \geq (q^T x)^2, x \circ (1 - z) = 0\}.$$  

**Theorem (Atamtürk and Gómez, 2019)**

$$\text{cl conv}(R) = \left\{(z, x, t) \in [0, 1]^n \times \mathbb{R}^{n+1} \mid t \geq (q^T x)^2, t \geq \frac{(q^T x)^2}{\sum_{i \in [n]} z_i} \right\}$$

- $\sum_{i=1}^{n} z_i \geq 1$ is a strong inequality separating 0 from the set $Z$

$$\text{conv}(Z \setminus \{0\}) = \{z \in [0, 1]^n : \sum_{i=1}^{n} z_i \geq 1\}$$
Special Case: Diagonal $Q$, General $Z \subset \{0, 1\}^n$

\[ R \equiv \{(z, x, t) \mid t_i \geq Q_{ii} x_i^2, i \in [n], x \circ (1 - z) = 0, z \in Z\} \]

Corollary (Wei, Gómez, Küçükyavuz, 2022)

\[
\text{cl conv}(R) = \left\{ (z, x, t) \mid t_i \geq \frac{Q_{ii} x_i^2}{z_i}, i \in [n], z \in \text{conv}(Z) \right\}.
\]

Xie and Deng (2020) show this for $Z = \{z \in \{0, 1\}^n : \sum_{i=1}^{n} z_i \leq k\}$. 
Numerical Results

Least squares regression with **strong hierarchy** constraints on pairwise interactions.

\[
\begin{align*}
\min_{z,x} & \quad \sum_{\ell=1}^{p} \left( y_{\ell} - \sum_{i=1}^{n} A_{\ell i} x_i - \sum_{i=1}^{n} \sum_{j=i}^{n} A_{\ell i} A_{\ell j} x_{ij} \right)^2 + \lambda \| x \|_2^2 + \mu \| z \|_1 \\
\text{s.t.} & \quad x_i (1 - z_i) = 0 \quad \forall i \\
& \quad x_{ij} (1 - z_{ij}) = 0 \quad i \leq j \\
& \quad z_{ij} \leq z_i \quad \forall i \\
& \quad z_{ij} \leq z_i, \ z_{ij} \leq z_j \quad i \leq j \\
& \quad z \in \{0, 1\}^\frac{n(n+3)}{2}
\end{align*}
\]

- \( z_i + z_j - z_{ij} \geq 1 \) is a strong inequality separating 0 from the set \( Z \)
Relaxation comparisons

- **Perspective**: Optimal perspective relaxation (Dong et al., 2015)

- **Rank1**: Rank-one relaxation (Atamtürk and Gómez, 2019)

- **Hier**: Hierarchical strengthening (the formulation we proposed)

- **Rank1 + Hier**: Combine these two methods
Hier (vs. Persp): Significant improvement in lower bound

Rank1+Hier (vs. Rank1): Gives the best optimality gap
- **Hier (vs. Persp):** Only a slight increase in solution time
- **Rank1+Hier (vs. Rank1):** No increase in solution time
Special Case: Tridiagonal $Q$

$$Q = \begin{pmatrix}
* & * & 0 & 0 & \ldots & 0 \\
* & * & * & 0 & \ldots & 0 \\
0 & * & * & * & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & * & *
\end{pmatrix}$$

Support graph of $Q$: Arc $(i, j)$ for $i \leq j$ with $Q_{ij} \neq 0$
Special Case: Tridiagonal $Q$

\[
\min_{x,z} a^T x + b^T z + \frac{1}{2} x^T Q x \\
\text{s.t. } x \odot (1 - z) = 0 \\
x \in \mathbb{R}^n, \ z \in \{0, 1\}^n
\]

Suppose $z = 1$

- Optimality condition: Solve $Qx = -a$

- Thomas Algorithm for tridiagonal $Q$ takes $O(n)$ time
Single indicator: $z_k \in \{0, 1\}$

Now suppose $z_j = 1$ for $j \in [n] \setminus \{k\}$:

$$\begin{align*}
\min_{x, z_k} & \quad a^T x + b_k z_k + \frac{1}{2} \sum_{i \in [n]} Q_{ii} x_i^2 + \sum_{i \in [n-1]} Q_{i,i+1} x_i x_{i+1} \\
\text{s.t.} & \quad x_k (1 - z_k) = 0 \\
& \quad x \in \mathbb{R}^n, \ z_k \in \{0, 1\}
\end{align*}$$
Proposition (Liu, Fattahi, Gómez, Küçükyavuz, 2022)

*MIQP with tridiagonal matrices can be solved by solving a shortest path problem.*

**Complexity:**
- Direct: $O(n^2)$ arcs $\times O(n)$ arc cost calculation $= O(n^3)$
- Improved: $O(n^2)$

Leads to a shortest path-based compact tight extended formulation.
Experiments

Can we leverage this efficient algorithm to solve the problem for non-tridiagonal $Q > 0$?
Sparse, Strictly Diagonally Dominant Matrix $Q$

**Idea:** Split $Q$ into tridiagonal submatrices: $T_1, \ldots, T_\ell$ and a remainder $R$ of off-tridiagonals

- Use convexification and Fenchel duality for off-tridiagonals

- Decomposes to path subproblems ($O(n^2)$ algorithm)
Convexification and Fenchel duality

Rewriting the problem
\[
\min_{x,z,t} \quad a^T z + b^T z + \frac{1}{2} \sum_{k=1}^\ell t_k + \frac{1}{2} \sum_{i=1}^n \sum_{j=i+2}^n |Q_{i,j}| (x_i \pm x_j)^2 \\
\text{s.t. } t_k \geq x^T T_k x, \quad k = 1, \ldots, \ell, \quad x \circ (1 - z) = 0, \quad x \in \mathbb{R}^n, \quad z \in \{0, 1\}^n, \quad t \in \mathbb{R}^\ell.
\]

Convexify the rank-one terms to obtain relaxation objective
\[
\min_{x,z,t} \quad a^T x + b^T z + \frac{1}{2} \sum_{k=1}^\ell t_k + \frac{1}{2} \sum_{i=1}^n \sum_{j=i+2}^n |Q_{i,j}| \left( \frac{(x_i \pm x_j)^2}{\min\{1, z_i + z_j\}} \right)
\]

Relax complicating terms via Fenchel dual to obtain relaxation
\[
\zeta_p = \min_{x,z,t} \quad \max_{\alpha,\beta} \quad a^T x + b^T z + \frac{1}{2} \sum_{k=1}^\ell t_k \\
+ \frac{1}{2} \sum_{i=1}^n \sum_{j=i+2}^n |Q_{ij}| \left( \alpha_{ij} (x_i \pm x_j) - \beta_{ij} z_i - \beta_{ij} z_j - f^* (\alpha_{ij}, \beta_{ij}, i, \beta_{ij}, j) \right)
\]
Fenchel Decomposition

Strong duality holds, so

\[
\zeta_p = \max_{\alpha, \beta} \left( -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=i+2}^{n} |Q_{ij}| f^* (\alpha_{ij}, \beta_{ij, i}, \beta_{ij, j}) + \min_{x, z, t} \{ \psi(x, z, t, \alpha, \beta) \} \right)
\]

- **Inner min** Independent tridiagonal problems

- **Outer max** Subgradient ascent
Given: noisy observations (orange)
Goal: find true values (blue)
Arc \((i,j)\): connection between variables \(i,j\) with \(Q_{ij} \neq 0\)
Computational Results

\[ n = 100 \]

<table>
<thead>
<tr>
<th>Noise</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Method</strong></td>
<td><strong>Time(s)</strong></td>
<td><strong>Gap</strong></td>
</tr>
<tr>
<td>Decomposition</td>
<td>0.1</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>Big-M</td>
<td>0.3</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

\[ n = 1600 \]

<table>
<thead>
<tr>
<th>Noise</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Method</strong></td>
<td><strong>Time(s)</strong></td>
<td><strong>Gap</strong></td>
</tr>
<tr>
<td>Decomposition</td>
<td>43.6</td>
<td>&lt;1%</td>
</tr>
<tr>
<td>Big-M</td>
<td>3600</td>
<td>3.9%</td>
</tr>
</tbody>
</table>

\[ \text{Gap} = \frac{(\text{Upper Bound} - \text{Lower Bound})}{\text{Upper Bound}} \]
Decomposition Method

$n = 1600$, high noise
Research Questions

- Can we exploit matrix and constraint structure to obtain stronger relaxations? (Part 1) ✅

- What does strong mean for MIQP?
  Can we leverage polyhedral theory for MIQP? (Part 2)
Agenda
For a subset \( S \in \mathbb{Z} \), \( z_i = 1 \) if \( i \in S \) and \( Q_S \) is the submatrix of \( Q \) indexed by \( S \) (similarly \( a_S, b_S, x_S \))

\[
\min_{x} a^T x + b_S + \frac{1}{2} x^T Q x = \min_{x_S} a_S^T x_S + b_S + \frac{1}{2} x_S^T Q_S x_S
\]

s.t. \( x_i = 0 \), \( \forall i \notin S \).

- \( x^*_S = -Q_S^{-1} a_S \).
- A combinatorial problem of selecting subset \( S \)

\[
\min_{S \subseteq [n]} \ b_S - \frac{1}{2} a_S^T Q_S^{-1} a_S
\]
Notation

- Given $S \subseteq \{1, \ldots, n\}$:
  - $e_S = n$-dimensional indicator vector of $S$
  - $Q_S = |S| \times |S|$ submatrix of $Q$ induced by $S$
  - $\hat{Q}_S^{-1} = n \times n$ matrix corresponding to $Q_S^{-1}$ in the rows/columns of $S$, and 0 elsewhere

Example: $Q = \begin{pmatrix} d_1 & 1 \\ 1 & d_2 \end{pmatrix}$

$d_1 d_2 > 1$
Structure of the convex hull (extended formulation)

\[ X \equiv \left\{ (z, x, t) \in Z \times \mathbb{R}^{n+1} \mid t \geq x^T Q x, \ x \circ (1 - z) = 0 \right\}. \]

\[ P \equiv \text{conv} \left( \{(e_S, \hat{Q}_S^{-1})_{S \in Z}\} \right). \]

**Theorem (Wei, Atamtürk, Gómez and Küçükyavuz, 2022)**

*If \( Q \) is positive definite, then*

\[ \text{cl conv}(X) = \left\{ (z, x, t) \in [0, 1]^n \times \mathbb{R}^{n+1} \mid \exists W \in \mathbb{R}^{n \times n}, \begin{pmatrix} W & x \\ x^T & t \end{pmatrix} \succeq 0, (z, W) \in P \right\}. \]

Can be extended to the psd/low rank case (a more compact extended formulation)
Preliminaries

Definition

Given a matrix $W \in \mathbb{R}^{p \times q}$, its pseudoinverse $W^\dagger \in \mathbb{R}^{q \times p}$ is the unique matrix satisfying: $WW^\dagger W = W$, $W^\dagger WW^\dagger = W^\dagger$, $(WW^\dagger)^\top = WW^\dagger$, $(W^\dagger W)^\top = W^\dagger W$.

Examples

- if $W$ is invertible then $W^\dagger = W^{-1}$
- $W = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$, $W^\dagger = \begin{pmatrix} 1/a & 0 \\ 0 & 0 \end{pmatrix}$

Lemma (Generalized Schur Complement)

$U = \begin{pmatrix} U_{11} & U_{12} \\ U_{12}^\top & U_{22} \end{pmatrix}$ with $U_{11} \in S^{m \times m}$ and $U_{22} \in S^{n \times n}$, and $U_{12} \in \mathbb{R}^{m \times n}$. Then $U \succeq 0$ if and only if $U_{11} \succeq 0$, $U_{11} U_{11}^\dagger U_{12} = U_{12}$ and $U_{22} - U_{12}^\top U_{11}^\dagger U_{12} \succeq 0$. 

Simge Küçükyavuz
Recall

Theorem (Wei, Atamtürk, Gómez and Küçükyavuz, 2022)

If $Q$ is positive definite, then

$$\text{cl conv}(X) = \{(z, x, t) \in [0, 1]^{n} \times \mathbb{R}^{n+1} \mid \exists W \in \mathbb{R}^{n \times n} \left( \begin{array}{c} W \\ x^T \\ t \end{array} \right) \succeq 0, (z, W) \in P \}.$$  

Proof idea: Optimizing over $\text{cl conv}(X)$ is equivalent to optimizing the original problem.
Proof of the Theorem

Optimizing over $\text{cl \, conv}(X)$

$$\min_{x, z, W} \quad a^T x + b^T z + t$$

subject to

$$\begin{pmatrix} W & x \\ x^T & t \end{pmatrix} \succeq 0$$

$$(z, W) \in P \equiv \text{conv}\left( \{(e_S, \hat{Q}_S^{-1}) | S \in Z\} \right)$$

- $z = e_S$ for some $S \in Z$
- $W = \hat{Q}_S^{-1} = \begin{pmatrix} Q_S^{-1} & 0 \\ 0 & 0 \end{pmatrix} \succeq 0$ and $W^\dagger = \begin{pmatrix} Q_S & 0 \\ 0 & 0 \end{pmatrix}$
- $WW^\dagger x = x \iff \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_S \\ x_{[n]\setminus S} \end{pmatrix} = \begin{pmatrix} x_S \\ x_{[n]\setminus S} \end{pmatrix} \iff x_{[n]\setminus S} = 0$
- $t \geq x^T W^\dagger x \iff t \geq x_S^T Q_S x_S$
Example: Quadratic with "Choose-one" constraint

\[ X_{C_1} = \{(z, x, t) \in \{0, 1\}^n \times \mathbb{R}^{n+1} \mid t \geq x^\top Q x, \ x \circ (1 - z) = 0, \sum_{i=1}^n z_i \leq 1\} \]

Corollary

\[ \text{cl} \ \text{conv}(X_{C_1}) = \left\{(z, x, t) \in \mathbb{R}_+^n \times \mathbb{R}^n \times \mathbb{R} \mid t \geq \sum_{i=1}^n Q_{ii} \frac{x_i^2}{z_i}, \sum_{i=1}^n z_i \leq 1 \right\}. \]

- \( P = \text{conv} \left( \left\{(0, 0), (e_{\{i\}}, \hat{Q}_{\{i\}}^{-1})_{i=1}^n \right\} \right) \)

- \( P = \{(z, W) \mid W_{ij} = 0, i \neq j, W_{ii} = \frac{z_i}{Q_{ii}}, i = 1, \ldots, n\} \)

\[
\begin{pmatrix}
W \\
x^\top \\
t
\end{pmatrix}
\geq 0, (z, W) \in P \iff 
\begin{pmatrix}
\frac{z_1}{Q_{11}} & \cdots & 0 & x_1 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & \frac{z_n}{Q_{nn}} & x_n \\
x_1 & \cdots & x_n & t
\end{pmatrix}
\geq 0 \iff t \geq \sum_{i=1}^n Q_{ii} \frac{x_i^2}{z_i} \]
Structure of the convex hull (original space)

Let

$$X = \{ (x, z, t) \in \mathbb{R}^n \times Z \times \mathbb{R} : t \geq x^T Q x, \ x \circ (1 - z) = 0 \}$$

Suppose a minimal description of $P$ is given by

$$\langle \Gamma_i, W \rangle - \gamma_i^T z \leq \beta_i, \quad i = 1, \ldots, m_1$$
$$\langle \Gamma_i, W \rangle - \gamma_i^T z = \beta_i, \quad i = m_1 + 1, \ldots, m.$$ 

Theorem (Wei, Atamtürk, Gómez, Küçükyavuz, 2022)

$$(x, z, t) \in cl \ conv(X) \text{ iff } z \in \text{conv} (Z), \ t \geq 0 \text{ and }$$

$$t \geq \frac{x^T (\sum_{i=1}^{m} \Gamma_i s_i) x}{\beta^T s + (\sum_{i=1}^{m} \gamma_i s_i)^T z}$$

for all $s \in \mathbb{R}_{+}^{m_1} \times \mathbb{R}^{m-m_1}$ such that $\sum_{i=1}^{m} \Gamma_i s_i \geq 0, \sum_{i=1}^{m} Tr(\Gamma_i) s_i \leq 1.$
Observations

- Semi-infinite conic quadratic program, but “finitely" generated by \((\Gamma_i, \gamma_i, \beta_i), i \in [m]\).

- The strongest conic quadratic inequality is given by

\[
t \geq \max_{s \in \mathbb{R}^{m_1}_+ \times \mathbb{R}^{m-m_1}} \frac{x^T (\sum_{i=1}^m \Gamma_i s_i) x}{\beta^T s + (\sum_{i=1}^m \gamma_i s_i)^T z}
\]

\[
\text{s.t. } \sum_{i=1}^m \Gamma_i s_i \geq 0, \sum_{i=1}^m \text{Tr}(\Gamma_i) s_i \leq 1.
\]

- How to work with \(P = \text{conv}\left\{ (e_S, \hat{Q}_S^{-1})_{S \in Z} \right\} \) in practice? We give an MILP formulation for \( \left\{ (e_S, \hat{Q}_S^{-1})_{S \in Z} \right\} \). Preliminary tests show that this MILP is faster than perspective for some instances.
Research Questions

- Can we exploit matrix and constraint structure to obtain stronger relaxations? (Part 1) ✓

- What does strong mean for MIQP? Can we leverage polyhedral theory for MIQP? (Part 2) ✓
Agenda
Conclusions

- We characterize the convex hulls of MIQPs with indicators

- Convexification reduces to finding a facial description of a polytope

- We can use any tools from MILP to do so

- We can use polyhedral theory to understand strength of convexifications

- Offers insights into design of algorithms
References


