Actively managing tracking error

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Abstract Managing tracking error on an *ex ante* basis requires an ability to assess the possible effects of trades on a fund's performance relative to its benchmark. This paper develops several simple diagnostic tools to help fund managers evaluate alternative trading strategies in terms of their potential for reducing tracking error. Moreover, risk reductions can be readily balanced against trading requirements and impacts on active return to identify desirable strategies.

Keywords: *tracking error, risk management, trading strategies, risk decomposition,* ex ante*, trade risk profile*

Introduction

In recent years, there has been increasing focus on tracking error as the means for measuring and controlling risk in actively managed funds. This is a result of several important investor trends

— a growing awareness of risk management methodologies
— the demand for additional risk reporting and disclosure from fund managers
— investor-mandated risk limits relative to standard or custom benchmarks.

While tracking error is routinely used to monitor a fund's performance, it often receives limited consideration in the investment decision-making process itself. Undoubtedly, this is due in part to a lack of appropriate tools for assessing the potential impact of trades on tracking error. This paper develops several simple diagnostic tools to assist in the proactive management of a fund's tracking error.

Tracking error, defined as the standard deviation of a fund's active return relative to a specified benchmark, measures the relative volatility of the fund.

Traditionally, it is computed in an *ex post* fashion from historical return data (eg six years of actual monthly returns for a fund and its benchmark). In contrast, incorporating tracking error in the investment process requires an *ex ante* view — the objective is to assess the fund's risk over some future period. In this case, the asset returns are stochastic, and the active return is a random variable.

In making investment decisions, it is common practice for fund managers first to identify which securities are to be traded (eg purchase shares of IBM, funded equally from cash and by selling long-term government bonds). This first-stage decision is referred to as selecting a trading or rebalancing strategy. Secondly, they determine the extent of trading to be done (eg how many shares of IBM should be purchased?). As part of this process, fund managers typically face questions such as

— Which position is currently the largest contributor to the tracking error?
— How is the tracking error likely to change as a result of a particular rebalancing strategy?
— What is the minimum tracking error that can be achieved by a given rebalancing strategy, and what is the resulting portfolio composition?

In response to the first question, several authors have suggested decomposing the tracking error into various components (Asad-Syed *et al.*, 1999; Amman and Tobler, 2000; Scherer, 2001; Mina, 2002). The remaining questions, however, which relate to the second-stage decision, have received less attention.

Various techniques for managing value-based (rather than return-based) risk measures, such as value at risk (VaR) and expected shortfall, have been reported in the literature (Litterman, 1996; Garman, 1997; Mausser and Rosen, forthcoming). These tools are based on the trade risk profile (TRP), which plots the portfolio risk as a function of the position size (in terms of units or dollar value) for a particular asset or sub-portfolio. From the TRP, it is possible to compute the risk minimising, or best hedge, position for an asset as well as the marginal risk, which indicates the instantaneous rate of change in the overall risk due to trading the asset. The marginal risk, in turn, can be used to obtain an additive risk decomposition.

This paper applies the concepts of the TRP and its associated risk analytics to tracking error. A key extension of previous results is the incorporation of trading strategies in the analysis. Specifically, a closed-form expression of tracking error is first obtained for any given rebalancing rule. From this expression, several simple analytics are then derived, which help fund managers to evaluate the impact of different investment decisions on the risk of their portfolios. An illustrative example, using an actual fund and market data, demonstrates how these tools can be applied in practice. Finally, several suggestions are offered for how these tools might be developed further.

Tracking error computation

This section expresses tracking error as a function of a specified trading strategy. In the following, the term 'asset' is used in a general sense to represent a tradable entity from the interpretation of the fund manager. Thus, an asset may represent an individual security (eg a particular stock or bond) or a group of securities that are considered to be one tradable unit (eg a given sector of an equity index).

Background

Suppose that a fund and its benchmark, denoted P and B, respectively, are able to take positions in a set of n assets. For a specified time horizon, the simple return of fund P is the value-weighted average of the simple returns of its underlying assets

$$R^P = \sum_{j=1}^{n} w_j^P r_j \tag{1}$$

where r_j is the return of asset j over the time horizon and

$$\sum_{j=1}^{n} w_j^P = 1$$

The return of the benchmark R^B is computed in a similar manner. The active return of the fund is the difference between its return and that of the benchmark

$$AR = R^P - R^B$$

Given an appropriate sample of returns (eg a historical sample for *ex post* analysis or a set of Monte Carlo scenarios for *ex ante* analysis), the tracking error is the standard deviation of the active return. Specifically, if one lets Γ denote the $n \times n$ covariance matrix of asset returns, the tracking error can be computed as

$$TE = \sqrt{\boldsymbol{w}^T \boldsymbol{\Gamma} \boldsymbol{w}} \tag{2}$$

where

$$\boldsymbol{w} = \boldsymbol{w}^P - \boldsymbol{w}^B \tag{3}$$

denotes the weight differentials, or 'active' weights.

Trading strategies

Recall that the objective is to determine how changing the composition of portfolio P (ie the weights $\boldsymbol{w}^P$) is likely to affect the tracking error in the future. As a first step, it is necessary to represent a trading strategy in mathematical terms.

Let $\boldsymbol{w}^{P,0}$ and $\boldsymbol{w}^0$ denote the existing portfolio weights and the active weights, respectively (the benchmark weights are assumed to remain fixed at $\boldsymbol{w}^B$ during the period of interest). A trading strategy is represented by a so-called w-rule $\boldsymbol{q}$, which expresses the portfolio weights in terms of the parameter $\boldsymbol{\theta}$ as follows

$$\boldsymbol{w}^P(\boldsymbol{q}, \theta) = \boldsymbol{w}^{P,0} + \boldsymbol{q\theta} \tag{4}$$

Substituting Equation (4) into Equation (3) yields a similar parameterisation of the active weights

$$\boldsymbol{w}(\boldsymbol{q}, \theta) = \boldsymbol{w}^0 + \boldsymbol{q\theta} \tag{5}$$

Since the total weight of assets in any portfolio must equal one, it follows that

$$\sum_{j=1}^{n} q_j = 0$$

For example, suppose the portfolio manager wants to increase the weight of asset j in the portfolio. Moreover, each increase of 0.01 in w_j^P is to be offset by decreases of 0.0025 and 0.0075 in the weights of assets i and k, respectively. This trading strategy can be represented by a w-rule in which $\boldsymbol{q}_j = 1$, $q_i = -0.25$, $\boldsymbol{q}_k = -0.75$ and $\boldsymbol{q}_m = 0$ for all other assets m.

Note that the preceding w-rule is not unique; $\boldsymbol{q}$ and $\alpha\boldsymbol{q}$ are effectively equivalent for any constant $\alpha \neq 0$. Thus, a normalised w-rule is defined as one that satisfies

$$\sum_{j=1}^{n} |q_j| = 1$$

It is straightforward to verify that any non-trivial w-rule (ie other than $\boldsymbol{q} = 0$) can be normalised as follows

$$N(\boldsymbol{q}) = \frac{\boldsymbol{q}}{Q^+ - Q^-}$$

where

$$Q^+ = \sum_{j \mid q_j > 0} q_j$$

and

$$Q^- = \sum_{j \mid q_j < 0} q_j$$

From the definition above, it follows that $N(\boldsymbol{q}) = N(\alpha \boldsymbol{q})$ for $\alpha \neq 0$. Thus, in the sequel it is assumed, without loss of generality, that all non-trivial w-rules are normalised.

Tracking error for a trading strategy

Substituting Equation (5) into Equation (2) computes the tracking error associated with the w-rule $\boldsymbol{q}$ as a function of θ

$$TE(\boldsymbol{q},\theta) = \sqrt{(\boldsymbol{w}^0 + \boldsymbol{q}\theta)^T \boldsymbol{\Gamma} (\boldsymbol{w}^0 + \boldsymbol{q}\theta)} \tag{6}$$

Equation (6) can be written more compactly as

$$TE(\boldsymbol{q},\theta) = \sqrt{a_q \theta^2 + 2b_q \theta + c} \tag{7}$$

where

$$a_q = \boldsymbol{q}^T \boldsymbol{\Gamma} \boldsymbol{q}$$
$$b_q = \boldsymbol{q}^T \boldsymbol{\Gamma} \boldsymbol{w}^0$$

and

$$c = (\boldsymbol{w}^0)^T \boldsymbol{\Gamma} \boldsymbol{w}^0$$

Note that c (ie the current tracking error) depends only on the existing positions in the portfolio, while $\boldsymbol{a_q}$ and b_q depend on the trading strategy, as reflected by the w-rule $\boldsymbol{q}$.

Tracking error analytics

From Equation (7), it is straightforward to compute a number of useful risk analytics that can guide fund managers in their investment decisions. These include various trade risk profiles, best hedges, marginal tracking error and risk contributions.

Trade risk profile

The TRP for a given trading strategy plots tracking error against the parameter θ. For any non-trivial w-rule, the TRP has the characteristic shape shown in Figure 1. Each non-trivial w-rule has an associated risk-minimising, or best hedge, position that occurs for some value θ^*. The potential reduction in tracking error represents one possible criterion for evaluating different trading strategies. Alternatively, one might consider the rate of risk reduction afforded by a given trading strategy, or the so-called marginal tracking error, which corresponds to the slope of the curve at the point $\theta = 0$.

Managers also may find it insightful to view the relationship between tracking error and other portfolio characteristics. As discussed below, it is possible to obtain alternative profiles that plot tracking error against various functions of θ, such as portfolio return, trading cost and the weights of selected positions.

Best hedge position

Equation (7) has a minimum (which is unique if Γ is positive definite) at

$$\theta_q^* = -\frac{b_q}{a_q}$$

The corresponding weights

$$\boldsymbol{w}^P(\boldsymbol{q},\theta_q^*) = \boldsymbol{w}^{P,0} + \boldsymbol{q}\theta_q^* \tag{8}$$

are referred to as the best hedge position for the portfolio P under the w-rule $\boldsymbol{q}$. The tracking error at the best hedge position

$$TE(\boldsymbol{q},\theta_p^*) = \sqrt{c - \frac{b_q^2}{a_q}}$$

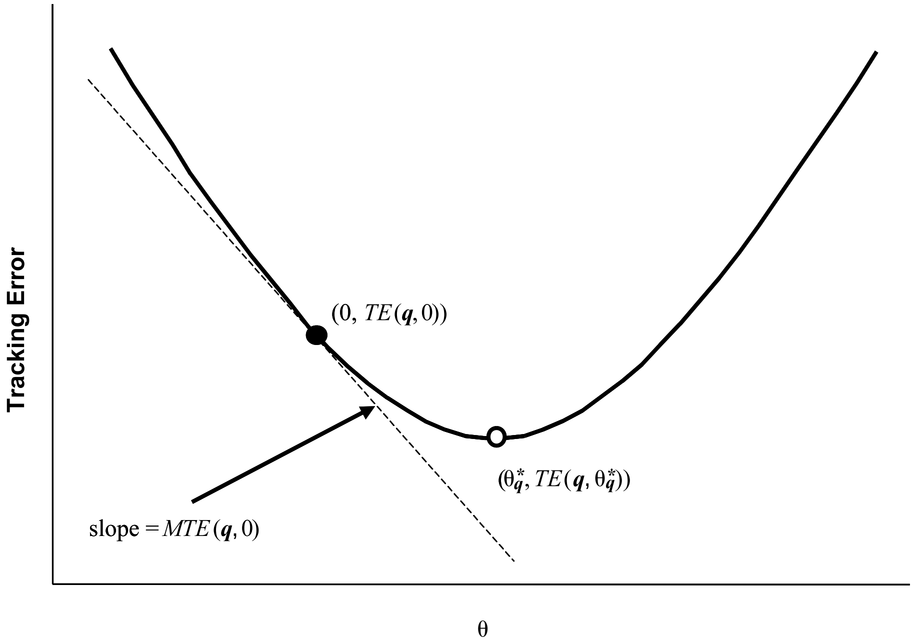

Figure 1 Trade risk profile

is the minimum risk that can be achieved by the trading strategy $\boldsymbol{q}$.

The w-rule that yields the greatest possible risk reduction does not necessarily represent the most desirable trading strategy for the portfolio manager. It is important to recognise the costs, in terms of return and volume of trading, associated with attaining the best hedge position. The former can be quantified by simply computing the difference between the expected returns at the best hedge and the existing positions. From Equations (1) and (8), the return difference is

$$\Delta R^P(\boldsymbol{q},\theta^*_q) = \theta^*_q \boldsymbol{q}^T \boldsymbol{r}$$

$MR(\boldsymbol{q}) = \boldsymbol{q}^{\mathrm{T}}\boldsymbol{r}$ is referred to as the marginal return of trading strategy $\boldsymbol{q}$.

The trading costs incurred to attain the best hedge position can be approximated by a suitable best hedge 'distance' metric, such as the total value of all trades (ie long and short trades do not offset). For a normalised w-rule, minimising the tracking error requires trades with a dollar value of

$$\Delta V^P(\boldsymbol{q},\theta^*_q) = |V^P\theta^*_q|$$

Alternatively, the required trading activity can be expressed as a proportion of the portfolio's value, which is simply the absolute value of θ^*_q.

Marginal tracking error

The marginal tracking error (MTE) for a given trading strategy is the derivative of tracking error with respect to θ (ie it corresponds to the slope of the curve in Figure 1 at the point $\theta = 0$)

$$MTE(\boldsymbol{q},0) = \left.\frac{dTE(\boldsymbol{q},\theta)}{d\theta}\right|_{\theta=0} = \left.\frac{a_q\theta + b_q}{TE(\boldsymbol{q},\theta)}\right|_{\theta=0} = \frac{b_q}{\sqrt{c}}$$

The MTE gives the instantaneous change in tracking error obtained by applying a given trading strategy. Thus, if $|MTE(\boldsymbol{q}',0)| > |MTE(\boldsymbol{q},0)|$, for example, trading strategy $\boldsymbol{q}'$ has a greater initial

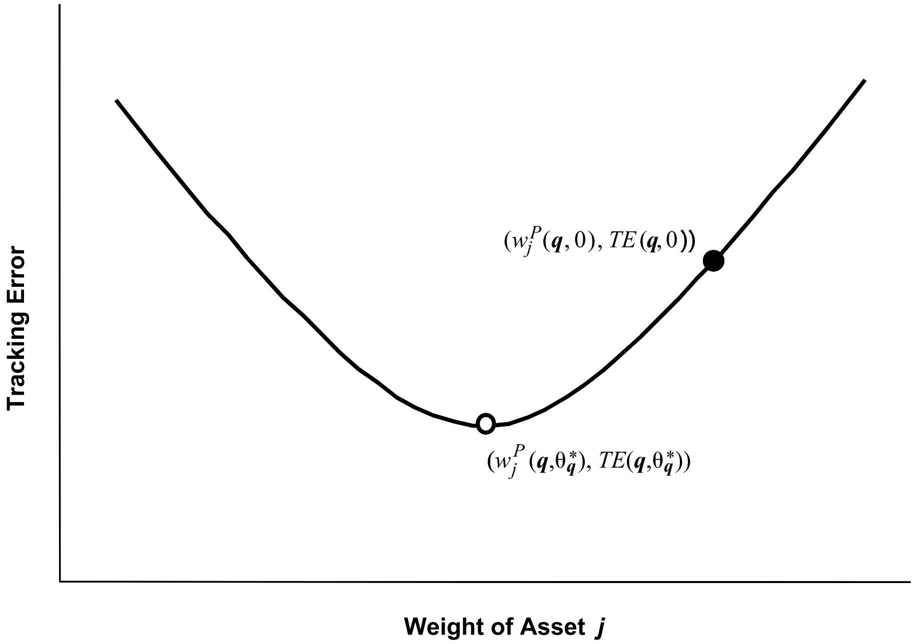

Figure 2 TRP for asset *j*

impact on tracking error than strategy $\boldsymbol{q}$ (although $\boldsymbol{q}'$ does not necessarily result in a smaller minimum tracking error than $\boldsymbol{q}$ at their respective best hedge positions). Note that the derivative can, of course, also be computed at points other than $\theta = 0$, if desired.

Alternative profiles

Suppose that a manager is interested in seeing how tracking error changes in relation to the weight of a particular position. This TRP can be obtained by simply replacing the independent variable θ in Figure 1 with the position weight $w_j^P(\boldsymbol{q},\theta)$. For example, Figure 2 shows a TRP for an 'active' asset j (ie $q_j \neq 0$). In contrast, the TRP for an 'inactive' asset j (ie $q_j = 0$) is a vertical line with a minimum at $(w_j^{P,0}, TE(\boldsymbol{q}, \theta^*_q))$.

Since weight is a linear function of θ (ie Equation 4), it follows that the TRP for an asset has the same characteristic shape as the curve in Figure 1.

The marginal tracking error is defined with respect to the weight of asset j to be the slope of its TRP at the initial position

$$\begin{aligned} MTE_j(\boldsymbol{q},0) &= \frac{1}{q_j}\left.\frac{dTE(\boldsymbol{q},\theta)}{d\theta}\right|_{\theta=0} \\ &= \frac{1}{q_j}\frac{b_q}{\sqrt{c}} \end{aligned} \tag{9}$$

Equation (9) gives the instantaneous rate of change in tracking error (ie corresponding to an infinitesimal change in weight) under a specified trading strategy. Note, however, that MTE_j is not a partial derivative in a strict sense, ie

$$MTE_j(\boldsymbol{q},\theta) \neq \frac{\partial TE(\boldsymbol{q},\theta)}{\partial w_j^P(\boldsymbol{q},\theta)}$$

since the weights of all positions other than j do not remain constant.

The marginal tracking error is distinguished from a related measure, known as 'tracking error delta', which indicates the change in tracking error due to a weight adjustment of a specified (non-infinitesimal) size. For example, suppose a manager is interested in the effect of increasing the weight of asset j by 0.01 under the w-rule $\boldsymbol{q}$. In this case, the tracking error delta is computed as

$$\Delta TE_j(\boldsymbol{q},0) = TE(\boldsymbol{q},\tilde{\boldsymbol{\theta}}) - TE(\boldsymbol{q},0)$$

where $\tilde{\boldsymbol{\theta}}$ satisfies

$$w_j^P(\boldsymbol{q},\tilde{\boldsymbol{\theta}}) = w_j^{P,0} + 0.01$$

Tracking error contribution

By identifying which positions constitute the most significant risks in a portfolio, risk contributions can help managers construct trading strategies. As noted previously, several papers describe how risk contributions can be calculated from an additive decomposition of the tracking error. Essentially, one uses the fact that the variance of the active return satisfies

$$\mathrm{var}(AR) = \sum_{j=1}^{n} w_j \,\mathrm{cov}(r_j, AR)$$

Since tracking error is the square root of the variance above, it follows that

$$TE = \sum_{j=1}^{n} w_j \frac{\mathrm{cov}(r_j, AR)}{TE}$$

as long as the tracking error is non-zero. The tracking error contribution of asset j is

$$CTE_j = w_j \frac{\mathrm{cov}(r_j, AR)}{TE} \tag{10}$$

From Equation (10), it follows that an asset's contribution can be positive, negative or zero, where the latter can occur only if the active weight is zero or if the asset's return is uncorrelated with the active return.

One can compute the tracking error contribution of asset j for any portfolio obtained under the trading strategy q as follows

$$CTE_j(\boldsymbol{q},\boldsymbol{\theta}) = (w_j + q_j\boldsymbol{\theta}).\frac{(\boldsymbol{w} + \boldsymbol{q}\boldsymbol{\theta})^T \boldsymbol{\Gamma}_{(j)}}{TE(\boldsymbol{q},\boldsymbol{\theta})}$$

Table 1 Fund and benchmark risk and return, %

Fund reurn (expected)	22.26
Fund return standard deviation	25.14
S&P500 return (expected)	5.98
S&P500 return standard deviation	20.39
Active return (expected)	16.28
Tracking error	12.54

where $\Gamma_{(j)}$ is the jth column of the variance covariance matrix. It follows that the total contribution of a group of assets is simply the sum of the individual asset contributions.

Example

For demonstration purposes, the methodology is used to improve the *ex ante* risk/return characteristics of an actual Large Growth mutual fund that is benchmarked against the S&P500 Index. Table 1 reports the (expected) annualised risk and return of the fund and the benchmark, computed using six years of historical monthly asset return data and based on their composition as of 31st December, 2003.

As indicated in Table 2, the fund's performance is driven mainly by the large overweight position in EBAY, which contributes approximately half of both the active return and the tracking error. The second most significant impact on tracking error is due to the portion of the benchmark (denoted SP500*) that is not held in the portfolio. The negative contribution implies that excluding these assets actually reduces the tracking error. Since the active weight of SP500* is negative, it follows from Equation (10) that $\mathrm{cov}(r_{SP500\star}, AR)$ is positive, or alternatively, that the covariance of its return with the return of the portfolio exceeds its covariance with the return of the benchmark, ie $\mathrm{cov}(r_{SP500\star}, R^P) > \mathrm{cov}(r_{SP500\star}, R^B)$.

The decomposition of tracking error suggests selling shares in EBAY as an

Table 2 Asset weights and contributions to risk and return

Asset	TE contrib. (CTE)	Fund weight (w^P)	S&P500 weight (w^B)	Active weight (w)	Asset return (r)	Fund return (w^Pr)	S&P500 return (w^Br)	Active return (wr)
EBAY	6.79	10.45	0.06	10.39	74.51	7.79	0.04	7.74
ERTS	1.17	5.3	0.25	5.05	29.01	1.54	0.07	1.47
DNA	1.07	5.61	0.00	5.61	35.67	2.00	0.00	2.00
AMZN	0.67	1.66	0.00	1.66	60.59	1.01	0.00	1.01
UNH	0.57	9.16	0.75	8.41	24.97	2.29	0.19	2.10
GS	0.47	4.07	0.15	3.92	13.16	0.54	0.02	0.52
TXN	0.43	3.17	0.01	3.16	15.37	0.49	0.00	0.49
VOD	0.29	2.51	0.00	2.51	4.27	0.11	0.00	0.11
BAC	0.28	5.9	0.08	5.82	3.76	0.22	0.00	0.22
LVLT	0.27	0.74	0.00	0.74	5.43	0.04	0.00	0.04
MSFT	0.23	5.26	0.09	5.17	3.47	0.18	0.00	0.18
DISH	0.22	0.88	0.00	0.88	58.26	0.51	0.00	0.51
QCOM	0.20	1.6	0.12	1.48	51.39	0.82	0.06	0.76
YHOO	0.18	1.17	0.29	0.88	50.04	0.59	0.15	0.44
TWX	0.17	1.54	0.15	1.39	21.57	0.33	0.03	0.30
CHTR	0.12	1.69	0.00	1.69	–25.62	–0.43	0.00	–0.43
NOK	0.09	1.06	0.00	1.06	27.52	0.29	0.00	0.29
KSS	0.09	1.97	0.22	1.75	10.68	0.21	0.02	0.19
WLP	0.08	1.43	0.05	1.38	21.28	0.30	0.01	0.29
SLM	0.08	3.3	0.11	3.19	14.50	0.48	0.02	0.46
AZO	0.08	1.77	0.25	1.52	14.67	0.26	0.04	0.22
PG	0.08	3.13	0.09	3.04	3.06	0.10	0.00	0.09
CFC	0.06	2.05	0.07	1.98	11.15	0.23	0.01	0.22
NXTL	0.05	0.5	0.16	0.34	23.27	0.12	0.04	0.08
NKE	0.02	4.62	0.05	4.57	9.19	0.42	0.00	0.42
LLY	0.02	0.56	0.24	0.32	–3.83	–0.02	–0.01	–0.01
MMM	0.01	3.26	0.25	3.01	8.11	0.26	0.02	0.24
VIA.B	0.00	0.68	0.00	0.68	5.68	0.04	0.00	0.04
Tbill	0.00	8.64	0.00	8.64	3.52	0.30	0.00	0.30
FRX	–0.01	2.55	0.25	2.30	35.90	0.92	0.09	0.83
KMX	–0.01	0.49	0.00	0.49	34.35	0.17	0.00	0.17
WFC	–0.06	3.28	0.38	2.90	5.18	0.17	0.02	0.15
SP500*	–1.18	0	95.93	–95.93	5.36	0.00	5.15	–5.15
Total	12.54	100	100.00	0.00		22.26	5.98	16.28

intuitive way of reducing risk. A particularly simple strategy might use the proceeds of such sales to buy Treasury Bills (ie an allocation to the 'cash' position). To evaluate this strategy, a manager can perform a so-called 'what-if' analysis to determine the effect of selling a given portion of the EBAY holdings. For example, reducing the weight of EBAY by 5 per cent (to 5.45 per cent) lowers the tracking error by 2.74 per cent (to 9.80 per cent). Upon obtaining this result, the manager then may decide to proceed with the given trade.

In contrast to this type of 'what-if' analysis, which considers only a single trade size, expressing the trading strategy as a w-rule allows a complete trade risk profile to be generated. In this case, the strategy is represented by the normalised trading rule $\boldsymbol{q1}$, which has only two non-zero elements: $q1_{\text{EBAY}} = -0.5$ and $q1_{\text{Tbill}} = 0.5$.

The resulting trade risk profile for EBAY (Figure 3) includes not only the 'what-if' trade (ie the triangular icon), but also shows that tracking error can be reduced further if more EBAY shares are sold. In particular, at the best hedge position, the weight of EBAY is reduced by 9 per cent, leading to a 3.57 per cent decline in tracking error. The corresponding parameter value $\theta^*_{q1} = 0.18067$ implies that 18.07 per cent of the portfolio's value must be traded in order to attain the best hedge

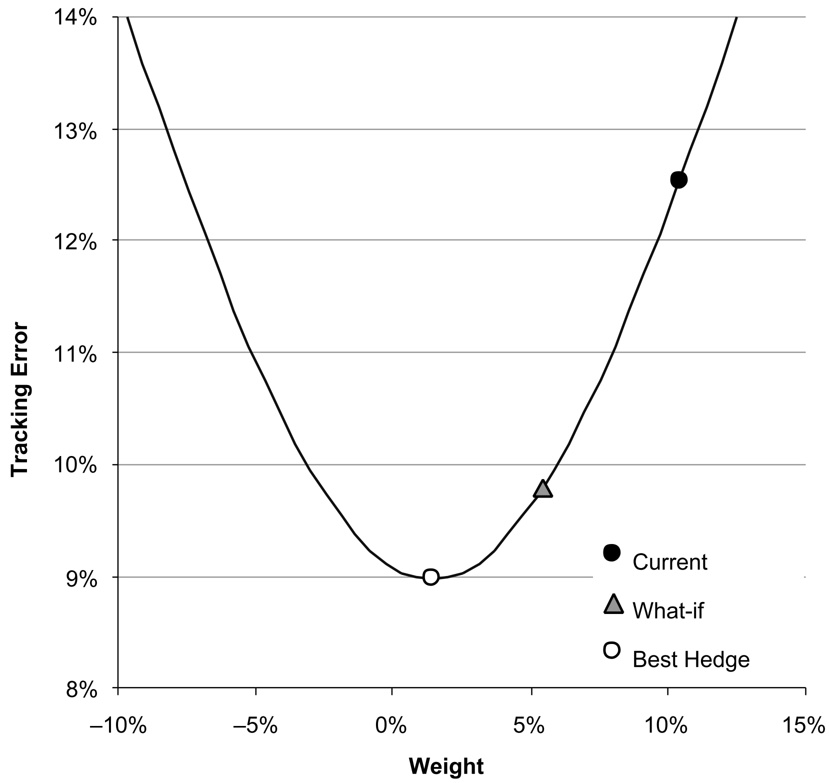

Figure 3 TRP for EBAY as defined by ***q1***

position. Both the what-if and the best hedge trades, however, reduce risk at the expense of active return (eg the return difference at the best hedge is $\Delta R^P(\boldsymbol{q1}, \theta^*_{q1}) = -6.41$ per cent). This is consistent with the fact that the given trading strategy has a negative marginal return ($MR(\boldsymbol{q1}) = -35.49$).

An alternative trading strategy might seek to trade various assets in a way that earns additional return while selling EBAY. A second trading rule, ***q2***, sells assets whose expected return is negative (CHTR and LLY) along with EBAY, and purchases AMZN, which has a relatively high expected return. Specifying $q2_{\mathrm{EBAY}} = -0.1$, $q2_{\mathrm{CHTR}} = -0.2$, $q2_{\mathrm{LLY}} = -0.2$ and $q2_{\mathrm{AMZN}} = 0.5$ results in a positive marginal return for ***q2***.

Perhaps surprisingly, however, one finds that ***q2*** has a positive marginal tracking error ($MTE(\boldsymbol{q2},0) = 13.78$) and the best hedge occurs at $\theta^*_{q2} = -0.07365$; reducing tracking error under this trading strategy entails purchasing, not selling, shares of EBAY. Thus, the positive marginal return cannot be exploited in this case.

Clearly, to reduce risk while simultaneously increasing expected return, a trading strategy requires a marginal tracking error and a marginal return that are of opposite sign. One way to obtain such a strategy is to identify a *w*-rule ***q*** whose marginal return is zero (ie by solving $\boldsymbol{q}^{\mathrm{T}}\boldsymbol{r} = 0$) and that significantly reduces risk, as might be reflected by a large marginal tracking error or an attractive best hedge. Given such a rule, it is then possible to make slight adjustments to ***q*** in order to obtain a marginal return that is of the desired sign.

Using this approach, one obtains the trading strategy ***q3***, which sells EBAY and

Table 3 Characteristics of trading strategies

	q1	*q2*	*q3*
Marginal tracking error	–33.88	13.78	–14.83
Marginal return	–35.49	28.74	2.34
Best hedge parameter (θ^*)	0.18067	–0.07365	0.10519
Best hedge tracking error	8.97	12.02	11.74
Change in tracking error	–3.57	–0.52	–0.81
Best hedge return	15.85	20.14	22.51
Change in return	–6.41	–2.12	0.25
Best hedge trade (% portfolio)	18.07	7.36	10.52
EBAY	–9.033	0.736	–3.945
AMZN	0.000	–3.682	0.000
DISH	0.000	0.000	2.630
YHOO	0.000	0.000	2.630
CHTR	0.000	1.473	–1.315
LLY	0.000	1.473	0.000
Tbill	9.033	0.000	0.000

CHTR and buys DISH and YHOO as follows: $\boldsymbol{q3}_{\mathrm{EBAY}} = -0.375$, $\boldsymbol{q3}_{\mathrm{CHTR}} = -0.125$, $\boldsymbol{q3}_{\mathrm{DISH}} = 0.250$ and $\boldsymbol{q3}_{\mathrm{YHOO}} = 0.250$. In this case, the marginal tracking error is negative while the marginal return is positive. Adopting the best hedge position reduces the tracking error by 0.81 per cent and increases the active return by 0.25 per cent.

The main characteristics of the three trading strategies are summarised in Table 3 (only those assets actually traded are listed). It is apparent that ***q1*** yields the greatest potential reduction in tracking error, but at considerable cost in terms of both return and trading volume. In contrast, ***q3*** delivers a smaller improvement in tracking error while increasing return, at a lower trading volume.

Comparing various trade risk profiles generated by the trading strategies provides further insights into their relative performance. For example, a plot of tracking error versus change in return (Figure 4) clarifies the risk/return trade-offs for alternative strategies. In particular, a trading strategy whose best hedge position is to the right of zero (eg ***q3***) improves both risk and return. Moreover, it follows that the current portfolio is inefficient (in the sense of tracking the benchmark) with respect to such strategies, since it is possible to obtain a higher return at the same level of tracking error.

Another important consideration when evaluating a strategy is the trading volume required to attain a certain reduction in risk. To analyse this aspect of the problem, Figure 5 plots tracking error against trading activity, expressed as a proportion of the portfolio value (ie $|\theta|$). It is apparent that ***q1*** is the most efficient strategy in this sense, since it produces the greatest risk reduction for a given trading volume.

Since the trading strategies were motivated largely by the tracking error contributions, it is interesting to examine the contributions at various best hedge positions. These contributions may then form the basis for additional refinements to the trading strategy. Figure 6 compares the relative tracking error contributions for the initial portfolio with those that exist at the best hedge positions for ***q1*** and ***q3*** (to improve clarity, only those assets that contribute at least 5 per cent of the total tracking error are shown).

While EBAY initially accounts for more than 50 per cent of the total tracking error, its contribution is almost zero (0.40 per cent) at the best hedge

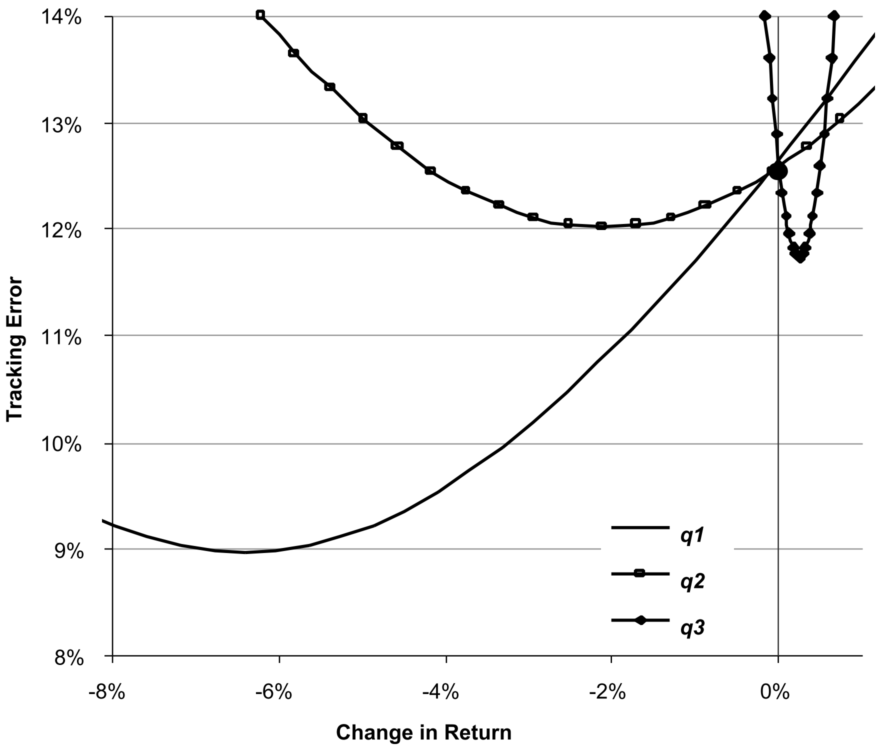

Figure 4 TRP's plotting tracking error against changes in return

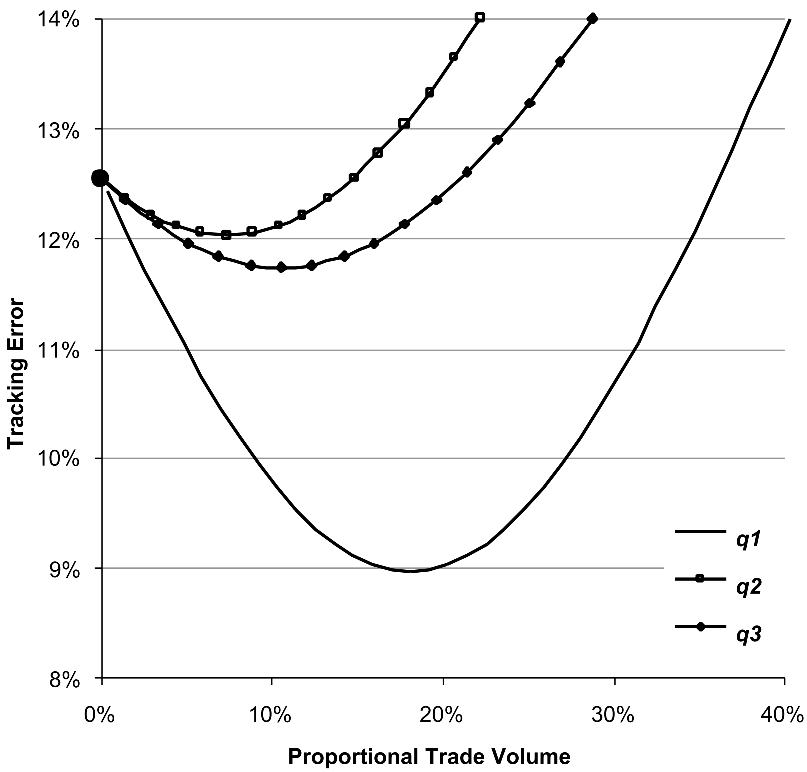

Figure 5 TRP's plotting tracking error against trading activity

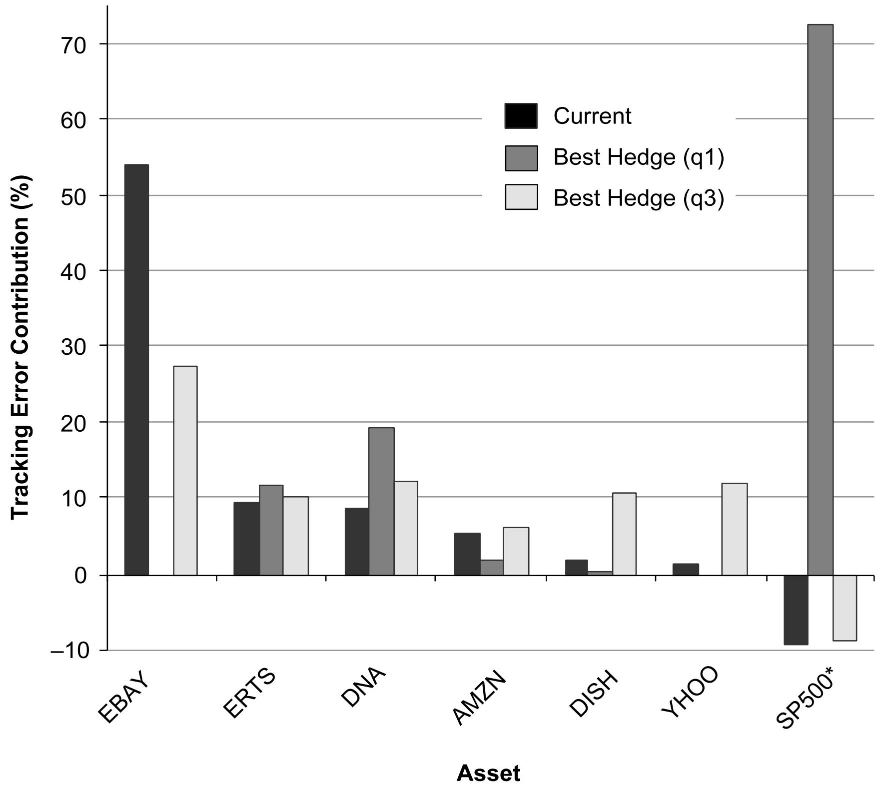

Figure 6 Relative contribution to tracking error

position for ***q1***. Since ***q1*** trades EBAY for a practically riskless asset in the form of T-bills, tracking error is minimised when the active weight of EBAY is close to zero in this case. The most significant contributor to tracking error at the best hedge position, by far, is the group of benchmark assets absent from the portfolio. Thus, adjusting the trading strategy to include these assets may be appropriate to obtain further risk reductions. In contrast, at the best hedge position for ***q3***, the risk is distributed more evenly among the assets in the portfolio.

Concluding remarks

In evaluating trading strategies, fund managers can benefit from an understanding of their potential impact on tracking error. The TRP, which can be constructed easily from a covariance matrix of asset returns, is the basis for several simple tools that assist in this analysis: best hedges, marginal risk and tracking error contributions.

The methodology described in this paper can be extended in several ways. For example, it is straightforward to incorporate simple trading restrictions, such as preventing short positions, in the analysis. In this case, the TRP is truncated to reflect only those portfolios that are acceptable. Alternatively, it is possible to construct a profile showing an asset's tracking error contribution as a function of the trade size.

This methodology also facilitates a consolidated risk analysis for multiple investment decisions. For example, a senior manager can analyse the trading strategies used in each sub-portfolio separately (ie multiple ***q***s and θ's or on an aggregate basis (ie sum of ***q***s and a single θ). Thus, it is possible to compare the best hedge position of each sub-portfolio with an overall best hedge for instance. The decomposition of trades into individual investment decisions, as proposed by Scherer (2001) in an *ex post* framework, is also an interesting possibility for *ex ante* analysis.

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