INTRODUCTION
Managing tracking error on an *ex ante* basis requires an ability to assess the possible effects of trades on a fund’s performance relative to its benchmark. Given a trading strategy, its potential for reducing tracking error must be balanced against trading costs and return expectations. This chapter presents several simple diagnostic tools to help fund managers evaluate alternative trading strategies in this regard. Specifically, expressing tracking error as a function of the trading strategy allows for the construction of various Trade Risk Profiles (TRPs), which clarify the relationships among tracking error, risk contributions, expected returns and trading activity. Moreover, such profiles can readily incorporate trading restrictions as necessary to ensure an acceptable portfolio structure. An illustrative example demonstrates the practical application of these techniques.

In recent years there has been an increased focus on tracking error as the means for measuring and controlling risk in actively managed funds. This is a result of several important investor trends:

- a growing awareness of risk management methodologies;
- the demand for additional risk reporting and disclosure from fund managers;
- investor-mandated risk limits relative to standard or custom benchmarks.
While tracking error is routinely used to monitor a fund’s performance, it often receives limited consideration in the investment decision-making process itself. Undoubtedly, this is due in part to a lack of appropriate tools for assessing the potential impact of trades on tracking error. This chapter describes several simple diagnostic tools to assist in the proactive management of a fund’s tracking error.

Tracking error, defined as the standard deviation of a fund’s active return relative to a specified benchmark, measures the relative volatility of the fund. Traditionally, it is computed in an *ex post* fashion from historical return data (e.g., six years of actual monthly returns for a fund and its benchmark). In contrast, incorporating tracking error in the investment process requires an *ex ante* view – the objective is to assess the fund’s risk over some future period. In this case, the asset returns are stochastic (the true returns are unknown since they have yet to be realised) and the active return is a random variable. Thus, *ex ante* analysis requires a model of potential future returns, which may or may not be derived from historical data, to provide a basis for evaluating investment decisions.

Making an investment decision is typically a two-stage process. The first stage involves selecting a trading strategy: identifying which, and in what proportion, securities are to be bought or sold (e.g., purchase shares of IBM, funded equally from cash and by selling long-term government bonds). Given a trading strategy, the second stage determines the extent of trading to be done (e.g., how many shares of IBM should be purchased?). As part of this process, fund managers typically face the following questions.

- Which position is currently the largest contributor to the tracking error?
- How is the tracking error likely to change as a result of a particular trading strategy?
- What is the minimum tracking error that can be achieved by a given trading strategy and what is the resulting portfolio composition?

In response to the first question, several authors have suggested decomposing the tracking error into various components (Asad-Syed *et al* 1999; Amman and Tobler 2000; Scherer 2001; Mina 2002).
The remaining questions, which relate to the second-stage decision, were considered recently by Burmeister et al (2005), who combined concepts from Value-at-Risk (VAR) management with concise representations of trading strategies to obtain relevant tools for managing tracking error. These tools are based on the TRP introduced by Litterman (1996), which plots the portfolio risk as a function of the position size for a particular asset or sub-portfolio. From the TRP, it is possible to compute the risk-minimising, or best hedge, position for an asset as well as the marginal risk, which indicates the instantaneous rate of change in the overall risk due to trading the asset. Moreover, it is straightforward to obtain TRPs that show the relationship between tracking error and other relevant portfolio characteristics, such as expected return and trading volume.

One limitation of the techniques proposed in Burmeister et al (2005) is that they assume trading activity is unrestricted. In practice, fund managers often face certain constraints when rebalancing their portfolios, which limit the extent to which a trading strategy may be executed (e.g., it may not be possible to take short positions in certain securities). This chapter shows that various trading restrictions can be included in the analysis through a simple extension of the existing methodology.

The following sections first review the results of Burmeister et al (2005), deriving relevant TRPs and associated risk analytics for evaluating trading strategies on an ex ante basis. Trading restrictions are then introduced and shown to translate into bounds on an underlying trading parameter. An illustrative example demonstrates the practical application of the methodology.

**TRACKING ERROR COMPUTATION**

This section expresses tracking error as a function of a specified trading strategy. In the following, the term *asset* broadly represents any tradable entity from the interpretation of the fund manager (i.e., an individual security or a group of securities that are considered to be one tradable unit, such as a sector of an equity index).

**Background**

Suppose that a fund and its benchmark, denoted \( P \) and \( B \) respectively, are able to take positions in a set of \( n \) assets. For a specified
time horizon, the simple return of the fund $P$ is the value-weighted average of the simple returns of its underlying assets

$$R^P = \sum_{j=1}^{n} w^P_j r_j$$

(1)

where $r_j$ is the return of asset $j$ over the time horizon and

$$\sum_{j=1}^{n} w^P_j = 1$$

The return of the benchmark ($R^B$) is computed in a similar manner. The active return of the fund is the difference between its return and that of the benchmark

$$AR = R^P - R^B$$

Tracking error is the standard deviation of the active return, as computed from an historical sample for ex post analysis or an appropriate model for ex ante analysis. Specifically, if $\Gamma$ denotes the $n \times n$ covariance matrix of asset returns, then the tracking error is

$$TE = \sqrt{w^T \Gamma w}$$

(2)

where

$$w = w^P - w^B$$

(3)

denotes the weight differentials or “active” weights.

Trading strategies

In order to determine how changing the composition of the portfolio $P$ (ie, the weights $w^P$) affects the tracking error in the future, it is first necessary to represent a trading strategy in mathematical terms.

Let $w^{P,0}$ and $w^0$ denote the existing portfolio weights and the active weights, respectively (the benchmark weights are assumed to remain fixed at $w^B$ during the period of interest). A trading strategy is represented by a so-called $w$-rule $q$, which expresses the portfolio weights in terms of the parameter $\theta$ as follows:

$$w^P(q, \theta) = w^{P,0} + q\theta$$

(4)
Substituting Equation (4) into Equation (3) yields a similar parameterisation of the active weights

$$w(q, \theta) = w^0 + q\theta$$  \hspace{1cm} (5)

Since the total weight of assets in any portfolio must equal one, it follows that

$$\sum_{j=1}^{n} q_j = 0$$

For example, suppose the portfolio manager intends to increase the weight of asset $j$ in the portfolio, with each increase of 0.01 in $w^j$ offset by decreases of 0.0025 and 0.0075 in the weights of assets $i$ and $k$, respectively. This trading strategy can be represented by a $w$-rule in which $q_j = 1$, $q_i = -0.25$, $q_k = -0.75$ and $q_m = 0$ for all other assets $m$.

Since the $w$-rules $q$ and $\alpha q$ are effectively equivalent for any constant $\alpha \neq 0$, it is useful to define a normalised $w$-rule as one that satisfies

$$\sum_{j=1}^{n} |q_j| = 1$$

Any non-trivial $w$-rule (ie, other than $q = 0$) can be normalised as follows

$$N(q) = \frac{q}{Q^* - Q^-}$$

where

$$Q^* = \sum_{j|q_j > 0} q_j$$

and

$$Q^- = \sum_{j|q_j < 0} q_j$$

From the definition above, it follows that $N(q) = N(\alpha q)$ for $\alpha \neq 0$. Thus, the sequel assumes, without loss of generality, that all non-trivial $w$-rules are normalised.
Tracking error for a trading strategy
Substituting Equation (5) into Equation (2) computes the tracking error associated with the \( w \)-rule \( q \) as a function of \( \theta \):

\[
TE(q, \theta) = \sqrt{(w^0 + q\theta)^T \Gamma (w^0 + q\theta)}
\]

Equation (6) can be written more compactly as

\[
TE(q, \theta) = \sqrt{a_q \theta^2 + 2b_q \theta + c}
\]

where

\[
a_q = q^T \Gamma q \\
b_q = q^T \Gamma w^0
\]

and

\[
c = (w^0)^T \Gamma w^0
\]

Note that \( c \) (i.e., the current tracking error) depends only on the existing positions in the portfolio while \( a_q \) and \( b_q \) depend on the trading strategy \( q \).

TRACKING ERROR ANALYTICS
From Equation (7) it is straightforward to compute a number of useful risk analytics that can guide fund managers in their investment decisions. These include various TRPs, best hedges, marginal tracking error and risk contributions.

Trade risk profile
The TRP for a given trading strategy plots tracking error against the parameter \( \theta \). For any non-trivial \( w \)-rule, the TRP has the characteristic shape shown in Figure 1. Each non-trivial \( w \)-rule has an associated risk-minimising, or best hedge, position that occurs for some value \( \theta^* \). The potential reduction in tracking error represents one possible criterion for evaluating different trading strategies. Alternatively, one might consider the rate of risk reduction afforded by a given trading strategy, or the so-called marginal tracking error, which corresponds to the slope of the curve at the point \( \theta = 0 \).
Managers also may find it insightful to view the relationship between tracking error and other portfolio characteristics. As discussed below, it is possible to obtain alternative profiles that plot tracking error against various functions of $\theta$, such as portfolio return, trading cost and the weights of selected positions.

**Best hedge position**

Equation (7) has a minimum (which is unique if $\Gamma$ is positive definite) at

$$
\theta^*_q = -\frac{b_q}{a_q}
$$

We refer to the corresponding weights

$$
\omega^p(q, \theta^*_q) = \omega^{p,0} + q\theta^*_q
$$

(8)

as the best hedge position for the portfolio $P$ under the $w$-rule $q$.

The tracking error at the best hedge position,

$$
TE(q, \theta^*_q) = \sqrt{c - \frac{b_q^2}{a_q}}
$$

is the minimum risk that can be achieved by the trading strategy $q$. 

Figure 1  Trade risk profile
The \( w \)-rule that yields the greatest possible risk reduction does not necessarily represent the most desirable trading strategy for the portfolio manager. It is important to recognise the costs, in terms of return and volume of trading, associated with attaining the best hedge position. The former can be quantified by simply computing the difference between the expected returns at the best hedge and the existing positions. From Equations (1) and (8), the return difference is

\[
\Delta R^p(q, \theta^*_q) = \theta^*_q q^r r
\]

We refer to \( MR(q) = q^r r \) as the marginal return of trading strategy \( q \).

The trading costs incurred to attain the best hedge position can be approximated by a suitable best hedge “distance” metric, such as the total value of all trades. For a normalised \( w \)-rule, minimising the tracking error entails trading a dollar value of

\[
\Delta V^p(q, \theta^*_q) = |V^p \theta^*_q|
\]

Alternatively, \( |\theta^*_q| \) expresses the required trading activity as a proportion of the portfolio’s value.

**Marginal tracking error**

The marginal tracking error (MTE) for a given trading strategy is the derivative of tracking error with respect to \( \theta \) (ie, it corresponds to the slope of the curve in Figure 1 at the point \( \theta = 0 \)):

\[
MTE(q, 0) = \frac{dTE(q, \theta)}{d\theta} \bigg|_{\theta=0} = \frac{a_q \theta + b_q}{TE(q, \theta)} \bigg|_{\theta=0} = \frac{b_q}{\sqrt{c}}
\]

The MTE gives the instantaneous change in tracking error that results from a given trading strategy. Thus, if \( |MTE(q’, 0)| > |MTE(q, 0)| \) for example, then trading strategy \( q’ \) has a greater initial impact on tracking error than strategy \( q \) (although \( q’ \) does not necessarily result in a smaller minimum tracking error than \( q \) at their respective best hedge positions).
Alternative profiles
Suppose that a manager is interested in seeing how tracking error changes in relation to the weight of a particular position. This TRP can be obtained by simply replacing the independent variable $w^j$ in Figure 1 with the position weight $w_j$. For example, Figure 2 shows a TRP for an “active” asset $j$ (ie, $q_j \neq 0$). In contrast, the TRP for an “inactive” asset $j$ (ie, $q_j = 0$) is a vertical line with a minimum at $(w_j^*, 0^*)$.

The marginal tracking error with respect to the weight of asset $j$ is the slope of its TRP at the initial position

$$MTE_j(q, 0) = \frac{1}{q_j} \frac{dTE(q, 0)}{dq} \bigg|_{q=0} = \frac{1}{q_j} \frac{b_j}{\sqrt{c}}$$

The marginal tracking error differs from a related measure, known as “tracking error delta”, which indicates the change in tracking error due to a weight adjustment of a specified (non-infinitesimal) size. For example, if a manager is interested in the effect of increasing the weight of asset $j$ by 0.01 under the $w$-rule $q$, the tracking error delta is computed as

$$\Delta TE_j(q, 0) = TE(q, \tilde{\theta}) - TE(q, 0)$$

where $\tilde{\theta}$ satisfies

$$w_j(q, \tilde{\theta}) = w_j^*, 0 + 0.01$$
Tracking error contribution

By identifying which positions constitute the most significant risks in a portfolio, risk contributions can help managers construct trading strategies. The tracking of error contributions relies on the fact that the variance of the active return satisfies

\[ \text{var}(AR) = \sum_{j=1}^{n} w_j \text{cov}(r_j, AR) \]

Since tracking error is the square root of the variance above, it follows that

\[ TE = \sum_{j=1}^{n} w_j \frac{\text{cov}(r_j, AR)}{TE} \]

as long as the tracking error is non-zero. The tracking error contribution of asset \( j \) is

\[ CTE_j = w_j \frac{\text{cov}(r_j, AR)}{TE} \] \hspace{1cm} (9)

From Equation (9) it follows that an asset’s contribution can be positive, negative or zero, where the latter can occur only if the active weight is zero or if the asset’s return is uncorrelated with the active return.

The tracking error contribution of asset \( j \) for any portfolio obtained under the trading strategy \( q \) is given by

\[ CTE_j(q, \theta) = (w_j + q_j \theta) \cdot \frac{(w + q \theta)^T \Gamma_{(j)}}{TE(q, \theta)} \]

where \( \Gamma_{(j)} \) is the \( j \)th column of the variance covariance matrix.

Trading constraints

While Equation (8) identifies the best hedge position for a given trading rule, the preceding analysis does not consider whether the
composition of the best hedge portfolio meets the requirements of the manager. For example, it may entail unacceptably large long or short positions in certain assets. In practice, a fund manager may want to impose certain restrictions, or constraints, on the portfolio’s structure when assessing a trading rule.

Since a trading rule expresses the weights of all assets in the portfolio in terms of a single parameter $\theta$ (Equation (5)), various restrictions on the portfolio can be translated into equivalent restrictions on $\theta$. This has the effect of limiting the range of acceptable values of $\theta$ to some interval $[\theta_{min}, \theta_{max}]$ (when there are no restrictions, the interval extends from negative infinity to positive infinity).

For example, suppose an asset $j$, whose initial weight is $w_{j,0}^{P,0} = 0.05$, must never represent more than 25% of a fund’s value. Given a trading rule with $q_j = 0.10$, it follows from Equation (5) that this restriction implies

$$0.05 + 0.10 \times \theta \leq 0.25$$

or, equivalently, $\theta \leq 2$.

Similarly, it is possible to place restrictions on the relative weights of two asset classes. Suppose that the manager of a balanced fund requires the total weight of stocks to be at least twice that of bonds. Consider a trading strategy that satisfies

$$\sum_{j \in \text{stocks}} q_j > 2 \sum_{j \in \text{bonds}} q_j$$

(11)

In this case, the resulting constraint

$$\frac{1}{\theta} \left[ \sum_{j \in \text{stocks}} w_{j,0}^{P,0} + \left( \sum_{j \in \text{bonds}} q_j \right) \theta \right] \geq 2 \left[ \sum_{j \in \text{stocks}} w_{j,0}^{P,0} + \left( \sum_{j \in \text{bonds}} q_j \right) \theta \right]$$

(12)

leads to a lower bound for $\theta$:

$$\theta \geq \frac{2 \sum_{j \in \text{stocks}} w_{j,0}^{P,0} - \sum_{j \in \text{bonds}} w_{j,0}^{P,0}}{\sum_{j \in \text{stocks}} q_j - 2 \sum_{j \in \text{bonds}} q_j}$$

(13)
More generally, any linear function of the weights can be handled in this manner, with each valid restriction yielding an upper or lower bound for \( \theta \). The interval \([\theta_{\text{min}}, \theta_{\text{max}}]\) is obtained by setting \( \theta_{\text{min}} \) to the largest lower bound and \( \theta_{\text{max}} \) to the smallest upper bound.

**EXAMPLE**

As an illustrative example, we use the methodology to improve the ex ante risk/return characteristics of an actual large growth mutual fund that is benchmarked against the S&P 500 Index. Here we are extending the example in Burmeister et al (2005) in two important ways. First, we aggregate the individual securities in the large growth portfolio and the S&P 500 Index into sectors and we restrict the trading to the sector level (individual securities within any given sector are traded in proportion to their sector weight). Second, we place limits on sector weights. Table 1 reports the (expected) annualised risk and return of both the fund and the benchmark aggregated into 9 sectors and a cash group based on their composition as of 31st December 2003. The risk and return are computed using six years of historical monthly asset returns.

As indicated in Table 1, the fund’s performance is driven mainly by the large overweight position in the Communications sector, which contributes approximately one half of the active return and approximately 70% the tracking error. The other two sectors that drive the return and tracking error of the portfolio are Consumer Non-Cyclical and Technology. Together, these three sectors account for approximately 85% of the return and 95% of the total tracking error for the portfolio. The active weight for both the Consumer Non-Cyclical and the Technology sectors is less than 2% suggesting that excess return and additional tracking error is a result of security selection rather than sector allocations.

The decomposition of tracking error suggests reducing the weight of the Communications sector as an intuitive way of reducing risk. A particularly simple strategy might sell a percentage of the Communications and invest the proceeds in cash. To evaluate this strategy, a manager can perform a so-called “what-if” analysis to determine the effect of selling a given percentage of the Communications sector. For example, reducing the weight of the Communication sector by 5% (to 20.92%) and increasing the
### Table 1  Sector weights and contributions to risk and return

<table>
<thead>
<tr>
<th>Sectors</th>
<th>Fund weight (w_F)</th>
<th>Fund sector return (r_F)</th>
<th>Fund return (w_F * r_F)</th>
<th>Benchmark weight (w_B)</th>
<th>Benchmark sector return (r_B)</th>
<th>Benchmark return (w_B * r_B)</th>
<th>Active weight</th>
<th>Active return contribution</th>
<th>Tracking error contribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic materials</td>
<td>0.0000</td>
<td>n/a</td>
<td>0.0000</td>
<td>0.0389</td>
<td>0.0002</td>
<td>-0.0002</td>
<td>0.48</td>
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<tr>
<td>Cash</td>
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<td>0.0352</td>
<td>0.0030</td>
<td>0.0026</td>
<td>0.0001</td>
<td>0.0838</td>
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<td>0.0106</td>
<td>0.0944</td>
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<tr>
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<td>100.00</td>
<td></td>
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</table>
weight of the Cash sector by 5% (to 5.86%) lowers the tracking error by 0.19% (to 12.3%). This strategy is represented by the normalised trading rule $q_1$, which has only two non-zero elements: $q_{1,\text{Communications}} = -0.5$ and $q_{1,\text{Cash}} = 0.5$. The resulting trade risk profile for the Communications sector (Figure 3) includes not only the “what-if” trade (ie, the triangular icon), but also shows that tracking error can be reduced further if more of the Communications sector is sold. In particular, at the best hedge position, the weight of the Communications sector is 124.78%, leading to a 3.24% decline in tracking error. In other words, we need to short the Communications sector to reach the best hedge.

If short sales are not permitted, we can constrain the weights of all sectors to be non-negative. This reduces the trade risk profile and shifts the best hedge to the point corresponding to $w_{1,\text{Communications}} = 0$ (Figure 4).

Although $q_1$ reduces tracking error, both the what-if and the best hedge trades reduce risk at the expense of active return (eg, the return difference at the constrained best hedge is $\Delta R^p(q_1, 0) = -10.82\%$).
This is consistent with the fact that the given trading strategy has a negative marginal return $(MR(q_1) = -0.2087)$.

An alternative trading strategy might seek to trade various sectors in a way that earns additional return while reducing risk. A second trading rule, $q_2$, sells the Financial sector and purchases the Technology sector, which has a slightly higher expected return. Specifying $q_{2, Financial} = -0.5$, $q_{2, Technology} = 0.5$ results in a positive marginal return for $q_2$.

However, $q_2$ has a positive marginal tracking error $(MTE(q_2, 0) = 0.0142)$ and the best hedge occurs at $\theta_{q_2}^* = -0.19$; reducing tracking error under this trading strategy entails purchasing, not selling, the Financial sector. Thus, the positive marginal return cannot be exploited in this case.

Clearly, to reduce risk while simultaneously increasing expected return, a trading strategy requires a marginal tracking error and a marginal return that are of opposite sign. One way to obtain such
a strategy is to identify a \( w \)-rule \( q \) whose marginal return is zero (ie, by solving \( q^T r = 0 \)) and that significantly reduces risk, as might be reflected by a large marginal tracking error or an attractive best hedge. Given such a rule, it is then possible to make slight adjustments to \( q \) in order to obtain a marginal return that is of the desired sign.

Using this approach, we obtain the trading strategy \( q_3 \), which sells the Technology sector and buys the Consumer Cyclical sector as follows: \( q_{Technology}^3 = -0.5, q_{ConsumerCyclical}^3 = 0.5 \). In this case, the marginal tracking error is negative while the marginal return is positive. Adopting the best hedge position reduces the tracking error by 0.514% and increases the return by 0.04%.

Comparing various trade risk profiles generated by the trading strategies provides further insights into their relative performance.
For example, a plot of tracking error versus return clarifies the risk/return tradeoffs for alternative strategies. Suppose we want to sell the Technology and Financial sectors and buy the Consumer Cyclical sector. However, we are unsure of the best relative weights to use when selling the Technology and Financial sectors. To start, we compare the following three strategies and plot the trade risk profile both as a function of weight (Figure 5) and expected return (Figure 6):

$q_4$: $q_{4_{\text{ConsumerCyclical}}} = 0.5$ and $q_{4_{\text{Technology}}} = -0.25$ and $q_{4_{\text{Financial}}} = -0.25$

$q_5$: $q_{5_{\text{ConsumerCyclical}}} = 0.5$ and $q_{5_{\text{Technology}}} = -0.1$ and $q_{5_{\text{Financial}}} = -0.4$

$q_6$: $q_{6_{\text{ConsumerCyclical}}} = 0.5$ and $q_{6_{\text{Technology}}} = -0.4$ and $q_{6_{\text{Financial}}} = -0.1$
By analysing these profiles we see that all three strategies increase return and decrease tracking error. Strategy \( q_6 \) is the best at reducing tracking error but gains the least return. In contrast, \( q_5 \) is the best at increasing expected return but reduces tracking error the least. After analysing these three strategies, a portfolio manager can adjust the relative percentages of the Technology and Financial sectors to generate slightly different profiles until a suitable strategy is determined.

While the trading strategies above were motivated largely by the tracking error contributions, it is also possible to compare strategies that trade sectors in the benchmark that are absent from the portfolio.

CONCLUDING REMARKS
In evaluating trading strategies, fund managers can benefit from an understanding of their potential impact on tracking error. The TRP, which can be constructed easily from a covariance matrix of asset returns, is the basis for several simple tools that assist in this analysis: best hedges, marginal risk and tracking error contributions. TRPs provide an analytical approach to comparing a wide range trading strategies to assist in the process of portfolio construction.

The methodology described in this chapter can be extended in several ways. For example, it is straightforward to incorporate other simple trading restrictions, such as setting caps or floors on linear combinations of asset weights (eg, \( w_{\text{Technology}} + w_{\text{Financial}} \leq 0.6 \)). Alternatively, it is possible to construct a profile showing an asset’s tracking error contribution as a function of the trade size.

This methodology also facilitates a consolidated risk analysis for multiple investment decisions. For example, a senior manager can analyse the trading strategies used in each sub-portfolio separately (ie, multiple \( q_s \) and \( \theta_s \)) or on an aggregate basis (ie, sum of \( q_s \) and a single \( \theta \)). Thus, for instance it is possible to compare the best hedge position of each sub-portfolio with an overall best hedge. The decomposition of trades into individual investment decisions, as proposed by Scherer (2001) in an \textit{ex post} framework, is also an interesting possibility for \textit{ex ante} analysis.

1 If the inequality in Equation (11) is reversed then the inequality in Equation (13) is reversed also, yielding an upper bound for \( \theta \). If Equation (11) is an equality, the restriction has no effect on \( \theta \).
REFERENCES


