QUANTUM PHASE TRANSITIONS IN MAGNETIC SYSTEMS

by

Omid Nohadani

A Dissertation Presented to the FACULTY OF THE GRADUATE SCHOOL UNIVERSITY OF SOUTHERN CALIFORNIA In Partial Fulfillment of the Requirements for the Degree DOCTOR OF PHILOSOPHY (PHYSICS)

December 2005

Copyright 2005

Omid Nohadani

Contents

De	edica	tion	ii			
Acknowledgments						
Τa	Table of Figures vi					
Abstract						
1	Intr	roduction	1			
2	Nui	nerical Methods	13			
	2.1	Introduction	13			
	2.2	Quantum Monte Carlo	15			
	2.3	Monte Carlo Simulations	16			
		2.3.1 Worldline Representation	20			
		2.3.2 Path-Integral Quantum Monte Carlo	24			
		2.3.3 Stochastic Series Expansion	26			
		2.3.4 Directed Loops Algorithm	38			
		2.3.4.1 Summary on Directed Loops	50			
		2.3.5 Operator Estimators	53			
3	Qu	antum Criticality	55			
	3.1	Introduction	55			
	3.2	Quantum Phase Transition	59			
	3.3	Scaling Analysis	63			
4	Fiel	d- and Pressure-induced Quantum Phase Transitions	69			
	4.1	Introduction	69			
	4.2	Ginzburg-Landau Theory	73			
	4.3	Model and Method	75			
	4.4	Zero-Temperature Phase Diagram	77			
	4.5	Field-Induced Quantum Phase Transition	82			

	4.6	Scaling of the Uniform Magnetization	83
	4.7	Scaling of the Order Parameter	84
	4.8	Conclusions	90
5	Bo	se-Einstein Condensation of Magnons	93
	5.1	Introduction	93
	5.2	Model and The Power-Law Scaling	95
	5.3	Quantum Monte Carlo Simulations	99
	5.4	Conclusions	104
6	Bos	e-Glass Phases in Disordered Quantum Magnets	105
	6.1	Introduction	105
	6.2	Numerical Modeling	107
	6.3	Bosonic Interpretation	109
	6.4	The Effects of Randomness on Antiferromagnets	113
	6.5	Relevance of Randomness and Harris Criterion	116
	6.6	Conclusions	117
7	Cor	nclusions	119
R	efere	nces	123
Bi	Bibliography		

List of Figures

1.1	Velocity-distribution data confirming the discovery of a new phase of matter, the Bose-Einstein condensate, out of a gas of rubidium atoms.	4
1.2	Schematic diagram of the energy ϵ as a function of applied external magnetic field h .	7
1.3	Schematic arrangement of the $TlCuCl_3$ crystal	8
2.1	Schematic example of a worldline configuration of a Heisenberg chain based on the checkerboard decomposition.	23
2.2	Schematic example of possible local update on two neighboring spin- $1/2$ sites, building a vertex.	32
2.3	Schematic example of an operator string configuration on a 4-site $spin=1/2$ chain with periodic boundary condition	34
2.4	Graphical representation of a vertex list and a global update example on a 4-site spin= $1/2$ chain with periodic boundary condition	35
2.5	The four paths on a vertex in the case of the entrance leg being the low-left one	36
2.6	The six possible vertices with non-zero matrix elements. \ldots .	40
2.7	All allowed vertices in the deterministic operator-loop construction for Heisenberg antiferromagnetic and ferromagnetic spin systems.	42
2.8	Two examples of vertices with their four possible paths, where the entrance leg is the lower left leg	44

2.9	All possible directed-loop segments (the lines with the arrows) for half of the different combinations of vertices and entrance legs in a XXZ model	45
2.10	Two different symmetries, which allow to relate the directed-loop equations for different quadrants of Figure 2.9 to each other	46
2.11	Different sectors of the phase space, for which optimized solutions to the directed-loop equations are provided in Table 2.1.	49
2.12	One exemplary vertex from a vertex list, connecting two neighboring spins via the bond Hamiltonian H_b .	50
3.1	Schematic phase diagram of a first-order phase transition	57
3.2	Phase diagrams of two different quantum phase transitions	62
4.1	Schematic zero-temperature phase diagram of a three-dimensional coupled dimer compound.	71
4.2	Layers of coupled dimers, with staggered and aligned arrangements of dimers.	72
4.3	Temperature dependence of the order parameter $m_{\rm s}^{\perp}$ in the isotropic spin-1/2 Heisenberg model at $h = 0$ on a cubic lattice with $L = 16$.	76
4.4	Scaling plot of the zero-temperature staggered magnetization in the aligned arrangement of Figure 4.2 (b).	79
4.5	Scaling plot of the zero-temperature staggered magnetization in the staggered arrangement of Figure 4.2 (a).	80
4.6	Staggered magnetization m_s of coupled dimer arrays as a function of the inter-dimer coupling strength.	81
4.7	Experimental data from Ref. [81] for the critical field $H = \Delta/g\mu_B$ as a function of applied pressure P .	82
4.8	Zero-temperature uniform magnetization of aligned dimer arrays for different inter-dimer couplings as a function of the magnetic field h .	85
4.9	Zero-temperature staggered transverse magnetization m_s^{\perp} in coupled aligned dimer arrays as a function of an applied magnetic field h .	86

4.10	Scaling plot and data collapse for the zero-temperature staggered transverse magnetization m_s^{\perp} of weakly coupled aligned dimers as a function of the applied magnetic field h for a coupling-ratio of $J'/J = 0.2.$	88
4.11	Extrapolation of finite-size data of staggered magnetization for dif- ferent magnetic fields close to the transition.	89
4.12	Scaling behavior of the zero-temperature staggered transverse magnetization as a function of the applied magnetic field h	90
4.13	Bulk staggered magnetization at zero temperature in a cubic anti- ferromagnet of aligned dimers	91
5.1	Phase diagram of 3D AF coupled dimers in a magnetic field. The solid line represents h_{c1} and the dot-dashed line h_{c2}	97
5.2	Scaled staggered magnetization at constant field $h = 0.8J$ and inter- dimer coupling $J' = 0.25J$, for a range of system sizes.	98
5.3	Critical temperature for field-induced order in 3D systems of coupled dimers for a range of values of J' .	100
5.4	Critical exponents α_1 at h_{c1} (a) and α_2 at h_{c2} (b), shown as a function of the fitting range x .	102
6.1	(a) Zero-temperature uniform and transverse staggered magnetiza- tion as a function of field; (b) Magnification of the "mini-condensation" surrounded by two neighboring Bose-Glass phases; (c) The effective bosonic random-potential.	108
6.2	Schematic response of the zero-temperature uniform and staggered magnetizations to an applied magnetic field.	111
6.3	Zero-temperature uniform and staggered magnetization as a function of field for different doping concentrations, for different inter-dimer couplings, and for different intra-dimer coupling strengths	114
6.4	Zero-temperature phase diagram of three-dimensional weakly coupled dimers with random intra-dimer coupling at a doping rate of $x \leq 15\%$	117

Abstract

Phase transitions in quantum antiferromagnets offer exciting and novel insights into the critical behavior of matter at ultra-low temperatures.

In this thesis, we apply the stochastic series expansion quantum Monte Carlo method to study the critical properties of ensembles of antiferromagnetically coupled spins subject to quantum phase transitions. The zero-temperature phase diagram, describing various phase transitions induced by an applied magnetic field, is constructed. The corresponding quantum critical points are determined to highest accuracy, allowing a conclusive interpretation of recent experimental measurements. Moreover, the scaling properties of uniform magnetization and staggered transverse magnetizationin magnetic fields are calculated, allowing the determination of the universality class of the system. The associated critical exponents are derived from Ginzburg-Landau theory as well. We find excellent agreement between the quantum Monte Carlo simulations and the analytical results, as well as previous bond-operator calculations.

Furthermore, the critical scaling exponent, which governs the power-law dependence of the transition temperature on the applied magnetic field, is extracted from the numerical data. We show that this exponent is independent of material specific inter-constituent interactions. Moreover, it converges to the value predicted for Bose-Einstein condensation of magnons. These results are of direct relevance to compounds such as TlCuCl₃ and KCuCl₃, and explain the broad range of exponents reported my exponents for field-induced ordering transitions.

Finally, we introduce geometric randomness into a model of coupled dimers. The calculations show that at finite randomness, field-induced quantum phase transitions into and out of ordered Bose-Einstein condensates pass through a Bose-Glass phase. The localization of the bosons and their finite compressibility manifests this unique regime. Once delocalized, the bosons condense, and long-range order sets in. We further detect that an intermediate magnetization plateau can occur for a parameter range, in which the spins of the doped bonds become fully polarized. This rich field-dependence is expected to be experimentally observable in weakly coupled dimer compounds with small doping and negligible spin-orbit coupling or directionality effects. The calculations in this thesis cover fundamental phases and transitions between them, as they can occur in antiferromagnetic quantum spin systems.

Chapter 7

Conclusions

In this thesis, quantum phase transitions in antiferromagnets were studied, using a combination of analytical and numerical techniques. Here, we summarize the results along with an outlook for possible investigation in the future.

The zero-temperature phase diagram of three-dimensional coupled dimer-spin systems was constructed using stochastic series expansion quantum Monte Carlo simulations. We developed massively parallel codes to simulate systems with up to 10,000 spins on more than 1,200 CPUs simultaneously, to warrant for the highest accuracy and reliable Monte Carlo results. In principle, stochastic series expansion quantum Monte Carlo calculations are finite-temperature simulations. In order to explore and understand the behavior of a large ensemble of spins in applied magnetic fields at ultra-low temperatures, we implemented the novel directed-loop algorithm. This effort unveiled the ultra-low temperature behavior of antiferromagnets through quantum Monte Carlo calculations. The resulting phase diagram featured a low-field dimer spin liquid phase at weak inter-dimer couplings, a partially polarized regime with long-range transverse magnetic order for intermediate magnetic fields $h_c \leq h \leq h_s$, and a fully polarized phase at high magnetic fields. Both of these field-induced quantum phase transitions were shown to be continuous. The numerical values of these exponents agree perfectly with the presented Ginzburg-Landau calculations as well as previous bond-operator mean-field theory. Moreover, the quantum critical point is determined to highest accuracy, such that we could demonstrate cubic scaling for $m_u \propto h^3$ at the quantum critical point, as it was reported recently from magnetization study on the TlCuCl₃ compound under hydrostatic pressure. Furthermore, for the first time we present corrections to mean-field scaling, emerging at the zero-field pressure-induced quantum phase transition in three-dimensional compounds. In particular, we show that the experimental data from Ref. [81] can be well fitted to a mean-field scaling law taking into account logarithmic corrections.

With the knowledge of the universality class of these systems, the temperature dependence of the field-induced phase transition was investigated. We showed that the critical scaling exponents, describing the power-law of the transition temperature close to the critical field h_c (h_s), are independent of the inter-dimer coupling ratio. This demonstrates that, despite large quantitative differences in their magnon dispersion relations, the physical properties of the coupled-dimer systems, such as KCuCl₃ or TlCuCl₃, display the same universal scaling behavior. The exponents are consistent with a description of the ordering transitions as the Bose-Einstein condensation of magnon excitations. The extraction of the exponents was shown to depend sensitively on the interval of data taken into account. We propose a novel technique to extract power-law exponents independent of the range of available data points. This method offers an explanation for the wide range of scaling exponents reported in current literature. Following our report, recent theoretical predictions as well as experimental measurements agree well with our calculations. [75, 74, 76] Several new compounds with different geometries and dimer arrangements were examined in high magnetic fields, confirming the presented calculations in this thesis.

Furthermore, we introduced randomness into a model of coupled dimers. The phase diagram found to be rich for weakly coupled dimers with random intra-dimer coupling strengths. Quantum Monte Carlo data shows that at finite randomness, a field-induced quantum phase transition into and out of an ordered Bose-Einstein condensate passes through a Bose-Glass phase. We clearly identify that the localization of the bosons and the finite compressibility manifests this unique regime. Once delocalized, the triplons condense and Néel-order sets in. We further detect that depending on coupling ratios, an intermediate plateau can occur, in which the spins of the doped bonds are fully polarized. This rich field-dependence is expected to be experimentally observable in weakly coupled dimer compounds with small doping and negligible spin-orbit coupling or directionality effects. We showed that Bose-Glass phases can occur in spin systems using the microscopic Heisenberg model.

For future investigations, we are eager to understand the competition of thermal and geometrical randomization close to quantum critical points. On the one hand, temperature drives a quantum phase transition away from its pure quantum character. At the same time, the correlation between the space and imaginary time direction changes its exponent, as discussed in Chapter 3. On the other hand, geometric randomness at higher dimension may break the Harris criterion and thus become relevant for the critical exponents and subsequently for the universality class. The introduction of disorder could result in new phases created due to geometrical randomness, as reported in Chapter 6, the nature of which could be destroyed or changed upon the increase of temperature. In experimental studies, the real compounds are never 100% clean and always contain impurities, which could be modeled through geometrical disorder. At the same time, ultra-low temperature experiments require tremendous efforts and the laws of thermodynamics prevent zero-temperature measurements. Therefore, theoretical predictions of finite-temperature phase transitions in impurity doped or randomized quantum magnets would shine new light on these still not completely resolved questions.

References

- A. Einstein, Sitzungsbericht der Preussischen Akademie der Wissenschaften, Physikalisch-Mathematische Klasse, p. 261 (1924); p. 3 (1925).
- [2] S. N. Bose, Z. Phys. **26**, 178 (1924).
- [3] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Science 269, 198 (1995)
- [4] W. Ketterle, Scientific American, May 2004, p. 120.
- [5] NIST BEC Lab, from http://www.bec.nist.gov/gallery.html.
- [6] C. J. Pethick, H. Smith, *Bose-Einstein Condensates in Dilute Atomic Gases*, (Cambridge University Press; 1st edition, 2001).
- [7] L. P. Pitaevskii, S. Stringari, *Bose-Einstein Condensation* (Oxford University Press, 2003).
- [8] M. Matsumoto, B. Normand, T. M. Rice, and M. Sigrist, Phys. Rev. Lett. 89, 077203 (2002).
- [9] S. Haas, J. Riera, and E. Dagotto, Phys. Rev. B 48, 3281 (1993).
- [10] S. Haas, J. Riera, and E. Dagotto, Phys. Rev. B 48, 13174 (1993).
- [11] S. Wessel and S. Haas, Phys. Rev. B **61**, 15262(2000).
- [12] S. Wessel and S. Haas, Phys. Rev. B **62**, 316(2000).
- [13] A. Läuchli, G. Schmid, and M. Troyer, Phys. Rev. B 67, 100409 (2003).
- [14] H. J. Schulz, Phys. Rev. Lett. 77, 2790 (1996).
- [15] A. W. Sandvik and D. K. Campbell, Phys. Rev. Lett. 83, 195 (1999).
- [16] Stefan Wessel, Critical Properties of Quantum Spin Liquids, PhD. Thesis, University of Southern California, 2001.

- [17] C++ Boost, http://www.boost.org.
- [18] M. Troyer, B. Ammon and E. Heeb, Lecture Notes in Computer Science 1505 (1998) 191.
- [19] F. Alet, P. Dayal, A. Grzesik, A. Honecker, M. Koerner, A. Laeuchli, S.R. Manmana, I.P. McCulloch, F. Michel, R.M. Noack, G. Schmid, U. Scholl-woeck, F. Stoeckli, S. Todo, S. Trebst, M. Troyer, P. Werner, S. Wessel, for the ALPS collaboration, Report cond-mat/0410407. Sources of this library are available at http://alps.comp-phys.org/.
- [20] J. M. Thijssen, *Computational Physics* (Cambridge University Press, 1999)
- [21] N. Metropolis, Z. Rosenbluth, M. Rosenbluth, A.H. Teller, and E. Teller, J. Chem. Phys. 21, 1087 (1953).
- [22] Tao Pang, An Introduction to Computational Physics, Cambridge University Press (1997).
- [23] R.-H. Swendsen and J.-S. Wang, Phys. Rev. Lett. 58, 86 (1987).
- [24] U. Wolff, Phys. Rev. Lett. **62**, 361 (1989).
- [25] U. Wolff, Nucl. Phys. B **322**, 759 (1989).
- See for example, H. De Raedt and A. Lagenkijk, Phys. Rep. 127 233 (1985);
 M. Suzuki (ed.), "Qantum Monte Carlo Methods in Equilibrium and Non-Equilibrium Systems", Springer, Berlin, Heidelberg (1987).
- [27] H. F. Trotter, Proc. Am. Math. Soc. 10, 545 (1959).
- [28] M. Suzuki, Prog. Theor. Phys. 56, 1454 (1976).
- [29] B. B. Beard and U.-J. Wiese, Phys. Rev. Lett. 77, 5130 (1996).
- [30] H. G. Evertz, G. Lana, and M. Marcu, Phys. Rev. Lett. 70, 875 (1993).
- [31] U.-J. Wiese and H.-P. Ying, Z. Phys. B **93**, 147 (1994).
- [32] S. Chandrasekharan, S. Scarlet, and U.-J. Wiese, cond-mat/9909451.
- [33] R. P. Feynman, *Statistical Mechanics, A Set of Lectures*, Perseus Books Group; 2nd edition (1998).
- [34] D. C. Handscomb, Proc. Cambridge Philos. Soc. 58, 594 (1962).
- [35] D. C. Handscomb, Proc. Cambridge Philos. Soc. 60, 115 (1964).
- [36] J. W. Lyklema, Phys. Rev. Lett. 49, 88 (1982).

- [37] E. Manousakis, Rev. Mod. Phys. 63, 1 (1991).
- [38] A. W. Sandvik and J. Kurkijärvi, Phys. Rev. B 43, 5950 (1991).
- [39] A. W. Sandvik, J. Phys. A **25**, 3667 (1992).
- [40] N. Kawashima, J. E. Gubernatis, and H. Evertz, Phys. Rev. B 50, 136 (1994).
- [41] M. Troyer and S. Sachdev, Phys. Rev. Lett. 81, 5418 (1998).
- [42] A. W. Sandvik, Phys. Rev. B 56, 11678 (1997).
- [43] A. W. Sandvik, Phys. Rev. B **59**, R14157 (1999).
- [44] A. Dorneich and M. Troyer, Phys. Rev. E 64, 066701(2001).
- [45] A. W. Sandvik, Phys. Rev. B 50, 15803 (1994).
- [46] N. V. Prokofév, B. V. Svistunov Teor. Fiz. 64, 853 (1996) [JETP Lett. 64, 911 (1996)].
- [47] I. S. Tupitsyn, Pisma Zh. Eks. Zh. Eks. Teor. Fiz. 114, 570 (1998) [JETP 87, 310 (1998)].
- [48] N. Kawashima and J. E. Gubernatis, Phys. Rev. Lett. **73**, 1295 (1994).
- [49] A. W. Sandvik, O. F. Syljuåsen, Proceedings of "The Monte Carlo Method in the Physical Sciences: Celebrating the 50th Anniversary of the Metropolis Algorithm", Los Alamos, June 9-11, 2003, Report cond-mat/0306542.
- [50] O. F. Syljuåsen and A. W. Sandvik, Phys. Rev. E 66, 046701 (2002).
- [51] O. F. Syljuåsen, Phys. Rev. E 67, 046701 (2003).
- [52] F. Alet, S. Wessel, and M. Troyer, Phys. Rev. E **71**, 036706 (2005).
- [53] A. W. Sandvik, in *Strongly correlated magnetic and superconducting systems*, edited by G. Sierra and M. A. M. Martin Delgado (Springer, Berlin, 1997).
- [54] M. E. Fisher Rev. Mod. Phys. 46, 597 (1974).
- [55] M. E. Fisher, Scaling, Universality and Renormalization Group Theory, in Lecture Notes in Physics, Vol. 186, Critical Phenomena, edited by F. J. W. Hahne (Springer, Berlin), 1983.
- [56] S. Sachdev, *Quantum Phase Transitions*, (Cambridge University Press, 1999).

- [57] J. Cardy, Scaling and Renormalization in Statistical Physics, (Cambridge University Press, 1999).
- [58] S. Sachdev, T. Senthil, and R. Shankar, Phys. Rev. B 50, 258 (1994).
- [59] M. A. Continentino, Quantum Scaling in Many-Body Systems, (World Scientific, 2001).
- [60] E. Brézin, J. C. Le Guillou, and J. Zinn-Justin, *Phase Transitions and Critical Phenomena*, 6, ed. C. Domb and M. S. Green, (Academic Press, London, 1976).
- [61] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena, Oxford University Press, 2002.
- [62] M. Vojta, Rep. Prog. Phys. 66, 2069 (2003).
- [63] A. V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B 49, 11919 (1994).
- [64] S. Chakravarty, B. I. Halperin, and D. R. Nelson, Phys. Rev. Lett. 60, 1057 (1988).
- [65] S. Chakravarty, B. I. Halperin, and D. R. Nelson, Phys. Rev. B 39, 2344 (1989).
- [66] Troyer and M. Imada, Computer Simulations in Condensed Matter Physics X, eds. D. P. Landau *et al.*, (Springer Verlag, Heidelberg, 1997).
- [67] W. Shiramura, K. Takatsu, H. Tanaka, K. Kamishima, M. Takahashi, H. Mitamura, and T. Goto, J. Phys. Soc. Jpn. 66, 1900 (1997).
- [68] A. Oosawa, M. Ishii, and H. Tanaka, J. Phys. Condens. Matter 11, 265 (1999).
- [69] N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer, and H. Mutka, Phys. Rev. B 63, 172414 (2001).
- [70] Ch. Rüegg, N. Cavadini, A. Furrer, H.-U. Güdel, K. Krämer, H. Mutka, A. Wildes, K. Habicht, and P. Vorderwisch, Nature (London) 423, 62 (2003).
- [71] H. Tanaka, A. Oosawa, T. Kato, H. Uekusa, Y. Ohashi, K. Kakurai, and A. Hoser, J. Phys. Soc. Jpn 70, 939 (2001).
- [72] T. Kato, K. Takatsu, H. Tanaka, W. Shiramura, M. Mori, K. Nakajima, and K. Kakurai, J. Phys. Soc. Jpn. 67, 752 (1998).
- [73] A. Oosawa, T. Takamasu, K. Tatani, H. Abe, N. Tsujii, O. Suzuki, H. Tanaka, G. Kido, and K. Kindo, Phys. Rev. B 66, 104405 (2002).

- [74] M. Jaime, V. F. Correa, N. Harrison, C. D. Batista, N. Kawashima, Y. Kazuma, G.A. Jorge, R. Stein, I. Heinmaa, S.A. Zvyagin, Y. Sasago, and K. Uchinokura, Phys. Rev. Lett. 93, 087203 (2004).
- [75] S. Sebastian, D. Yin, P. Tanedo, G. A. Jorge, N. Harrison, M. Jaime, Y. Mozharivskyj, G. Miller, J. Krzystek, S. A. Zvyagin, and I. R. Fisher, Phys. Rev. B 71, 212405 (2005).
- [76] N. Kawashima, J. Phys. Soc. Jpn. Vol.**73** 3219 (2004).
- [77] S. Wessel, M. Olshanii, and S. Haas, Phys. Rev. Lett. 87, 206407 (2001).
- [78] O. Nohadani, S. Wessel, B. Normand, and S. Haas, Phys. Rev. B 69, 220402 (2004).
- [79] T. Giamarchi and A. M. Tsvelik, Phys. Rev. B 59, 11398 (1999). T. Nikuni,
 M. Oshikawa, A. Oosawa, and H. Tanaka, Phys. Rev. Lett. 84, 5868 (2000).
- [80] A. Oosawa, M. Fujisawa, T. Osakabe, K. Kakurai, and H. Tanaka, J. Phys. Soc. Jpn. Vol.72, 1026 (2003).
- [81] K. Goto, M. Fujisawa, T. Ono, H. Tanaka, and Y. Uwatoko, J. Phys. Soc. Jpn. Vol.73, 3254 (2004).
- [82] M. Matsumoto and M. Sigrist, J. Phys. Soc. Jpn. 74, 2310 (2005).
- [83] I. S. Aranson and L. Kramer, Rev. of Mod. Phys. 74, 99 (2002).
- [84] M. Matsumoto, B. Normand, T. M. Rice, and M. Sigrist, Phys. Rev. B 69, 054423 (2004).
- [85] B. Normand, M. Matsumoto, O. Nohadani, S. Wessel, S. Haas, T.M. Rice, and M. Sigrist, J. Phys.: Condens. Matter 16, No 11, S867-S873 (2004).
- [86] M. Troyer and U.-I. Wiese, Phys. Rev. Lett. **94** (2005) 170201.
- [87] A. W. Sandvik, Phys. Rev. B 56, 11678 (1997).
- [88] A. W. Sandvik, Phys. Rev. Lett. 80, 5196 (1998).
- [89] M. Troyer, M. Imada, and K. Ueda, J. Phys. Soc. Jpn. 66, 2957 (1997).
- [90] T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science 303, 1490 (2004); and references therein.
- [91] G. Chaboussant, P. A. Crowell, L. P. Levy, O. Piovesana, A. Madouri, and D. Mailly, Phys. Rev. B 55, 3046 (1997).

- [92] M. B. Stone, Y. Chen, J. Rittner, H. Yardimci, D. H. Reich, C. Broholm, D. V. Ferraris, and T. Lectka, Phys. Rev. B 65, 064423 (2002).
- [93] M. Mito, H. Akama, T. Kawae, K. Takeda, H. Deguchi, and S. Takagi, Phys. Rev. B 65, 104405 (2002).
- [94] M. Hasenbusch and T. Toeroek, unpublished (cond-mat/9904408).
- [95] A. E. Leanhardt, Y. Shin, A. P. Chikkatur, D. Kielpinski, W. Ketterle, and D. E. Pritchard, Phys. Rev. Lett. 90, 100404 (2003).
- [96] A. E. Leanhardt, Y. Shin, A. P. Chikkatur, D. Kielpinski, W. Ketterle, and D. E. Pritchard, 89, 040401 (2002).
- [97] J. Fortàgh, H. Ott, S. Kraft, A. Gnther, and C. Zimmermann, Phys. Rev. A 66, 041604(R) (2002).
- [98] S. Kraft, A Günther1, H. Ott, D. Wharam, C. Zimmermann, and J. Fortàgh, J. Phys. B 35, L469 (2002).
- [99] D. W. Wang, M. D. Lukin, and E. Demler, Phys. Rev. Lett. 92, 076802 (2004).
- [100] L. Ammor, N. H. Hong, B. Pignon, and A. Ruyter, Phys. Rev. B 70, 224509 (2004).
- [101] A. Oosawa and H. Tanaka, Phys. Rev. B 65, 184437 (2002).
- [102] Y. Shindo and H. Tanaka, J. Phys. Soc. Jpn. Vol. 73, 2642 (2004).
- [103] O. Nohadani, S. Wessel, and S. Haas, Phys. Rev. Lett. 95, 227201 (2005).
- [104] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B 40, 546 (1989).
- [105] O. Nohadani, S. Wessel, and S. Haas, Phys. Rev. B 72, 024440 (2005).
- [106] I. Fischer and A. Rosch, Report cond-mat/0412284.
- [107] M. Sigrist and A. Furusaki, J. Phys. Soc. Jpn. 65, 2385 (1996).
- [108] V. N. Popov, Functional Integrals and Collective Excitations, (Cambridge University Press, Cambridge, 1987).
- [109] O. Nohadani, S. Wessel, S. Haas, and A. Honecker, in preparation.
- [110] T. Moriya, Phys. Rev. 120, 91 (1960).
- [111] J. Sirker, A. Weiße and O.P. Sushkov, Europhys. Lett., 68 (2), 275 (2004).

- [112] J. T. Chayes, L. Chayes, D. S. Fisher, and T. Spencer, Phys. Rev. Lett. 57, 2999 (1986).
- [113] D. S. Fisher and M. P. A. Fisher, Phys. Rev. Lett. 61, 1847 (1988).
- [114] N. Prokofév and B. Svistunov, Phys. Rev. Lett. **92**, 015703 (2004).
- [115] S. Wiseman and E. Domany, Phys. Rev. Lett. 81, 22 (1998).