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Calculation of the Polarization Observables of the radiative capture $d + p \rightarrow {}^3\text{He} + \gamma$

by

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The vector and tensor polarization observables of the capture reaction $d + p \rightarrow {}^3\text{He} + \gamma$ are calculated for energies up to 100 MeV within the framework of coupled integral equations employing separable versions of the Paris, the Bonn-A, and Bonn-B potentials. The Ernst-Shakin-Thaler (EST) approximation is used for the separable representation of the two-body inputs in the Alt-Grassberger-Sandhas (AGS) three-nucleon equations. In the calculations Mesonic exchange currents, final state interaction, and $E2$ -contributions are taken into account, which are necessary for a good agreement between theory and experiment.

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**Berechnung von Polarisations-Observablen
beim radioaktiven Einfang $d + p \rightarrow {}^3\text{He} + \gamma$**

von
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1 Introduction

The investigation of the three-nucleon problem is of high importance for checking the quality of nuclear interactions. Realistic potentials are well adjusted to the experimental two-nucleon data, but they still do not provide all experimental three-nucleon data. Three-nucleon scattering calculations, thus, can point to the real nature of the nuclear forces. Of an also high interest are calculations of the three-nucleon photodisintegration or of the inverse radiative capture process. There the matrix elements contain the continuum states as well as the three-body bound states. Hence, they enable a better test of the nucleon-nucleon interaction than scattering and break-up calculations.

Technically, our calculations are based on the Faddeev-type AGS formalism [1], adjusted to photonuclear processes [2], as done already by Gibson and Lehman [3]. After separable expansion of the two-body potentials this momentum space formulation of the three-body problem – introduced by Alt, Grassberger and Sandhas – is most directly reduced to effective two-body equations. It is therefore particularly suitable for numerical treatments than, as will be shown in Section 2.

Detailed, highly accurate [4] calculations of the cross sections of photonuclear and radiative capture reactions of 3H and 3He were done by Schadow and Sandhas [5–8]. They show at low energies a pronounced potential dependence which, however, is to be contrasted with considerable experimental uncertainties. At higher energies the potential dependence vanishes, but the resulting theoretical curve lies between the two different data sets available. This situation and the big error bars call for new and more accurate measurements. Our goal in this work is to treat besides the differential cross section the vector and tensor polarization observables of the radiative capture reaction $p + d \rightarrow {}^3He + \gamma$ by using realistic nucleon-nucleon potentials. In fact, to get additional information on three-nucleon sensitivities, a detailed treatment of the polarization is necessary. Moreover, the spin distribution of the participating particles is not visible through the cross section measurements. Because of the discrepancy of the available realistic potentials concerning the triton binding energy, the calculated observables provide a further useful test of the employed potentials.

Calculations of different deuteron tensor analyzing powers of the capture reaction ${}^1H(\vec{d}, \gamma){}^3He$ at low energy ($E_d = 10$ MeV) have been done by Fonseca and Lehman [9]. There, the Paris two-nucleon interaction was used in the three-body equations and the electromagnetic transition is calculated with the Siegert $E1$ operator. They suggested to incorporate $E2$ -contributions in order to clear up the occurring discrepancies between some of their calculations and the corresponding experimental data sets. Ishikawa and Sasakawa have calculated the tensor analyzing power A_{yy} by employing different choices of realistic potentials and including a three-nucleon potential [10].

The calculation of the original two-dimensional three-body scattering problem is nu-

merically quite demanding [11]. In order to reduce the complexity of the problem, it is, therefore, standard to expand the original non-separable potential into series of separable terms. The resulting one-dimensional integral equations are manageable with a considerably reduced computational effort.

Januschke has calculated polarization observables of the $n - d$ scattering for the Paris potential using the W-matrix representation of the two-body T -matrix [12]. There, the high quality of the W-matrix approach even for realistic nucleon-nucleon interactions is demonstrated.

In this work, we employ three different realistic potentials. These are the Paris [13], Bonn-A [14], and Bonn-B [15] potentials. We use for them the separable expansion developed by Ernst, Shakin, and Thaler (EST) [16,17]. This expansion of the mentioned potentials developed by the Graz group (PEST, BAEST, and BBEST) [18,19], have led to the first fully reliable realistic results in the three-nucleon problem [20,21]. In the following applications to photoprocess an improved parametrization by Haidenbauer is used [22]. Depending on the demanded accuracy one can choose for each partial wave different ranks of approximation, which increase the dimension of the system of equations.

The parametrization of the EST expansion of different potentials yields different binding energies. In Groningen [7] it was shown that the peak heights of the cross section calculations are correlated to the triton binding energy and that the chosen number of partial waves plays an crucial role, so that the peak heights for different potentials with the same number of partial waves lie on a straight line. In other words, the peak height does not appear as an independent observable. Most of the calculated polarization observables also do not show a significant dependence on the different realistic potentials. We notice, however, that it is not enough to employ only the Born-term, i.e., the inhomogeneity in the coupled integral equations. The incorporation of the final state interaction (FSI) by solving the whole set of equations is necessary, as observed in [5–7].

In Section 3, we show the role of meson exchange currents (MEC) taken into account by means of the Siegert theorem and electric multipoles $E1$ and $E2$. The numerical results are presented in Section 5. The transition matrices employed in this work are delivered by Schadow. As shown in Ref. [6,7], the asymmetry caused by the $E2$ -contribution is necessary to describe the experimental data of the cross section. Our polarization calculations confirm this result. The dominance of the $E1$ transition is illustrated in Section 5 by various observables. There are still some discrepancies at the extreme forward and backward angles, which call for the calculation of $M1$ -contributions.

6 Conclusions

As in Ref. [7] discussed, taking mesonic exchange currents (MEC) via Siegert's theorem involves an approximation. But, at least in the deuteron case it has been shown that the results obtained in this way fully agree with calculations based on a Hamiltonian which includes the MEC contributions explicitly [52] up to 100 MeV. In Ref. [5–8] it is shown that for cross section calculations taking account of MEC, FSI, and $E2$ leads to a satisfactory agreement with the experimental data sets. In this work, we could confirm these outcomes by calculating the polarization observables.

The presented curves for the various polarization observables demonstrate the dominance of the $E1$ -contribution. Taking into account of $E2$ yields the necessary asymmetry. In Ref. [7] it was shown that the peak heights of the photodisintegration cross sections are correlated to the triton binding energy and that the chosen number of partial waves plays an crucial role. We could confirm this result for the inverse radiative capture reaction.

We have presented for T_{20} , T_{21} , A_{yy} , and the differential cross section excellent agreements to the experimental data sets at various energy ranges and employing different potentials. This achieved results insure a proper comparison of different potentials. The calculated cross sections, polarization observables, and peak heights, however, do not provide a sensitive test of the quality of the employed potentials in their EST representation. Thus, it should be emphasized that the EST method provide a sufficiently accurate approximation of the nucleon-nucleon potentials. The reduction of the original two-dimensional integral equations into a set of one-dimensional equations provides, therefore, quite accurate results. With this method the computational effort is considerably reduced. Taking account of higher partial waves, i.e., $j \leq 2$ stabilize the numerical results.

The big error bars for some observables, occurred for instance in figures 38 or 23, do not allow a real judgment, so that new, more accurate measurement would be needed. The discrepancies at the extreme forward and backward angles call for the calculation of $M1$ -contributions.

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