# Corrections to Numerical Optimization, Second Edition <br> Published August 2006 <br> (Last updated May 27, 2008) 

1. p. 5 , line -11 . "from a a finite" $\rightarrow$ "from a finite"
2. p. 9 , line 18. "n the 1940 s " $\rightarrow$ "in the 1940 s "
3. p. 23 , line -5 . " $\nabla f$ " $\rightarrow$ " $\nabla^{2} f$ "
4. p. 25, line 1. "...,is" $\rightarrow$ "is, respectively, (6.25) and"
5. p. 26 , line 8. "positive definite $p_{k}$ " $\rightarrow$ "positive definite"
6. p. 32 , line 8 . " $k=0,1, \ldots$ " $\rightarrow$ " $k=1,2, \ldots$ "
7. pp. 34-35, Figures 3.4 and 3.5. "desired slope" $\rightarrow$ "minimum acceptable slope"
8. p. 40 , line -9 . "will be able" $\rightarrow$ "will not be able"
9. p. 49 , line 15 . "For a proof this result" $\rightarrow$ "For a proof of this result"
10. p. 49, line 15. "For problems in which $\nabla f^{*}$ is close to singular" $\rightarrow$ "For problems in which $\nabla^{2} f\left(x^{*}\right)$ is close to singular"
11. p. 55, Example 3.2. Replace formula (3.52) by

$$
L=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\frac{1}{9} & \frac{2}{3} & 1 & 0 \\
\frac{2}{9} & \frac{1}{3} & \frac{5}{7} & 1
\end{array}\right], \quad B=\left[\begin{array}{cccc}
0 & 3 & 0 & 0 \\
3 & 4 & 0 & 0 \\
0 & 0 & \frac{7}{9} & 0 \\
0 & 0 & 0 & \frac{45}{63}
\end{array}\right] .
$$

Also, make the replacement "Note that both diagonal blocks in $B$ are $2 \times 2$ " $\rightarrow$ "Note that the leading diagonal block in $B$ is $2 \times 2$ "
12. p. 63 , line 8. Remove the paragraph "Another strategy ... Goldfarb [132])"
13. p. 75 , line -3 . "In the latter case, we compute the appropriate" $\rightarrow$ "When $\left\|p^{\mathrm{U}}\right\| \leq \Delta$, the appropriate value of $\tau$ is obtained from

$$
\tau=\frac{\Delta}{\left\|p^{\mathrm{U}}\right\|}
$$

Otherwise, when $\left\|p^{\mathrm{U}}\right\|<\Delta<\left\|p^{\mathrm{B}}\right\|$, we compute the appropriate"
14. p. 80 , line -3 . Delete "for some $t \in(0,1)$,".
15. p. 81, line 1. "to denote the Lipschitz" $\rightarrow$ "to denote half the Lipschitz"
16. p. 84 , line -5 . " $\lambda \neq \lambda_{j} " \rightarrow " \lambda \neq-\lambda_{j} "$
17. p. 85 , on the line after (4.40). "whcih" $\rightarrow$ "which"
18. p. 90, line 9. "global minimum" $\rightarrow$ "global minimizer".
19. p. 93, line 9. "neighborhhod" $\rightarrow$ "neighborhood"
20. p. 99, line 1. "the sequence $\{\|g\|\}$ " $\rightarrow$ "the sequence $\left\{\left\|g_{k}\right\|\right\}$ "
21. p. 99, Exercise 4.6. "positive definite" $\rightarrow$ "symmetric positive definite".
22. p. 145, lines 14-15. Item 2 should read "If $y_{k}=B_{k} s_{k}$, then the trivial updating formula $B_{k+1}=B_{k}$ satisfies the secant condition."
23. p. 158, formula (6.57) should be

$$
\tilde{M}_{k}=\frac{\left\|\tilde{y}_{k}\right\|^{2}}{\tilde{y}_{k}^{T} \tilde{s}_{k}} \leq \frac{\left(1+\bar{c} \epsilon_{k}\right)^{2}}{1-\bar{c} \epsilon_{k}} .
$$

24. p. 158, formula (6.58) should be

$$
\tilde{M}_{k} \leq 1+\frac{3 \bar{c}+\bar{c}^{2} \epsilon_{k}}{1-\bar{c} \epsilon_{k}} \epsilon_{k} \leq 1+c \epsilon_{k}
$$

25. p. 162, exercise 6.5 should read "Prove that if $y_{k} \neq B_{k} s_{k}$ and $\left(y_{k}-B_{k} s_{k}\right)^{T} s_{k}=0$, then there is no symmetric rank-one updating formula that satisfies the secant condition.
26. p. 167, line 9 . The first line of this displayed multiline formula should be

$$
\nabla f_{k+1}=\nabla f_{k}+\nabla^{2} f_{k} p_{k}+\int_{0}^{1}\left[\nabla^{2} f\left(x_{k}+t p_{k}\right)-\nabla^{2} f\left(x_{k}\right)\right] p_{k} d t
$$

(The quantities in the integral should be Hessians, not gradients.)
27. p. 171, line 8 of Algorithm 7.2. Remove "in (4.5)".
28. p. 176, eq (7.14). " $Q_{j}$ " $\rightarrow$ " $Q_{j}^{T}$ ".
29. p. 192, line 5. "its area is $q^{2 "} \rightarrow$ "its area is $q^{-2 "}$
30. p. 232, line 6. " $k=1,2, \ldots " \rightarrow " k=0,1,2, \ldots$ "
31. p. 238, line 18. "toward this value" $\rightarrow$ "toward the best vertex".
32. p. 238, line 19. "after some defining some notation" $\rightarrow$ "after defining some notation".
33. p. 238, line -9 . Should be

$$
\bar{x}=\frac{1}{n} \sum_{i=1}^{n} x_{i} .
$$

34. p. 239, line 12. " $f_{-1 / 2}=\bar{x}(-1 / 2) " \rightarrow f_{-1 / 2}=f(\bar{x}(-1 / 2)) "$
35. p. 239, line 17. " $f_{1 / 2}=\bar{x}(1 / 2)$ " $\rightarrow$ " $f_{1 / 2}=f(\bar{x}(1 / 2))$ "
36. p. 239, line 23. "three-dimensional" $\rightarrow$ "two-dimensional"
37. p. 240, caption of Figure 9.4. "simplex method in $\mathbb{R}^{3 "} \rightarrow$ "simplex method in $\mathbb{R}^{2 "}$
38. p. 253, line -11 . "less sentitive to" $\rightarrow$ "less sensitive to"
39. p. 255, line -3 . "can applied to study" $\rightarrow$ "can be applied to study"
40. p. 260, lines 2 and 4. " $\lambda I " \rightarrow \sqrt{\lambda} I$ ".
41. p. 269, Exercise 10.1. Delete the phrase ", and let $y \in \mathbb{R}^{m}$ be a vector"
42. p. 269, Exercise 10.5. "Assume also that the $r_{j}$ are bounded on $\mathcal{D}$, that is, there exists $M>0$ such that $\left|r_{j}(x)\right| \leq M$ for all $j=1,2, \ldots, m$ and all $x \in \mathcal{D}$." $\rightarrow$ "Assume also that the $r_{j}$ and $\nabla r_{j}$ are bounded on $\mathcal{D}$, that is, there exists $M>0$ such that $\left|r_{j}(x)\right| \leq M$ and $\left\|\nabla r_{j}(x)\right\| \leq M$ for all $j=1,2, \ldots, m$ and all $x \in \mathcal{D} . "$
43. p. 276, formula (11.11) should be

$$
\begin{equation*}
w\left(x_{k}, x^{*}\right)=\int_{0}^{1}\left[J\left(x_{k}+t\left(x^{*}-x_{k}\right)\right)-J\left(x_{k}\right)\right]\left(x_{k}-x^{*}\right) d t . \tag{1}
\end{equation*}
$$

44. p. 279, line 11. "at most $1 / 2$ " $\rightarrow$ "at most $3 / 4$ ".
45. p. 294, line -7 . " $\int_{0}^{1} \beta_{L}\left\|p_{k}\right\|^{2} d t " \rightarrow \int_{0}^{1} t \beta_{L}\left\|p_{k}\right\|^{2} d t$ ".
46. p. 295, line -2 . "not be increased" $\rightarrow$ "not be decreased".
47. p. 303, line 1. "decreasing in $\lambda$ " $\rightarrow$ "decreasing in $\lambda>0$ "
48. p. 314, line -7 . "it s easy to identify vectors $d$ that satisfies" $\rightarrow$ "it is easy to identify vectors $d$ that satisfy"
49. p. 315, line -9. "closed convex set" $\rightarrow$ "closed set"
50. p. 317, line -2 . "sequence are $\left(d=(0, \alpha)^{T}\right.$ " $\rightarrow$ "sequence are $\left(d=(0, \alpha)^{T}\right.$ with $\alpha \geq 0 "$
51. p. 324, line 14. "positive scalars such" $\rightarrow$ "positive scalars such that"
52. p. 324, line -8 . "At $t=0, z=x^{*}$, and the Jacobian of $R$ at this point is" $\rightarrow$ "At $t=0$, we have $z=x^{*}$, and the Jacobian of $R$ with respect to $z$ at this point is"
53. p. 325 , Replace the paragraph starting on line 1 and ending on line 10 (that is, "It remains to verify...." through "proof of (ii) is complete") with the following paragraph: In fact, the solution $z$ of (12.40) is an implicit function of $t$; we can write it as $z(t)$, and note that $z_{k}=z\left(t_{k}\right)$. The implicit function theorem states that $z$ is a continuously differentiable function of $t$, with

$$
z^{\prime}(0)=-\nabla_{z} R\left(x^{*}, 0\right)^{-1} \nabla_{t} R\left(x^{*}, 0\right)
$$

and we can use (12.40) and (12.41) to deduce that $z^{\prime}(0)=d$. Since $z(0)=x^{*}$, we have that

$$
\frac{z_{k}-x^{*}}{t_{k}}=\frac{z(0)+t_{k} z^{\prime}(0)+o\left(t_{k}\right)-x^{*}}{t_{k}}=d+\frac{o\left(t_{k}\right)}{t_{k}}
$$

from which it follows that (12.29) is satisfied (for $x=x^{*}$ ), Hence, $d \in T_{\Omega}\left(x^{*}\right)$ for an arbitrary $d \in \mathcal{F}\left(x^{*}\right)$, so the proof of (ii) is complete.
54. p. 325 , line -11 . "at which all feasible sequences" $\rightarrow$ "at which all feasible sequences approaching $x "$
55. p. 328 , line 6. " $2 t$ " $\rightarrow$ " $2 \alpha$ " in the second equation of this line.
56. p. 329 , formula (12.51). " $A\left(x^{*}\right)^{T} \lambda^{*} " \rightarrow " A\left(x^{*}\right)^{T} \lambda$ "
57. p. 333 , formula (12.63). replace the term

$$
\frac{1}{2} t_{k}^{2} w^{T} \nabla_{x x}^{2} \mathcal{L}\left(x^{*}, \lambda^{*}\right)
$$

by

$$
\frac{1}{2} t_{k}^{2} w^{T} \nabla_{x x}^{2} \mathcal{L}\left(x^{*}, \lambda^{*}\right) w
$$

58. p. 333 , line -7 . "condition (12.65) by" $\rightarrow$ "condition (12.65) can be replaced by"
59. p. 336 , line -3 . The matrix in the formula should be

$$
\left[\begin{array}{cc}
-0.8 & 0 \\
0 & 1.4
\end{array}\right]
$$

60. p. 337, add after line 12: "where $\left|\mathcal{A}\left(x^{*}\right)\right|$ denotes the cardinality of $\mathcal{A}\left(x^{*}\right)$."
61. p. 341, statement of Lemma 12.9. "Then t the normal cone" $\rightarrow$ "Then the normal cone"
62. p. 341 , lines 16 and 19. In these two displayed formulae, replace $\Rightarrow$ by $\Leftrightarrow$.
63. p. 344, line 3. " $q: \mathbb{R}^{n} \rightarrow \mathbb{R} " \rightarrow " q: \mathbb{R}^{m} \rightarrow \mathbb{R} "$
64. p. 344 , formula (12.84) should be

$$
\begin{equation*}
\max _{\lambda \in \mathbf{R}^{m}} q(\lambda) \quad \text { subject to } \lambda \geq 0 \tag{2}
\end{equation*}
$$

65. p. 351, in formula (12.96), replace $x^{6} \sin (1 / x)=0$ by $x^{6} \sin (1 / x)$.
66. p. 443, line 15. "from from" $\rightarrow$ "from".
67. p. 444, line 14. "if does not" $\rightarrow$ "if it does not".
68. p. 455 , line 15. "to obtain $\hat{y}$ " $\rightarrow$ "to obtain $\hat{z}$ ".
69. p. 461, line 15. "the scaled $n \times n$ projection matrix" $\rightarrow$ "the $n \times n$ matrix".
70. p. 468, line -6 . "positive definite" $\rightarrow$ "positive semidefinite".
71. p. 488, line -13. "else (ii) $\Delta t^{* *} \rightarrow$ "else (ii) if $\Delta t^{*}$ ".
72. p. 600, line -6 . "is a nonnegative multiple" $\rightarrow$ "is a multiple"
73. p. 602, line 16. "(i) the whole space $\mathbb{R}^{n "} \rightarrow$ "the whole space $\mathbb{R}^{2 "}$
74. p. 609, line 14. "set $x=P^{T} z " \rightarrow$ "set $x=P z$ "
75. p. 615. lines -12 to -9 . Replace this sentence by the following: "By combining these expressions, we find that the difference between this result and the true value $x-y$ may be as large as a quantity that is bounded by $\mathbf{u}(|x|+|y|+|x-y|)$ (ignoring terms of order $\mathbf{u}^{2}$ )."
76. p. 616, displayed formula on line -4 . " $\approx$ " $\rightarrow$ " $\leq "$
77. p. 617, formula (A.32). " $\approx " \rightarrow " \leq "$
78. p. 618, line 7. This displayed formula should be

$$
\left\|x_{k}-\hat{x}\right\| \leq \epsilon, \quad \text { for some } k \geq K
$$

79. p. 620, line 16. "have $\left(1+(0.5)^{k}\right)-1 \mid=(0 .)^{k "} \rightarrow$ "have $\left|\left(1+(0.5)^{k}\right)-1\right|=(0.5)^{k "}$
80. p. 629 , line -1 . " $1 / \sqrt{13}$ " $\rightarrow$ " $1 / \sqrt{3}$ "

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