## Corrections to Numerical Optimization, Second Edition Published August 2006 (Last updated May 27, 2008)

- 1. p. 5, line -11. "from a finite"  $\rightarrow$  "from a finite"
- 2. p. 9, line 18. "n the 1940s"  $\rightarrow$  "in the 1940s"
- 3. p. 23, line –5. " $\nabla f" \rightarrow$  " $\nabla^2 f"$
- 4. p. 25, line 1. "..., is"  $\rightarrow$  "is, respectively, (6.25) and"
- 5. p. 26, line 8. "positive definite  $p_k$ "  $\rightarrow$  "positive definite"
- 6. p. 32, line 8. "k = 0, 1, ..."  $\rightarrow$  "k = 1, 2, ..."
- 7. pp. 34-35, Figures 3.4 and 3.5. "desired slope"  $\rightarrow$  "minimum acceptable slope"
- 8. p. 40, line -9. "will be able"  $\rightarrow$  "will not be able"
- 9. p. 49, line 15. "For a proof this result"  $\rightarrow$  "For a proof of this result"
- 10. p. 49, line 15. "For problems in which  $\nabla f^*$  is close to singular"  $\rightarrow$  "For problems in which  $\nabla^2 f(x^*)$  is close to singular"
- 11. p. 55, Example 3.2. Replace formula (3.52) by

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{9} & \frac{2}{3} & 1 & 0 \\ \frac{2}{9} & \frac{1}{3} & \frac{5}{7} & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & \frac{7}{9} & 0 \\ 0 & 0 & 0 & \frac{45}{63} \end{bmatrix}.$$

Also, make the replacement "Note that both diagonal blocks in B are  $2 \times 2$ "  $\rightarrow$  "Note that the leading diagonal block in B is  $2 \times 2$ "

- 12. p. 63, line 8. Remove the paragraph "Another strategy ... Goldfarb [132])"
- 13. p. 75, line -3. "In the latter case, we compute the appropriate"  $\rightarrow$  "When  $||p^{U}|| \leq \Delta$ , the appropriate value of  $\tau$  is obtained from

$$\tau = \frac{\Delta}{\|p^{\mathrm{U}}\|}$$

Otherwise, when  $||p^{U}|| < \Delta < ||p^{B}||$ , we compute the appropriate"

- 14. p. 80, line -3. Delete "for some  $t \in (0, 1)$ ,".
- 15. p. 81, line 1. "to denote the Lipschitz"  $\rightarrow$  "to denote half the Lipschitz"

- 16. p. 84, line -5. " $\lambda \neq \lambda_j$ "  $\rightarrow$  " $\lambda \neq -\lambda_j$ "
- 17. p. 85, on the line after (4.40). "which"  $\rightarrow$  "which"
- 18. p. 90, line 9. "global minimum"  $\rightarrow$  "global minimizer".
- 19. p. 93, line 9. "neighborhhod"  $\rightarrow$  "neighborhood"
- 20. p. 99, line 1. "the sequence  $\{||g||\}$ "  $\rightarrow$  "the sequence  $\{||g_k||\}$ "
- 21. p. 99, Exercise 4.6. "positive definite"  $\rightarrow$  "symmetric positive definite".
- 22. p. 145, lines 14-15. Item 2 should read "If  $y_k = B_k s_k$ , then the trivial updating formula  $B_{k+1} = B_k$  satisfies the secant condition."
- 23. p. 158, formula (6.57) should be

$$\tilde{M}_k = \frac{\|\tilde{y}_k\|^2}{\tilde{y}_k^T \tilde{s}_k} \le \frac{(1 + \bar{c}\epsilon_k)^2}{1 - \bar{c}\epsilon_k}$$

24. p. 158, formula (6.58) should be

$$\tilde{M}_k \le 1 + \frac{3\bar{c} + \bar{c}^2 \epsilon_k}{1 - \bar{c} \epsilon_k} \epsilon_k \le 1 + c \epsilon_k$$

- 25. p. 162, exercise 6.5 should read "Prove that if  $y_k \neq B_k s_k$  and  $(y_k B_k s_k)^T s_k = 0$ , then there is no symmetric rank-one updating formula that satisfies the secant condition.
- 26. p. 167, line 9. The first line of this displayed multiline formula should be

$$\nabla f_{k+1} = \nabla f_k + \nabla^2 f_k p_k + \int_0^1 [\nabla^2 f(x_k + tp_k) - \nabla^2 f(x_k)] p_k \, dt$$

(The quantities in the integral should be Hessians, not gradients.)

- 27. p. 171, line 8 of Algorithm 7.2. Remove "in (4.5)".
- 28. p. 176, eq (7.14). " $Q_j$ "  $\rightarrow$  " $Q_j^T$ ".
- 29. p. 192, line 5. "its area is  $q^{2"} \rightarrow$  "its area is  $q^{-2"}$
- 30. p. 232, line 6. " $k = 1, 2, \dots$ "  $\rightarrow$  " $k = 0, 1, 2, \dots$ "
- 31. p. 238, line 18. "toward this value"  $\rightarrow$  "toward the best vertex".

32. p. 238, line 19. "after some defining some notation"  $\rightarrow$  "after defining some notation".

33. p. 238, line -9. Should be

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i.$$

- 34. p. 239, line 12. " $f_{-1/2} = \bar{x}(-1/2)$ "  $\rightarrow$  " $f_{-1/2} = f(\bar{x}(-1/2))$ "
- 35. p. 239, line 17. " $f_{1/2} = \bar{x}(1/2)$ "  $\rightarrow$  " $f_{1/2} = f(\bar{x}(1/2))$ "
- 36. p. 239, line 23. "three-dimensional"  $\rightarrow$  "two-dimensional"
- 37. p. 240, caption of Figure 9.4. "simplex method in  $\mathbb{R}^{3"} \to$  "simplex method in  $\mathbb{R}^{2"}$ "
- 38. p. 253, line -11. "less sentitive to"  $\rightarrow$  "less sensitive to"
- 39. p. 255, line -3. "can applied to study"  $\rightarrow$  "can be applied to study"
- 40. p. 260, lines 2 and 4. " $\lambda I$ "  $\rightarrow$  " $\sqrt{\lambda}I$ ".
- 41. p. 269, Exercise 10.1. Delete the phrase ", and let  $y \in \mathbb{R}^m$  be a vector"
- 42. p. 269, Exercise 10.5. "Assume also that the  $r_j$  are bounded on  $\mathcal{D}$ , that is, there exists M > 0 such that  $|r_j(x)| \leq M$  for all  $j = 1, 2, \ldots, m$  and all  $x \in \mathcal{D}$ ."  $\rightarrow$  "Assume also that the  $r_j$  and  $\nabla r_j$  are bounded on  $\mathcal{D}$ , that is, there exists M > 0 such that  $|r_j(x)| \leq M$  and  $||\nabla r_j(x)|| \leq M$  for all  $j = 1, 2, \ldots, m$  and all  $x \in \mathcal{D}$ ."
- 43. p. 276, formula (11.11) should be

$$w(x_k, x^*) = \int_0^1 \left[ J(x_k + t(x^* - x_k)) - J(x_k) \right] (x_k - x^*) \, dt. \tag{1}$$

- 44. p. 279, line 11. "at most 1/2"  $\rightarrow$  "at most 3/4".
- 45. p. 294, line –7. " $\int_0^1 \beta_L ||p_k||^2 dt$ "  $\rightarrow$  " $\int_0^1 t \beta_L ||p_k||^2 dt$ ".
- 46. p. 295, line -2. "not be increased"  $\rightarrow$  "not be decreased".
- 47. p. 303, line 1. "decreasing in  $\lambda$ "  $\rightarrow$  "decreasing in  $\lambda > 0$ "
- 48. p. 314, line -7. "it s easy to identify vectors d that satisfies"  $\rightarrow$  "it is easy to identify vectors d that satisfy"
- 49. p. 315, line -9. "closed convex set"  $\rightarrow$  "closed set"
- 50. p. 317, line -2. "sequence are  $(d = (0, \alpha)^T$ "  $\rightarrow$  "sequence are  $(d = (0, \alpha)^T$  with  $\alpha \geq 0$ "
- 51. p. 324, line 14. "positive scalars such"  $\rightarrow$  "positive scalars such that"
- 52. p. 324, line -8. "At t = 0,  $z = x^*$ , and the Jacobian of R at this point is"  $\rightarrow$  "At t = 0, we have  $z = x^*$ , and the Jacobian of R with respect to z at this point is"

53. p. 325, Replace the paragraph starting on line 1 and ending on line 10 (that is, "It remains to verify...." through "proof of (ii) is complete") with the following paragraph:

In fact, the solution z of (12.40) is an implicit function of t; we can write it as z(t), and note that  $z_k = z(t_k)$ . The implicit function theorem states that z is a *continuously* differentiable function of t, with

$$z'(0) = -\nabla_z R(x^*, 0)^{-1} \nabla_t R(x^*, 0)$$

and we can use (12.40) and (12.41) to deduce that z'(0) = d. Since  $z(0) = x^*$ , we have that

$$\frac{z_k - x^*}{t_k} = \frac{z(0) + t_k z'(0) + o(t_k) - x^*}{t_k} = d + \frac{o(t_k)}{t_k},$$

from which it follows that (12.29) is satisfied (for  $x = x^*$ ), Hence,  $d \in T_{\Omega}(x^*)$  for an arbitrary  $d \in \mathcal{F}(x^*)$ , so the proof of (ii) is complete.

- 54. p. 325, line -11. "at which all feasible sequences"  $\rightarrow$  "at which all feasible sequences approaching x"
- 55. p. 328, line 6. "2t"  $\rightarrow$  " $2\alpha$ " in the second equation of this line.
- 56. p. 329, formula (12.51). " $A(x^*)^T \lambda^*$ "  $\rightarrow$  " $A(x^*)^T \lambda$ "
- 57. p. 333, formula (12.63). replace the term

$$\frac{1}{2}t_k^2 w^T \nabla_{xx}^2 \mathcal{L}(x^*, \lambda^*)$$

by

$$\frac{1}{2}t_k^2w^T \nabla_{xx}^2 \mathcal{L}(x^*,\lambda^*)w$$

58. p. 333, line -7. "condition (12.65) by"  $\rightarrow$  "condition (12.65) can be replaced by"

59. p. 336, line -3. The matrix in the formula should be

$$\left[\begin{array}{rrr} -0.8 & 0 \\ 0 & 1.4 \end{array}\right]$$

- 60. p. 337, add after line 12: "where  $|\mathcal{A}(x^*)|$  denotes the cardinality of  $\mathcal{A}(x^*)$ ."
- 61. p. 341, statement of Lemma 12.9. "Then t the normal cone"  $\rightarrow$  "Then the normal cone"
- 62. p. 341, lines 16 and 19. In these two displayed formulae, replace  $\Rightarrow$  by  $\Leftrightarrow$ .
- 63. p. 344, line 3. "q:  $\mathbb{R}^n \to \mathbb{R}$ "  $\to$  "q:  $\mathbb{R}^m \to \mathbb{R}$ "
- 64. p. 344, formula (12.84) should be

$$\max_{\lambda \in \mathbf{R}^m} q(\lambda) \qquad \text{subject to } \lambda \ge 0.$$
(2)

- 65. p. 351, in formula (12.96), replace  $x^6 \sin(1/x) = 0$  by  $x^6 \sin(1/x)$ .
- 66. p. 443, line 15. "from from"  $\rightarrow$  "from".
- 67. p. 444, line 14. "if does not"  $\rightarrow$  "if it does not".
- 68. p. 455, line 15. "to obtain  $\hat{y}" \to$  "to obtain  $\hat{z}".$
- 69. p. 461, line 15. "the scaled  $n \times n$  projection matrix"  $\rightarrow$  "the  $n \times n$  matrix".
- 70. p. 468, line -6. "positive definite"  $\rightarrow$  "positive semidefinite".
- 71. p. 488, line -13. "else (ii)  $\Delta t^*$ "  $\rightarrow$  "else (ii) if  $\Delta t^*$ ".
- 72. p. 600, line -6. "is a nonnegative multiple"  $\rightarrow$  "is a multiple"
- 73. p. 602, line 16. "(i) the whole space  $\mathbb{R}^{n} \to$  "the whole space  $\mathbb{R}^{2}$ "
- 74. p. 609, line 14. "set  $x = P^T z$ "  $\rightarrow$  "set x = P z"
- 75. p. 615. lines -12 to -9. Replace this sentence by the following: "By combining these expressions, we find that the difference between this result and the true value x y may be as large as a quantity that is bounded by  $\mathbf{u}(|x| + |y| + |x y|)$  (ignoring terms of order  $\mathbf{u}^2$ )."
- 76. p. 616, displayed formula on line -4. " $\approx$ "  $\rightarrow$  " $\leq$ "
- 77. p. 617, formula (A.32). " $\approx$ "  $\rightarrow$  " $\leq$ "
- 78. p. 618, line 7. This displayed formula should be

$$||x_k - \hat{x}|| \le \epsilon$$
, for some  $k \ge K$ .

79. p. 620, line 16. "have  $(1 + (0.5)^k) - 1| = (0.)^{k}$ "  $\rightarrow$  "have  $|(1 + (0.5)^k) - 1| = (0.5)^k$ " 80. p. 629, line -1. " $1/\sqrt{13}$ "  $\rightarrow$  " $1/\sqrt{3}$ "

Thanks to Carlos Henao, Jorgen Sand, Marc Steinbach.