Stochastic Constraints and Multiple Objectives

- Deal with multiple objectives by pushing everything but one into the constraint set;
- "Construct" the pareto frontier;
- We will discuss the former.

A Motivating Example

$$
\begin{aligned}
\text{minimize} \quad & \mathrm{E}[\mathrm{Y}_1(\mathrm{x}_1, \mathrm{x}_2)] \\
\text{subject to} \quad & \mathrm{E}[\mathrm{Y}_2(\mathrm{x}_1, \mathrm{x}_2)] \geq 1 - \epsilon, \\
& \mathrm{x}_1 \geq 0, \mathrm{x}_2 \geq 0,
\end{aligned}
$$

where

- $\mathrm{Y}_1, \mathrm{Y}_2$ are the total cost in a year, and fraction of a year when service is met respectively;
- $\mathrm{x}_1, \mathrm{x}_2$ are re-order level and quantity respectively.

Issues

- The problem is well defined.
- If we are in a simulation environment, checking the constraint can be a big issue.
- What is our philosophy on the returned solution? Suboptimality in the returned solution is usually okay. Is infeasibility of our returned solution okay?

Possible Model Restatements

- Redefine constraint as $E[Y_2] \geq 1 - \epsilon$;
- Redefine constraint as $\Pr\{Y_2 \geq 1 - \epsilon\} \geq 0.95$.
- Redefine problem as $\text{Min} E[Y_1] + c[1 - \epsilon - E[Y_2]]^+$ after setting $\infty \times 0 = 0$.

None of the above variations seem to really address the issue fully. What do we mean by an algorithm succeeding?

A Possible Remedy?

Give an algorithm that guarantees that:

1. (Infinite Time Performance)

$$\lim_{m\to\infty} \Pr\{(E_{Y_2}[Y_2(X^*_m)] \geq 1-\epsilon) \cap (|E_{Y_1}[Y_1(X^*_m)] - \nu^*| < \delta)\} = 1,$$

   where X^*_m is the (random) solution returned by the algorithm; ν^* is the optimal value; and δ is like an indifference parameter. (The parameter δ might be specified as part of the problem.)

2. (Finite Time Performance)

$$\Pr\{(E_{Y_2}[Y_2(X^*_m)] \geq 1-\epsilon) \cap (|E_{Y_2}[Y_2(X^*_m)] - \nu^*| < \delta)\} \geq 1-\epsilon.$$

Research Priorities

1. Proper formulation of stochastic constraints. Deal with equality and inequality constraints explicitly. (Specify constraint types in NAO, hard, and soft.)
2. Develop algorithms to solve for the efficient frontier.
3. Develop algorithms that can handle stochastic constraints.
4. Develop criteria for algorithm performance in (i) infinite-time and (ii) finite-time. (For instance, plot curves akin to Pasupathy and Henderson [1] can be plotted for each constraint slack, i.e., $1 - \epsilon - \mathrm{E}[\mathrm{Y}_2]$, and the objective function value.)

Barriers to Practice

1. General lack of probability and statistics education.
2. “Incorrect” application of deterministic methods.
3. When does a practitioner’s lack of knowledge matter? He/she does not know.
4. Formulation is hard.
5. Implementation is hard: (i) not tuning parameter; (ii) algorithm selection.
6. Communication is hard (e.g., pictures).

Next Steps

1. Massive NSF funding.
2. WSC'11 paper motivating stochastic constraints: (i) formulation; (ii) implementation; and (iii) communication.
3. Develop criteria for algorithm evaluation.
4. We use the evaluation criteria.

R. Pasupathy and S. Henderson.
A testbed of simulation-optimization problems.
In L. Perrone, F. Wieland, J. Liu, B. Lawson, D. Nicol, and R. Fujimoto, editors, Proceedings of the 2006 Winter Simulation Conference. Institute of Electrical and Electronics Engineers: Piscataway, New Jersey, 2006.