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RESEARCH ARTICLE

Optimising customer equity through engagement

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ABSTRACT
Customer lifetime value (CLV) is the discounted sum of future cash flows due to a relationship with a single customer and customer equity (CE) is the sum of CLVs from all current and future customers of a company. Maximising CE is a central goal for customer relationship management. This paper presents a framework to use Markov chain models to optimise CE, allowing for actions taken at the segment level such as new customer acquisition, retention, and win-back, and controlling for engagement. The framework is tailored for subscription services but applies more generally. We derive closed-form expressions for the finite horizon case and partial derivatives for sensitivity analysis. Finally, we give an empirical example illustrating sensitivity analysis and how optimisation using gradient descent can guide strategic decisions by estimating the optimal levels of the decision variables.

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KEYWORDS
Customer lifetime value; customer equity; Markov processes; customer relationship management; budget allocation; churn management

1. Introduction

Organisations across different industries maintain large databases with information about their current, prospective and former customers. Customer relationship management (CRM) studies how to use databases to manage relationships. One core CRM task is to allocate resources across different customer groups to maximise long-term profitability. This task is both difficult and important. Managerial actions taken today can have long-term effects. For example, a customer acquired today may generate cash flows long into the future, and improvements to the customer experience should extend the duration of relationships (i.e., improved retention) with existing customers. There are trade-offs between new customer acquisition and current customer retention in that an organisation with a goal to achieve some level of revenue will have to allocate its resources between retaining customers longer and adding new ones.

To make these decisions, *customer lifetime value* (CLV) and *customer equity* (CE) have been proposed. CLV is the discounted sum of expected future cash flows due to a relationship with a single customer (Pfeifer et al., 2005):

$$\widehat{\mathbf{CLV}} = \sum_{t=0}^{T} \frac{\mathbb{E}(\tilde{V}_t)}{(1+d)^t},$$

where $\tilde{V}_t$ is a random variable giving the net contribution during a discrete time period (e.g., month) t, d is the period discount rate, and T is the length of the future horizon (Blattberg et al., 2009). We add a hat to CLV and CE to denote that they are expected (predicted) values. Net contribution $\tilde{V}_t$ will depend on the relationship duration, which will be determined by how well the firm retains its customers (customer retention), as well as customer revenues and service costs. For registered customers who have not yet been acquired $\tilde{V}_0$ would also include acquisition costs. CE is "a combination of a firm's current customer assets and the value of the firm's potential customer assets" (Hogan et al., 2002). It is the sum of CLVs from all current and future customers within a firm (Aravindakshan et al., 2004). Having estimated CLVs, one can obtain the CE by summing them over customers.

CLV and CE are forecasts, projecting future cash flows from relationships. The overarching goal of this paper is to move CE beyond forecasting to (1) understand its sensitivity to different managerial actions and (2) optimise it over these decisions. The managerial actions we study occur at the segment level, where the firm is developing some new touchpoint targeted at some segment of customers. While many models have been proposed to estimate $\tilde{V}_t$, we focus on extending one particular approach that uses Markov chain models (MCM) (Pfeifer & Carraway, 2000) and add optimisation. Details of MCM are summarised in Section 3, but to describe our contribution we introduce minimal notation here. Customers are partitioned into k segments (or

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Table 1. A summary of relevant CLV and resource allocation literature.

Exemplary Study	Type of Model: Retention	Type of Model: Migration	Model Validation?	Sensitivity Analysis?	Objective of Optimisation	Include win-back Cost?
Pfeifer and Carraway (2000)		✓	No	No	–	No
Blattberg and Deighton (1996), Dong et al. (2007) and Pfeifer and Ovchinnikov (2011)	✓		No	No	CE	No
Berger and Bechwati (2001)		✓	No	Yes	CE	No
Rust et al. (2004)		✓	Yes	Yes	CLV	No
Jonker et al. (2004), Simester et al. (2006) and Venkatesan and Kumar (2004)		✓	Yes	No	CLV	No
Ching et al. (2004), Koosha and Albadvi (2015) and Ovchinnikov et al. (2014)		✓	No	Yes	CE	No
Reinartz et al. (2005)	✓		Yes	Yes	Profit	No
Koosha and Albadvi (2020) and Tirenni et al. (2007)		✓	Yes	Yes	CE	No
Ma et al. (2008)		✓	No	Yes	CLV	No
Albadvi and Koosha (2011)		✓	No	robustness check	CLV	No
Buhl et al. (2011)	✓		No	Yes	CLV	No
Carr et al. (2016)		✓	Yes	No	CE	No
Memarpour et al. (2019)		✓	Yes	Yes	CE + CLV	No
Ascarza and Hardie (2013) and Ascarza et al. (2018)		✓	Yes	No	–	No
This paper		✓	Yes	Yes	CE	Yes

states), each generating some level of cash flow $\mathbf{v}$, a k-vector. They migrate between states over time according to a $k \times k$ transition matrix $\mathbf{P}$. We also allow for some number $\mathbf{a}$ (k-vector) of new customers to enter the system each period. For example, the firm may decide to acquire a fixed number of new customers each period. Customer equity can thus be considered a function $\widehat{\mathrm{CE}}(\mathbf{a}, \mathbf{P}, \mathbf{v})$.

Any attempt at optimisation must address the problem that parameters $\mathbf{a}$, $\mathbf{v}$ and $\mathbf{P}$ have different units of measurement. For example, how to compare the effect on CE of adding a new customer with changing some probability in the transition matrix? Which action would have a greater return over time? Moreover, the parameters are not decision variables in that a manager has only indirect control over them. For example, a manager could launch some initiatives, which, in turn, improve retention probabilities. To address these issues we express the parameters as functions of other variables that could be decision variables or variables with a common unit (e.g., dollars or engagement behaviors). CE becomes a composition of functions.

We make the following contributions to MCM models of CE: (1) allow for acquisition and win-back over time; (2) allow for MCM parameters to be functions of decision variables; (3) derive sensitivities to know which actions will have the greatest effects on CE; and (4) optimise CE over actions targeted at segments of customers.

2. Literature review

MCM for CLV was proposed by Pfeifer and Carraway (2000) (hereafter PC). MCMs are flexible and can handle a wide variety of CRM situations such as *retention* and *migration*. The *customer retention model* handles situations where customers who are not retained are considered lost for good, and so nonresponse signals the end of the firm's relationship with the customer. Managing churn is essential for CRM (Routh et al., 2021). In contrast, the *customer migration model* allows situations where nonresponse does not necessarily signal the end of the relationship. Other researchers call the former situation "lost-for-good" and the latter "always-a-share" (Rust et al., 2004; Venkatesan & Kumar, 2004). PC developed a formula for the expected present value over T periods for a single customer:

$$\widehat{\mathbf{CLV}} = \sum_{t=0}^{T} \left[(1+d)^{-1}\mathbf{P}\right]^t \mathbf{v},$$

where $\mathbf{P}$, $\mathbf{v}$ and d were defined above.

Table 1 summarises previous research on estimating and maximising CE or CLV.

Different authors assume different business situations, but none handles the question we are facing: in subscription service where customers transition through a lifecycle depending on customer engagement/experience, how much to spend on engagement drivers that affect customer acquisition, retention and win-back to maximise CE?

Blattberg and Deighton (1996) (hereafter BD) offered a general approach to optimise the allocation of the budget between acquisition and retention expenditures, but not how to determine the optimal level of spending on the activities independently. Building on their work, Berger and Bechwati (2001) (BB) proposed a model to optimally allocate a budget between acquisition and retention. BD and BB both assumed a "lost-for-good" scenario, which is inappropriate for our situation where customers who cancel might resubscribe later. Ching et al. (2004) extend BB with a stochastic dynamic programming model and use an MCM to solve the optimal allocation of a promotion budget to maximise CLV. Memarpour et al. (2019) extend this further by considering budget constraints across customer segments. Both assume the customer migration situation, but focus on a setting of promotions and sales, in contrast to our situation where customer engagement is the key.

CLV/CE-based resource allocation models generally involve two steps: (1) predict the high-level components of CLV from lower-level drivers, and (2) compute CLV/CE by predicting the high-level components. Rust et al. (2001) presented a framework that enables trade-offs between brand choice and competition. They first analysed low-level drivers that impact brand-switching patterns and then considered variables such as frequency of category purchase, average quantity of purchase, and brand-switching patterns combined with contribution margin as key high-level components of CE. Venkatesan and Kumar (2004) (VK) developed a resource allocation model that determines how much to invest in distinct communication channels. They identified variables such as purchase frequency, contribution margin, and marketing costs as the high-level components of CLV. By modeling the purchase frequency and contribution margin as a function of channel contacts (a low-level driver), they computed and maximised CE. Extending VK, Reinartz et al. (2005) examined resource allocation across contact channels more comprehensively. A majority of the proposed approaches are discussed in a non-contractual setting, where managers are interested in predicting future customer activity. The high-level components relate to customers' purchase activities (e.g., purchase frequency, recency, inter-purchase time, etc.) and margin contributions (VK). Therefore, these models cannot be applied to our contractual subscription situation, where customers pay the same amount at the same time in each period of their contract and there is no need to model purchase activities and margin contributions based on lower-level drivers. More recent work proposed hidden Markov models (HMM) to predict churn in contractual settings. Ascarza and Hardie (2013) developed a joint model of usage and churn. Ascarza et al. (2018) used an HMM-based framework to capture silent and overt churn. However, this stream of work focuses on comparing the predictive performance of the proposed models with benchmark models rather than our goal of optimising CE over decisions.

Furthermore, churned customers are different than new customers because they had experience with the firm both conceptually (Bogomolova, 2010; Stauss & Friege, 1999) and empirically (Kumar et al., 2015, 2018). Previous studies constructed models to predict regained customers' behaviours. For example, Kumar et al. (2015) showed that the behavior of a reacquired customer can be predicted from the first-lifetime behavior. Kumar et al. (2018) proposed a mixture cure-competing risks model of second-lifetime duration. However, existing research on resource allocation using CLV/CE as an objective concentrates on acquisition, retention (Backiel et al., 2016; Lima et al., 2009), or a combination of them (Berger & Bechwati, 2001; Dong et al., 2007; Kumar & George, 2007). Customer win-back, where a firm reacquires customers who have defected (Thomas et al., 2004), has been largely neglected in the literature. In subscription-based businesses, win-back is especially important and the companies that do it well enjoy dramatic results (Griffin & Lowenstein, 2001). Recognising the importance of win-back, we propose a CE model and develop a framework for resource allocation that considers acquisition, retention and win-back.

3. Markov chain models with costs

This section extends PC by introducing an MCM for estimating CE based on acquisition, retention and win-back. PC focuses on estimating CLV for a single customer, while we further aggregate all the individual CLVs and allow monthly acquisitions to obtain CE and optimise it. Two key elements of the MCM are the transition matrix and value vector. We extend their formulations to handle subscription services.

A transition matrix corresponds to a state diagram. Figure 1 shows a general scenario for the following situation: a firm provides a subscription service and customers transition through a lifecycle. The time unit is a month. A customer can be classified into four types of states: *registered users* provide their email address and have access to limited service for free, e.g., subscribing to a free newsletter. *New customers* start paying for the service as a trial, perhaps at a discounted price. The length of the trial period is l months and there are l corresponding states for new customers. *Established customers* continue the subscription after trial and pay the regular subscription fee every month. We allow for different engagement levels by including u states of established customers. We refer some established customers as *at-risk customers* who do not use the service they are paying for and are thereby more likely to cancel the subscription (Zhou et al., 2021). *Churned customers* cancel the subscription and stop paying. A customer moves from one state to another with certain transition probabilities. The model allows the firm to implement different strategies for

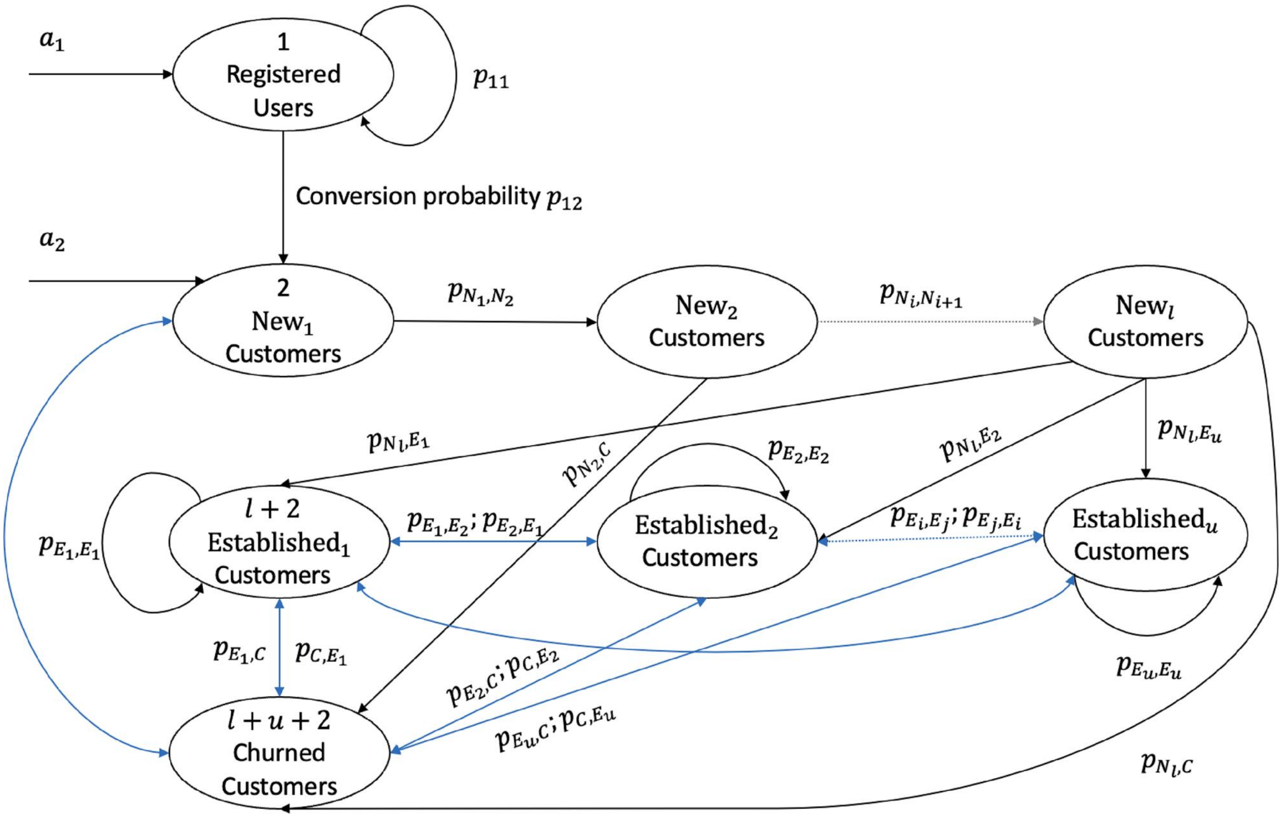

Figure 1. Diagram of the general firm's relationship with a single customer. All the black paths are one-way and the blue paths are two-way.

customers in different states, e.g., special touch-points could be developed for "at-risk subscribers" versus "new subscribers." A company can increase the number of states to accommodate additional variation across customers. The key is to include leading indicators of churn such as engagement (e.g., at-risk customers) in defining the states.

The MCM is specified by a $k \times k$ transition matrix $\mathbf{P} = [p_{ij}]$, where $k = l + u + 2$ and $p_{ij} = \mathsf{P}(s_{t+1} = j | s_t = i)$ is the probability of a customer migrating from state i in the current period to state j in the next period, where s_t is the state of a customer at period t and $\sum_{j=1}^{k} p_{ij} = 1 \forall i$. The transition matrix of this MCM is:

$$
\begin{array}{ccccccccccccc}
 & \text{Registered} & & \text{New} & & & & & \text{Established} & & & \text{Churned} & \text{States} \\
 & R & N_1 & N_2 & N_3 & \cdots & N_l & E_1 & E_2 & \cdots & E_u & C & \\
\mathbf{P} = & \left[\begin{array}{c|cccccc|cccc|c}
p_{RR} & p_{RN_1} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\
\hline
0 & 0 & p_{N_1N_2} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & p_{N_1C} \\
0 & 0 & 0 & p_{N_2N_3} & \cdots & 0 & 0 & 0 & \cdots & 0 & p_{N_2C} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & p_{N_lE_1} & p_{N_lE_2} & \cdots & p_{N_lE_u} & p_{N_lC} \\
\hline
0 & 0 & 0 & 0 & \cdots & 0 & p_{E_1E_1} & p_{E_1E_2} & \cdots & p_{E_1E_u} & p_{E_1C} \\
0 & 0 & 0 & 0 & \cdots & 0 & p_{E_2E_1} & p_{E_2E_2} & \cdots & p_{E_2E_u} & p_{E_2C} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & p_{E_uE_1} & p_{E_uE_2} & \cdots & p_{E_uE_u} & p_{E_uC} \\
\hline
0 & p_{CN_1} & 0 & 0 & \cdots & 0 & p_{CE_1} & p_{CE_2} & \cdots & p_{CE_u} & p_{CC}
\end{array}\right] & \begin{array}{c} R \\ N_1 \\ N_2 \\ \vdots \\ N_l \\ E_1 \\ E_2 \\ \vdots \\ E_u \\ C \end{array}
\end{array}
$$

In general, p_{ii} denotes the self-transition probability of state i. In a contractual setting, the trial period has a fixed length, so a new customer must move to another state in the next period after trial, which makes the self-transition probability of new customers deterministic. If the length of the trial period is greater than one month, one strategy is to add additional states to monitor which month of the trial period they are in Dwyer (1997), e.g., one state represents a new customer in the first month during the trial period, another state is for the second trial month, and so on. This is our motivation for constructing l states for a new customer. As shown in the figure, a customer can only move from trial period i to $i+1$ or directly churn, except in the last trial period, when customers can transition to any of the established states or churn. All blue paths are two-way. It is possible that an established customer's engagement level changes from month to month, thereby transitioning between established-customer states. The transition probability of moving from state $i (i \neq C)$ to the churned state $(i = C)$ is the *churn probability* p_{iC} (last column in the matrix). The *retention probability* of customers in state i is $r_i = 1 - p_{iC}$. The transition probability of moving from the churned state back to any one of states i $(i \neq C)$ is the *win-back probability* of customers in state i, which is denoted as $w_i = p_{Ci}$ (last row in the matrix). As shown in the figure, a churned customer can re-start the subscription as a new or established customer depending on the subscription price (trial or full) and engagement level. In addition to retention and win-back, the model accommodates acquisition by allowing a_1 registered users and a_2 new customers to enter the system each month.

3.1. Cost functions

Managers often think in terms of spending levels on customer acquisition, retention and win-back. In each subscription period, a firm expects to receive revenue h_i from a customer in state i. For customers in state i, it spends R_i per customer on retention, A_i to acquire each customer, and W_i on win-back per customer. This leads to the question, what is the expected CE (e.g., over the next three years)? Motivated by research that considers CE as the sum of two net present values for the returns from acquisition and retention spending (BD, BB) we further include the returns from win-back spending. Building on BD, we propose three cost functions modeling the relationship between acquisition cost and number of acquired customers, the relationship between retention cost and retention probability and the relationship between win-back cost and win-back probability.

Following BD's decision calculus, we use three concave curves to relate spending to different model parameters: the first curve relates acquisition spending (A) to the number of acquired customers (a), the second relates retention spending per customer (R) to the retention probability (r), and the third relates win-back spending per customer (W) to win-back probability (w). Each curve has two parameters: the first is a shape parameter (k_1, k_2, k_3, respectively) and the second is a ceiling—the largest possible number of customers the company could reasonably acquire (a_{ceiling}), retain (r_{ceiling}), win-back (w_{ceiling}) in a given time period. There is no limit to their acquisition (retention, win-back) spending. Shape parameters k_1, k_2, k_3 are positive constants determined from the managers' judgement.

Unlike BD, who focused on the acquisition curve for acquisition rate, we focus on the number of acquired customers, which is calculated by multiplying the total number of prospects M, a known constant obtained from data, by the acquisition rate α. Extending BD, we allow for a win-back curve. The three spending curves $A(a)$, $R(r)$ and $W(w)$ are:

$$A(a) = -\frac{1}{k_1}\ln\left(1-\frac{a}{a_{\text{ceiling}}}\right), 0 \le a < a_{\text{ceiling}}, \quad k_1 > 0, a_{\text{ceiling}} > 0, \tag{1}$$

$$R(r) = -\frac{1}{k_2}\ln\left(1-\frac{r}{r_{\text{ceiling}}}\right), 0 \le r < r_{\text{ceiling}}, \quad k_2 > 0, 0 < r_{\text{ceiling}} \le 1, \tag{2}$$

$$W(w) = -\frac{1}{k_3}\ln\left(1-\frac{w}{w_{\text{ceiling}}}\right), 0 \le w < w_{\text{ceiling}}, \quad k_3 > 0, 0 < w_{\text{ceiling}} \le 1. \tag{3}$$

Figure 2 shows an example of the retention spending curve $R(r)$, where the shape is determined by k_2 and the vertical asymptote is controlled by the ceiling retention probability. The other curves have similar shapes.

As we described previously, r and w can be expressed in terms of transition probabilities, and thus cost functions can also be expressed accordingly. For simplicity, we assume that the transition probabilities are constant over time[1]. This assumption can be relaxed for future work discussed in Section 6 and the general case derivation is in Supplementary Appendix B.3. Define $\mathbf{r} = (r_1, r_2, \ldots, r_k)^\top$, where r_i is the retention probability of state i; $\mathbf{w} = (w_1, w_2, \ldots, w_k)^\top$, where w_i is the win-back probability of state i; and $\mathbf{a}_t = (a_{t1}, a_{t2}, \ldots, a_{tk})^\top$, where a_{ti} is the number acquired customers in state i acquired to the system in period t. Most $a_{ti} = 0$ because there will usually be only one or two states where customers enter the system. For example, in the MCM above, only registered users and new subscribers can enter the system, which leads to $a_{ti} = 0, i \notin \{R, N_1\}, \forall t$.

After determining the cost functions, we express the value vector as the difference between net contribution and expenditures. For subscription services, the subscription fee is usually the net contribution and usually is constant in each period. We define $\mathbf{h} = (h_1, h_2, \ldots, h_k)^\top$, where h_i is the monthly subscriber revenue from subscribers in state $i = 1, 2, \ldots, k$. Note that in general, the h_i's are different. For example, registered users have zero revenue, while new subscribers may have discounted subscription fees. Therefore, the value vector is:

$$\mathbf{v}_t = \mathbf{h} - (\mathbf{A}_t + \mathbf{R} + \mathbf{W}), \tag{4}$$

where $\mathbf{A}_t = A(\mathbf{a}_t) = (A(a_{t1}), A(a_{t2}), \ldots, A(a_{tk}))^\top = (A(a_{t1}), A(a_{t2}), 0, \ldots, 0))^\top$ since only states 1 and 2 for registered users and new subscribers have acquisition, while the rest states do not allow acquisition; $\mathbf{R} = R(\mathbf{r}) = (R(r_1), R(r_2), \ldots, R(r_k))^\top$; and $\mathbf{W} = W(\mathbf{w}) = (W(w_1), W(w_2), \ldots, W(w_k))^\top$.

The transition probabilities and the expected revenue are computed assuming homogeneous groups of customers because we study the managerial actions that are taken at the segment level. When the probabilities are not homogeneous, one can add states. For example, news organisations often create newsletters targeted at groups of customers with similar reading habits. Thus, all customers within a segment are classified into the same state according to their engagement levels and can be represented by a single transition probability matrix $\mathbf{P}$ and the value vector. Similar assumptions can also be found in previous studies, e.g., Berger and Bechwati (2001) discussed promotion vehicles for market segments, Ching et al. (2004) analysed customers' states for the promotion and non-promotion periods and found the optimal promotion strategy, and Jonker et al. (2004)

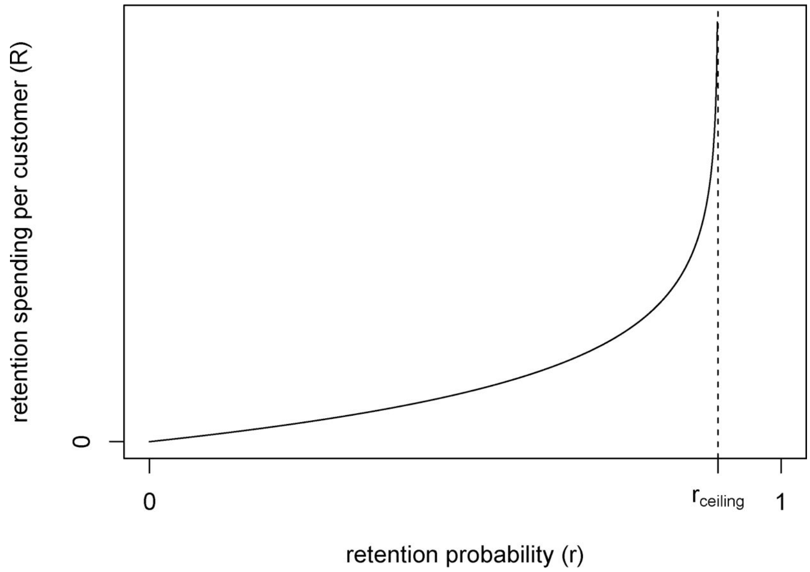

Figure 2. $R(r)$: retention spending per customer w.r.t. retention probability.

segmented customers into homogeneous groups and determined the optimal policy towards each segment.

3.2. Finite horizon

We consider the finite horizon scenario in three cases. The first case does not allow for any new customers to be added to the system over time, i.e., $\mathbf{a}_t = \mathbf{0}\ \forall t$. The second allows for a constant number of customers to be added each period over time, i.e., $\mathbf{a}_t = \mathbf{a}\ \forall t$. The third allows the acquisition strategy to vary over time, where a different number of customers is acquired over time. In the rest of the paper, we present the results for the second case and give details for the other two in Supplementary Appendices B.1 and B.3.

Suppose that the company will acquire the same number of customers every period, i.e., $\mathbf{a}_t{=}\mathbf{a}\ \forall t$. Then the cost for acquisition $\mathbf{A}_t = \mathbf{A} = A(\mathbf{a}) = (A(a_1), A(a_2), 0, \ldots, 0)^\mathsf{T}$ is also constant over time. Thus the value vector is constant and can be simplified to

$$\mathbf{v}_t = \mathbf{v} = \mathbf{h} - (\mathbf{A} + \mathbf{R} + \mathbf{W}) \quad \forall t \tag{5}$$

The expected number of customers in each state at time t is

$$\mathbf{n}_t^\mathsf{T} = \begin{cases} \mathbf{n}_0^\mathsf{T} & t = 0 \\ \mathbf{n}_0^\mathsf{T}\mathbf{P}^{(t)} + \mathbf{a}^\mathsf{T}\sum_{m=0}^{t-1}\mathbf{P}^{(m)} & t \geq 1 \end{cases} \tag{6}$$

where $\mathbf{n}_0 = (n_{01}, n_{02}, \ldots, n_{0k})^T$ is the initial number of customers by state. Assuming a constant monthly discount rate d, we can derive the closed-form formula for the expected CLV and CE respectively (derivation in Supplementary Appendix B.2),

$$\widehat{\mathbf{CLV}} = \sum_{t=0}^{T}\frac{\mathbf{P}^{(t)}\mathbf{v}}{(1+d)^t} = \sum_{t=0}^{T}\left(\frac{\mathbf{P}}{1+d}\right)^t \mathbf{v} = \left[\mathbf{I} - \left(\frac{\mathbf{P}}{1+d}\right)^{T+1}\right]\left(\mathbf{I} - \frac{\mathbf{P}}{1+d}\right)^{-1}\mathbf{v} \tag{7}$$

$$\widehat{\mathrm{CE}} = \mathbf{n}_0^\mathsf{T}\left[\mathbf{I} - \left(\frac{\mathbf{P}}{1+d}\right)^{T+1}\right]\left(\mathbf{I} - \frac{\mathbf{P}}{1+d}\right)^{-1}\mathbf{v} + \mathbf{a}^\mathsf{T}\sum_{t=1}^{T}\left(\frac{1}{(1+d)^t}\sum_{m=0}^{t-1}\mathbf{P}^{(m)}\right)\mathbf{v}. \tag{8}$$

4. Sensitivity analysis and optimisation

Sensitivity analysis quantifies how a change in one variable is associated with CE. This section derives closed-form expressions for computing partial derivatives of CE w.r.t. a and p_{ij} for the finite horizon scenario and a constant number of new customers. The partial derivative of CE w.r.t. each acquisition scalar component $a_i, i = 1, 2, \ldots, k$ are (details in Supplementary Appendix B.2):

$$\frac{\partial\widehat{\mathrm{CE}}}{\partial a_i} = \mathbf{n}_0^\mathsf{T}\left[\mathbf{I} - \left(\tfrac{\mathbf{P}}{1+d}\right)^{T+1}\right]\left(\mathbf{I} - \tfrac{\mathbf{P}}{1+d}\right)^{-1}\frac{\partial\mathbf{v}}{\partial a_i} + \mathbf{v}^\mathsf{T}\left[\sum_{t=1}^{T}\left(\frac{1}{(1+d)^t}\sum_{m=0}^{t-1}\mathbf{P}^{(m)}\right)\right]^\mathsf{T}\frac{\partial\mathbf{a}}{\partial a_i} + \mathbf{a}^\mathsf{T}\sum_{t=1}^{T}\left(\frac{1}{(1+d)^t}\sum_{m=0}^{t-1}\mathbf{P}^{(m)}\right)\frac{\partial\mathbf{v}}{\partial a_i} \tag{9}$$

Sensitivities for the transition probabilities are given by:

$$
\begin{aligned}
\frac{\partial \widehat{\mathrm{CE}}}{\partial p_{ij}} = & -\frac{\mathbf{n}_0^{\mathsf{T}}}{(1+d)^{T+1}}\left[\sum_{m=1}^{T+1}\mathbf{P}^{(m-1)}\frac{\partial \mathbf{P}}{\partial p_{ij}}\mathbf{P}^{(T+1-m)}\right]\left(\mathbf{I}-\frac{\mathbf{P}}{1+d}\right)^{-1}\mathbf{v} \\
& +\mathbf{n}_0^{\mathsf{T}}\left[\mathbf{I}-\left(\tfrac{\mathbf{P}}{1+d}\right)^{T+1}\right]\left(\mathbf{I}-\tfrac{\mathbf{P}}{1+d}\right)^{-1}\left[\frac{\partial \mathbf{v}}{\partial p_{ij}}-\frac{\partial\left(\mathbf{I}-\frac{\mathbf{P}}{1+d}\right)}{\partial p_{ij}}\left(\mathbf{I}-\frac{\mathbf{P}}{1+d}\right)^{-1}\mathbf{v}\right] \\
& +\mathbf{a}^{\mathsf{T}}\left(\sum_{t=1}^{T}\frac{1}{(1+d)^t}\sum_{m=0}^{t-1}\sum_{l=1}^{m}\mathbf{P}^{(l-1)}\frac{\partial \mathbf{P}}{\partial p_{ij}}\mathbf{P}^{(m-l)}\right)\mathbf{v} \\
& +\mathbf{a}^{\mathsf{T}}\sum_{t=1}^{T}\left(\frac{1}{(1+d)^t}\sum_{m=0}^{t-1}\mathbf{P}^{(m)}\right)\frac{\partial \mathbf{v}}{\partial p_{ij}}
\end{aligned}
\tag{10}
$$

These partial derivatives are w.r.t. variables in different units. To compute the partial derivatives w.r.t. respective costs (i.e., real decision variables that are under managers' control), we can apply the chain rule:

$$
\frac{\partial \widehat{\mathrm{CE}}}{\partial A_i} = \frac{\partial \widehat{\mathrm{CE}}}{\partial a_i}\cdot\frac{da_i}{dA_i}, \quad \frac{\partial \widehat{\mathrm{CE}}}{\partial R_i} = \frac{\partial \widehat{\mathrm{CE}}}{\partial p_{ij}}\cdot\frac{dp_{ij}}{dR_i},
$$

$$
\text{and} \quad \frac{\partial \widehat{\mathrm{CE}}}{\partial W_i} = \frac{\partial \widehat{\mathrm{CE}}}{\partial p_{ij}}\cdot\frac{dp_{ij}}{dW_i}.
$$

The next step is to optimise CE, which we do with gradient descent (GD) (Nocedal & Wright, 2006). According the partial derivatives, we find that the magnitude of the partials are very large (e.g., $> 10^6$), indicating that a tiny change in some decision variables can lead to a drastic change in CE. Thus, we need to be careful about choosing the step size for GD. We provide a numerical example to compute the partials in Section 5 for later discussion. For GD to work well, we start with a relatively small step size (e.g., values $<10^{-8}$) and use a backtracking line search to adjust the step size in the procedure for optimisation. Another issue is scaling since we are dealing with variables having different units, the number of acquired customers, and transition probabilities. To avoid the problem, we transform all the variables into measurements with common units such as dollars.

5. Empirical example

This section demonstrates our framework with the four-state MCM. The context is of great societal importance, namely local news subscription services. The rise of the Internet and social media has broken the historic business model for news production (Mierzejewska et al., 2017; Picard, 2008). Platforms, primarily Google and Facebook, have become the go-to source for information and direct consumers to news stories. They also claim an increasingly large share of advertising revenue. The result is that the creators of news stories face declining advertising revenue, thus limiting their capacity to produce news. Newsroom employment has plummeted by 70% between 2005 and 2022 (Abernathy, 2023). One consequence is "news deserts:" Out of 3,142 US counties, 204 do not have any news outlet and 1,562 have only one, which is usually a weekly newspaper (Abernathy, 2023). Fewer local stories are being produced and there is a real danger that the major producers of news will become "ghost newspapers" that reproduce commodity news from a few large, national news organisations without local journalists covering local affairs. This causes unforeseen consequences, e.g., communities where newspapers closed subsequently had increases in government spending and the costs for bonds increased (Gao et al., 2020).

5.1. Data source

We have clickstream data from a local news site (Site A) in the US that can be matched with subscriber payment history. Our samples consist of 2,326 digital-only subscribers. We have 11 months (2018-10 to 2019-08) to track when subscribers started and stopped their subscription and their online reading behaviors, such as how often they read, how many page views they read, time spent on reading, etc. All subscribers joined as new customers and started with a trial period. After the trial period, subscribers decide either to start the regular subscription, become established customers, or cancel the subscription, becoming churned. Established customers can cancel their subscriptions and become churned at any time. Some customers started and churned multiple times. Churned customers can rejoin only as new customers.

We estimate the subscription fees for different customers using the payment history and we measure engagement levels from the online behaviors. We also worked with news organizations to develop the Medill Subscriber Engagement Index (SEI) system that includes over 100 US news outlets' subscription and customer funnel data over the past three years (Jacob, 2021). The system provides benchmark metrics to approximate the ceiling parameters in our spending curves.

Due to special considerations with Site A the example we use to illustrate the model will be somewhat simpler than the general model. The behaviors of registered users were not recorded and so we do not include a state for registered users. The trial period is one month, so new customers only have one state. We use *regularity*, the number of days per month with reading, to create two engagement levels because it is a strong indicator of churn (Zhou et al., 2021). We obtain established customers with high regularity (reading three days or more per month) and *at-risk* customers with low regularity (reading two or fewer days per month). Hence, the general framework is reduced to the four-state MCM, which includes new, established, at-risk and churned customers.

5.2. Example

Site A charges new, established, at-risk and churned customers \$5.00, \$12.00, \$12.00 and \$0.00 for monthly-subscription, respectively. Its current allocation strategy is to spend $A = \$4.46$ acquiring each of $a = 100$ new customers in each month, i.e., $\mathbf{a} = (100, 0, 0, 0), \mathbf{A} = (\$4.46, 0, 0, 0)$; spend $R_1 = \$5.35, R_2 = \$3.26$ and $R_3 = \$4.00$ retaining each new, established, at-risk subscriber every month respectively; and spend $W = \$0.98$ recapturing each churned customers back to established every month. Additionally, managers judge that the best Site A can do is to acquire $a_{\text{ceiling}} = 500$ new customers among $M = 1,000$ prospects every month (giving acquisition probability $\alpha_{\text{ceiling}} = 0.5$) and to achieve the retention probability of $r_{\text{ceiling}} = 0.99$ and win-back probability of $w_{\text{ceiling}} = 0.08$. Further assume that the monthly discounted rate is $d = 0.01$ and initially there are 2,000 new, 5,000 established, 3,000 at-risk and 1,000 churned customers. The transition matrix is:

$$\begin{array}{cccccc} & \text{new} & \text{established} & \text{at-risk} & \text{churned} & \text{state} \\ \mathbf{P} = & \multicolumn{4}{c}{\left[\begin{array}{cccc} 0 & \boxed{0.75} & 0.20 & 0.05 \\ 0 & \boxed{0.82} & 0.03 & 0.15 \\ 0 & 0.30 & \boxed{0.60} & 0.10 \\ 0 & 0.05 & \boxed{0} & 0.95 \end{array}\right]} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

where the entries highlighted in the box are fixed (cannot be changed)[2]. Therefore, we can express the transition matrix as follows:

$$\begin{array}{cccccc} & \text{new} & \text{established} & \text{at-risk} & \text{churned} & \text{state} \\ \mathbf{P} = & \multicolumn{4}{c}{\left[\begin{array}{cccc} 0 & 0.75 & p_{13} & 0.25 - p_{13} \\ 0 & 0.82 & p_{23} & 0.18 - p_{23} \\ 0 & p_{32} & 0.60 & 0.40 - p_{32} \\ 0 & w & 0 & 1 - w \end{array}\right]} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

where p_{13} is the transition probability from new to at-risk, p_{23} is the transition probability from established to at-risk, p_{32} is the transition probability from at-risk to established, and w is the win-back of churned to established. In other words, there are five decision variables under Site A's control: (1) acquisition spending per customer, which is determined by a, (2) retention spending per new customer, which is determined by p_{13}, (3) retention spending per established customer, which is determined by p_{23}, (4) retention spending per at-risk customer, which is determined by p_{32}, and (5) win-back spending on bringing each established customer back to life, which is determined by w. We are interested in the following questions:

Q1. Given the current setting, what is the CE over $T = 36$ months?

Q2. How will the change in a variable (e.g., increasing p_{23} from 0.03 to 0.10) affect the change in CE?

Q3. If Site A plans to spend \$50,000 to create and send out newsletters to engage more customers, how will the CE change?

Q4. If a firm aims at maximising CE over 36 months, how should it allocate resources across acquisition, retention and win-back?

We assume that the current number of acquired customers is $a = 100$ per month, the current retention probabilities of new, established, and at-risk are $r_1 = 0.95, r_2 = 0.85, r_3 = 0.90,$ respectively, and the current win-back probability is $w = 0.05$. Given the current (acquisition, retention or win-back) spending, the current counts/probabilities, and the corresponding ceilings, we can estimate the three shape parameters by plugging the current spending, rate and ceiling to the three proposed curves $A(a)$, $R(r)$ and $W(w)$ and solving the equations, giving $k_1 = 0.05, k_2 = 0.6, k_3 = 1$. Table 2 compares different strategies and summarises answers to Q1–Q4. For Q1, if Site A uses the current allocation strategy for the next three years, then $\widehat{\text{CE}}$ is \$987,044 using (8).

Sensitivity analysis helps to answer Q2 and Q3. Suppose that all probabilities in the transition matrix are held constant except for p_{23}, which increases to 0.10, e.g., Site A takes some action to reduce the churn probability of established customers, causing a new retention probability of established customers and the corresponding retention spending: $r_2' = 0.92, R_2' = 4.42$. Thus, $\widehat{\text{CE}}$ becomes \$1,283,191. More specifically, if Site A increases retention spending per established subscriber from \$3.26 to \$4.42, $\widehat{\text{CE}}$ in the long-term increases 30% from \$987,044 to \$1,283,191. A small increase in the retention spending per established customer (\$1.16) can generate a large increase in $\widehat{\text{CE}}$ (\$296,147). The numerical result can be computed using the partial derivatives in (10). In practice to increase engagement managers must help customers develop reading habits and improve user experiences. Kim et al. (2021) found that regularity is a manifestation of habit and is negatively associated with subscription cancellation. They also showed that reading local news content (as opposed to commoditised content that can be found elsewhere), using ad blockers, and subscribing to newsletters help retain subscribers. Companies might also encourage app usage rather than browser use to improve the user experience (Peña et al., 2023).

Q3 asks about the long-term effects of a specific intervention, such as creating a newsletter. We need to know how such a newsletter will affect the churn probability, which could be determined either with a test or analysing an existing newsletter with a churn model. For this example, we use results from Zhou et al. (2021), which studied churn for those who do

Table 2. Comparison between different strategies to the current setting in the empirical example.

Strategies	Acquisition	Retention: New	Retention: Established	Retention: At-risk	Win-back	CE (Compare to current)
Current setting	100	0.95	0.85	0.90	0.05	\$987,044
	\$4.46	\$5.35	\$3.26	\$4.00	\$0.98	
Increase p_{23} to 0.10	100	0.95	0.92	0.90	0.05	\$1,283,191
	\$4.46	\$5.45	\$4.42	\$4.00	\$0.98	Increase 30.00%
Newsletter intervention	100	0.9625	0.8844	0.9239	0.05	\$1,153,145
	\$4.46	\$5.97	\$3.73	\$4.51	\$0.98	Increase 16.83%
Optimal strategy	334	0.9600	0.9575	0.9578	0.0590	\$1,736,550
	\$22.06	\$5.83	\$5.69	\$5.71	\$1.34	Increase 75.93%

Cost function parameters: $a_{\text{ceiling}} = 500, k_1 = 0.05, r_{\text{ceiling}} = 0.99, k_2 = 0.6, w_{\text{ceiling}} = 0.08,$ and $k_3 = 1$.

or do not subscribe to a sports newsletter. They found a logistic regression coefficient of −0.30, which indicates that customers who subscribe to sports newsletters are less likely to churn. For instance, if Site A plans to pay \$50,000 to hire staff to create the newsletter, then the log-odds of churn will decrease by 0.30. For simplicity, we assume that the newsletter has the same effect on all types of customers (in reality it might vary by segment). The transition matrix provides the current churn probabilities of new, established, at-risk as 0.05, 0.15, 0.10, respectively, and the corresponding odds of churn can be computed as $\frac{p}{1-p}$, where p is the probability of cancellation. After the intervention, the new odds of churn become $\exp(-0.30) = 0.7408$ multiplied by the original odds. We can easily convert the new odds to churn probabilities and calculate new retention probabilities of new, established, at-risk customers to be $r_1' = 0.9625$ $r_2' = 0.8844, r_3' = 0.9239$. Such a change corresponds to new retention spending: $R_1' = 5.97, R_2' = 3.73, R_3' = 4.51$. $\hat{\text{CE}}$ becomes \$1,153,145. More specifically, if the company spends \$50,000 to create a newsletter, $\widehat{\text{CE}}$ increases 16.83% from \$987,044 to \$1,153,145. The \$50,000 spending on newsletters increases retention probabilities and eventually generates a large increase in $\widehat{\text{CE}}$ (\$166,101).

To answer Q4, we use GD to maximise CE:

$$\begin{aligned} &\text{maximise}_{a, p_{13}, p_{23}, p_{32}, w} && \hat{\text{CE}}(\mathbf{a}, \mathbf{P}, \mathbf{v}) \\ &\text{subject to} && a \geq 0 \\ & && 0 \leq p_{13} \leq 0.25 \\ & && 0 \leq p_{23} \leq 0.18 \\ & && 0 \leq p_{32} \leq 0.40 \\ & && 0 \leq w \leq 1 \\ &\text{where} && \mathbf{a} = (a, 0, 0, 0)^T, \mathbf{P} = [p_{ij}], i, j = 1, \ldots, 4. \\ & && \mathbf{v} \text{ is a function of } \mathbf{a}, \mathbf{P} \end{aligned} \tag{11}$$

GD finds the optimal levels: $a^* = 334, p_{13}^* = 0.2100, p_{23}^* = 0.1375, p_{32}^* = 0.3578, w^* = 0.0590$ and $\widehat{\text{CE}}^* = \$1,736,549.79$ (last row in Table 2). The optimal strategy is to spend \$22.06 on acquiring 334 customers every month; spend \$5.83, \$5.69 and \$5.71 in retaining each new, established and at-risk customer, while maintaining the corresponding retention probability as 0.96, 0.9575, 0.9578; and spend \$1.34 on winning each churned customer back to life. Following this strategy, Site A expects to achieve a goal of \$1,736,550 in CE, which is a large improvement (increase 75.93%) compared to the current strategy.

6. Summary and discussion

This paper proposes a framework to use MCMs to optimise CE. Extending existing MCMs for CE, we allow for acquisition (adding customers to the system during each period), win-back over time, and for MCM parameters (e.g., transition probabilities) to be functions of decision variables that are under organisations' control. We derive closed-form expressions and derivatives for sensitivity analysis to help determine which actions will have the greatest effects on CE. Finally, we use gradient descent to optimise CE over actions, which provides strategic guidance on setting the optimal levels of decision variables. The framework is flexible in that it allows managers to add states and apply different strategies for customers in different states. The particular architecture of states should depend on the context. The contribution lies in optimising, rather than forecasting CLV/CE. Thus, we mainly consider transition probabilities to be functions of decision variables and illustrate how to conduct sensitivity analysis and optimisation.

The study has limitations that future research can address. MCM of CE is based on transition probabilities, the number of customers added to the system during each period, and the period cash flows/rewards. The proposed framework optimises over the first two and further studies can investigate optimising over the third. Our focus is on subscription services, where cash flows and rewards are usually constant subscription fees, but the approach can be extended and applied more generally to deal with situations where cash flows vary or are functions of decision variables. The framework can be extended to further incorporate different covariates in the transition probabilities to capture customer heterogeneity. Doing so may improve the predictive accuracy of CLV (Ascarza & Hardie, 2013). In terms of optimisation, if the included covariates are functions of decision variables, the current method can easily be applied by adding another layer of the chain rule to compute the partial derivatives.

The sensitivity analysis in this paper examines how levels of decision variables affect CE. The three (acquisition, retention and win-back) costs are key components of CE in that they directly determine the value vector. We find from some preliminary results that both CE and the partials are sensitive to the constant parameters (ceilings and shape parameters) in the cost curves. Certain invalid values of those parameters can even result in failure of the optimisation. Thus, another possibility of sensitive analysis is to investigate how the change in those constant parameters can affect CE. In addition, existing research applies BD's spending curves directly without validating their shapes. Future research can test the shape or propose other possible cost functions.

For illustration purpose, we ignore the dependency between the parameters in MCM. When we discuss the effect of one action, we assume that it only affects one of the parameters and thus impacts CE, while other parameters keep unchanged. However, the real situation can be much more complicated in that those parameters can be correlated. For example, if a news company wants to increase ad revenue, such action will not only decrease the retention probabilities (in that irritating ads bring negative experience and increase customers' churn rate), but also increase the revenue and thus increase the value vector. Future research could model the dependencies between parameters. Moreover, the assumption that the transition probabilities (retention and win-back rates) are constant over time can be relaxed. Future work can develop optimisation algorithms that deal with time-varying transition probabilities.

Furthermore, this study presents the algorithm that solves for the optimal allocation strategy within a certain periods of time (e.g., 3 years). However, managers might want to re-examine the current strategy routinely and make improvements over time. For example, we present a two-period optimisation strategy in Supplementary Appendix C and find that it outperforms the simple one-period constant strategy. We are also curious about whether one constant strategy is robust or we need to consider multiple different strategies. Further research can investigate using dynamic programming for optimisation. Finally, our goal was to optimise CE rather than forecasting. While improved forecast accuracy would be desirable, it would require comparing our model with the best existing models, and may be better accomplished with forecasting competitions such as the Makridakis competitions in time series (e.g., Makridakis et al., 2020). There has only been one contest for CLV (Ascarza & Hardie, 2013; Malthouse, 2009), and given the amount of research on CLV since this competition, it is time for another one.

Notes

1. When the decision variables are held constant then the transition probabilities will not change.
2. Empirical data shows that for Site A, the highlighted transition probabilities usually don't change much over time.

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