Option Pricing, Risk, and Planning Models

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- Risk and utility
- Observations from finance
- Option basics
- Applications in capacity plans
- General constraints

Modeling Steps

- Identify problem
- Determine objectives
- Specify decisions
- Find operating conditions
- Define metrics
  - How to measure objectives?
  - How to quantify requirements, limits?
  - How to include effect of uncertainty?
- Formulate

General Multistage Model

- **FORMULATION:**

  MIN $E\left[\sum_{t=1}^{T} f_t(x_t, x_{t+1})\right]$
  s.t. $x_t \in X_t$
  x_t nonanticipative
  $P[h_t(x_t, x_{t+1}) \leq 0] \geq a$ (chance constraint)

EXAMPLES:

- Linear functions, continuous variables
- Linear plus integer variables
- Nonlinear objective, continuous variables

WHY??

Utility Function Approach

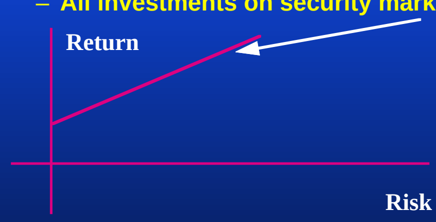
- **Observation:**
  - Most decision makers are adverse to risk
- **Assume:**
  - Outcomes can be described by a utility function
  - Decision makers want to maximize expected utility
- **Difficulties:**
  - Is the decision maker the sole stakeholder?
  - Whose utility should be used?
  - How to define a utility?
  - How to solve?
- **Alternative to decision maker - investor**

Measuring Investor Value

- SUPPOSE RISK NEUTRAL?
- (expected cost) objective
  - RESULT: Does not correspond to preference
  - Difficult to assess real value this way
- RESOLUTION:
  - Assume investors prefer lower risk
  - Investors can diversify away unique risk
  - Only important risk is market - contribution to portfolio
- CONSEQUENCE: Capital asset pricing model (CAPM)

Basics of CAPM

- RISK/RETURN TRADEOFF:
  - Investors can diversify
  - Firms need not diversity
  - All investments on security market line



NEED:Portfolio contribution - symmetric risk
How to determine?

Determing Risk Contribution

- USE CORRELATION?
  - Can measure for known markets (beta values)
  - If capacitated, depends on decisions
    - » Constrained resources
    - » Correlations among demands
- ALTERNATIVES?
  - Option Theory
    - » Allows for non-symmetric risk
    - » Explicitly considers constraints -
    - » As if selling excess to competitors at a given price

Use of Options

- Capacity limits potential sales
- View: option sold to competitor

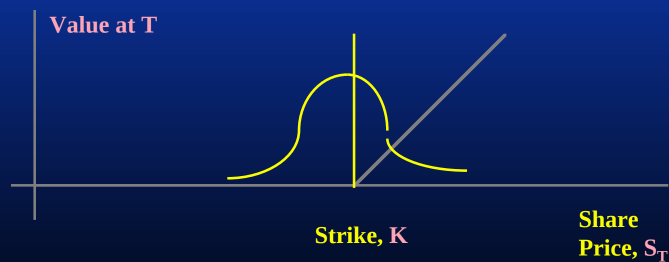
RESULTS FROM FINANCE:

- Assumption: risk free hedge
  - Can evaluate as if risk neutral
  - As in Black-Scholes model
- Steps in modeling:
  - Adjust revenue to risk-free equivalent
  - Discount at riskless rate

Valuing an Option

- (European) Call Option on Share assuming:
  - Buy at K at time T;Current time: t; Share price: S_t
  - Volatility: σ; Riskfree rate: r_f; No fees; Price follows Ito process
- Valuing option:
  - Assume risk neutral world (annual return=r_f independent of risk)
  - Find future expected value and discount back by r_f

Call value at t = $C_t = e^{-r_f(T-t)}\int(S_T-K)^+ dF_f(S_T)$



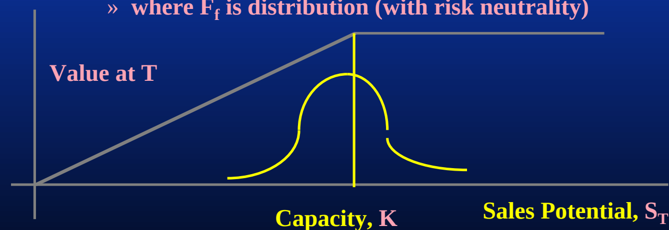
Relation to Capacity Evaluation

- What is the value of a plant with capacity K?
  - Discounted value of production up to K?
- Problems:
  - Production is limited by demand also (may be > K)
  - How to discount?
- Resolution:
  - Model as an option
  - Assume:
    - » Market for demand (substitutes)
    - » Forecast follows Ito process
    - » No transaction costs

∞ => Model like share minus call

Computing Capacity Value

- Goal: Production value with capacity K
  - Compute uncapacitated value based on CAPM:
    - » $S_t = e^{-r(T-t)} \int c_T S_T dF(S_T)$
    - » where c_T=margin,F is distribution (with risk aversion),
    - » r is rate from CAPM (with risk aversion)
  - Assume S_t now grows at riskfree rate, r_f ; evaluate as if risk neutral:
    - » Production value = $S_t - C_t = e^{-r_f(T-t)} \int c_T \min(S_T,K) dF_f(S_T)$
    - » where F_f is distribution (with risk neutrality)



Alternative Computation

- Approach:
  - Shift bounds instead of distribution
  - Replace F_f by F (riskfree to risk averse)
  - $F_f(A) = F(e^{(r-r_f)(T-t)} A)$ for any A
- Result:
  - $C_t = e^{-r_f(T-t)} \int c_t (S_T-K)^+ dF_f(S_T)$
  - $= e^{-(r-r_f)(T-t)} e^{-r_f(T-t)} \int e^{(r-r_f)(T-t)} c_t (S_T-K)^+ dF_f(S_T)$
  - $= e^{-r(T-t)} \int c_t (S_T - e^{(r-r_f)(T-t)} K)^+ dF(S_T)$
- Advantages:
  - No forecast changes
  - Extends to general models

General Models

- START: Eliminate constraints on production
  - Demand/market uncertainty remains
  - Can value unconstrained revenue with market rate, r:

$$e^{-rt}\ c_t\ x_t$$

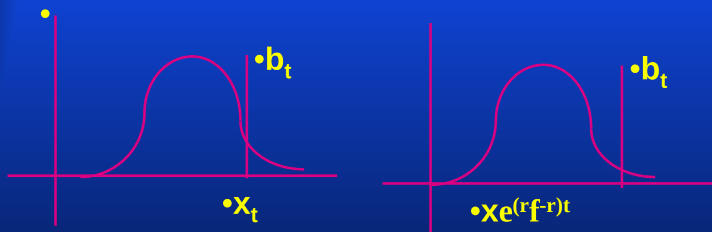
IMPLICATIONS OF RISK NEUTRAL HEDGE:
Can model as if investors are risk neutral
=> value grows at riskfree rate, r_f

Future value: $e^{-rt}c_t\ e^{r_f t}\ x_t$

BUT: This new quantity is constrained

CONSTRAINT MODIFICATION

- FORMER CONSTRAINTS: $A_t\ x_t \leq b_t$
- NOW: $A_t\ x_t e^{(r_f - r)t} \leq b_t$
-



New Period t Problem

- **WANT TO FIND (present value):**

$$e^{-r_f t} \int \text{MAX} [\, c_t\, x_t e^{-rt}\, e^{r_f t} \;|\; A_t\, x_t e^{-rt}\, e^{r_f t} \le b]$$

EQUIVALENT TO:

$$e^{-rt} \int \text{MAX} [\, c_t\, x \,|\, A_t\, x \le b e^{(r-r_f)t} \,]$$

MEANING: To compensate for lower risk with constraints, constraints expand and risky discount is used

Extreme Cases

- **ALL SLACK CONSTRAINTS:**

$$e^{-rt} \int \text{MAX} [\, c_t\, x \,|\, A_t\, x \le b e^{(r-r_f)t}]$$

becomes equivalent to:

$$e^{-rt} \int \text{MAX} [\, c_t\, x \,|\, A_t\, x \le b]$$

i.e. same as if unconstrained - risky rate

NO SLACK:

becomes equivalent to:

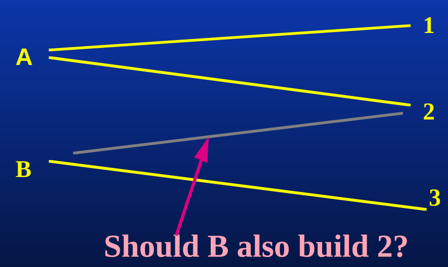
$$e^{-rt} \int [c_t\, x |\, x = B^{-1} b e^{(r-r_f)t}] = c_t\, B^{-1} b e^{-r_f t}$$

i.e. same as if deterministic- riskfree rate

Example: Capacity Planning

- What to produce?
- Where to produce? (When?)
- How much to produce?

EXAMPLE: Models 1,2, 3 ; Plants A,B



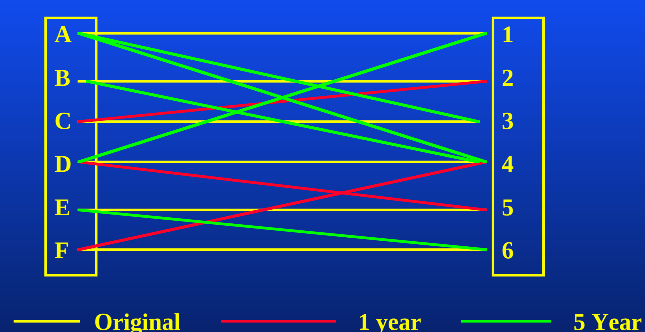
Stochastic Programming Model

- Key: Maximize the Added Value with Installed Capacity
  - Must choose best mix of models assigned to plants
  - Maximize Expected Value[$\Sigma_{i,t}$ e^{-rt}Profit (i) Production(i,t)
  - - CapCost(i at j,t)Capacity (i at j,t)]
  - subject to: MaxSales(i,t) $\geq \Sigma_j$ Production(i at j,t)
  - Σ_i Production(i at j,t) $\leq e^{(r-rf)t}$ Capacity (i,t)
  - Production(i at j,t) $\leq e^{(r-rf)t}$ Capacity (i at j,t)
  - Production(i at j,t) ≥ 0
- Need MaxSales(i,t) - random
  - Capacity(i at j,0) - Decision in First Stage (now)
- FIRST: Construct sales scenarios

EXAMPLE: Flexible Capacity- where?

- Find new capacity for next model year
- Model Data: from Graves/Jordan
- Vary: Model Lifetimes
  - Longer => More flexibility
- Start: 1 Year for all models (given all dedicated facilities)

Five Year Lifetime



- Note: new additions for 5 year
- Additional model years => more flexibility

Conclusions

- Utilility Modeling for Financial Objectives
  - Use investors’ preference
  - Problems with constraints
- Incorporating Constraints
  - Use risk neutral method from option theory
  - Effect:
    - » Discount objective with market rate
    - » Adjust unique linear constraints with discount factor ratio
    - » Maintain linear model with risk aversion
- Natural capacity planning interpretation
- Need for interpretation in other areas