Production-Financing Interactions and Optimal Capital Structure

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Themes

- Production decisions should reflect:
  - proper consideration of risk and market effects
  - method for financing operations
- Financing decisions should likewise depend on production decisions
- Integrated models can address both issues

Outline

- General background
- Two-stage model (news vendor)
  - Adjustment for risk
  - Financing without bankruptcy cost
  - Financing with bankruptcy cost
- Multi-period models
- Assumptions and revisions
- Conclusions

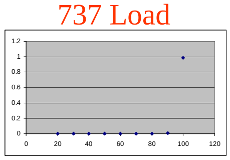
Background

- Traditional production models
  - Assume a fixed discount rate
  - No constraints on cash
- Why no fixed discount rate?
  - Risk depends on production decision
  - Discount rate should depend on risk
- How can financing change decision?
  - Orders and production require payments
  - Cost of payments depends on financing (reserves, borrowing, stretching payables, …)

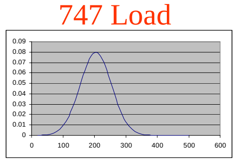
Why Change Rate?

- Consider a 737 (capacity 100) or 747 (capacity 400) for DTW-LGA route?
- Suppose demand has mean of 200 with standard deviation of 50
- 737 distribution: 100 passengers 98% of time
- 747 distribution: complete demand curve
- Should the same discount rate apply to both?

737 Load



747 Load



Why Consider Financing?

- Order tires for this month's production
- Cost $100/set
- Pay?
  - Stretch until receive payment? (5%/month)
  - Borrow (Limited? Interest to pay)
  - Investors?
- Cost of tires changes depending on form of financing

Simplified Model: News Vendor

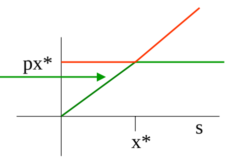
- Problem: how much to produce now for sales in future?
- Parameters: [no salvage value]
  - c – cost of production
  - p – selling price
  - F – distribution of demand
- Production: x* that maximizes over $x \geq 0$

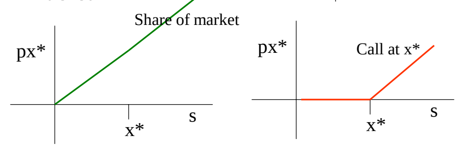
  $-cx + p(\int_0^x s\, dF(s) + x \int_x^\infty dF(s))$
- Solution:

  $F(x^*)=(p-c)/p$ or $x^*=F^{-1}((p-c)/p)$
- Problem: Have not discounted future and have not considered risk in cash flows

Future Payoff Function

- Future payoff as function of demand
- Note: Form of share minus call





Valuing the Future Payoff

- General Approaches
  - Capital asset pricing model (CAPM)
  - Option pricing models
- CAPM
  - Assume investors can diversify (without cost)
  - Observe market risk premium and correlation of cash flow to market
- Option pricing framework (no-arbitrage, risk-neutral, martingale pricing)
  - Assume market instruments can produce risk-free return
  - Use probabilities consistent with single risk-free rate (risk-neutral probabilities)

CAPM Approach

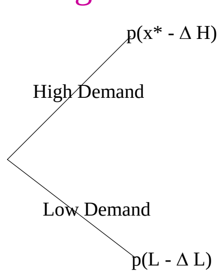
- Parameters:
  - S – uncertain future cash flow
  - r_f – risk-free rate
  - r_m – market rate of return
  - $\lambda = (r_m - r_f)/\sigma_m^2$ (market price of unit of risk)
  - E(S) – expected future cash flow
  - $Cov(S, r_m)$ – covariance of cash flow and market return
- Present value (period of length one)
  - $(E(S) - \lambda\,(Cov(S,r_m)))/(1+r_f)$
- Key: Finding covariance of cash flow and market return
- For news vendor, explored in Singhal (1988)

No-Arbitrage Pricing

- Idea: can trade market share (or equivalent cash flow) and firm cash flow to eliminate risk
- Suppose two demand outcomes, High:H (>x*) and Low: L (<x*)
- Firm's future payoff:px* if High, pL if Low
- Full market share returns: pH if High, pL if Low
- By selling Δ of market, can equate the payoffs:

  $p(x^*-\Delta H) = p(L - \Delta L)$

  where $\Delta=(x^*-L)/(H-L)$



Risk-neutral Pricing

- The cash flow of firm minus Δ of the market share is risk-free
- Can find the p.v. of cash flow given price of market share and risk-free bond (no-arbitrage assumption)

  PV $= \Delta$ (Market share) + Risk-free bond

  $= \Delta\,(pE(S)e^{-r}) + (pL(H-x^*)e^{-rf})/(H-L)$
- Observe: PV does not depend on probabilities – can use equivalent probability distribution that provides the same return on each investment (risk-neutral probabilities)
- Here:

  $Prob_f(H)H + (1-Prob_f(H))L =$ (Market share price)(e^{rf})
- Result: no-arbitrage $\Leftrightarrow$ risk-neutral probabilities

Risk-neutral Pricing

- If market is arbitrage free, then can find a risk-neutral equivalent measure, F_f, for news vendor
- Result:

$$F_f(x^*)=(p-e^{rf}c)/p \text{ or } x^*=F_f^{-1}((p-e^{rf}c)/p)$$

- How to find F_f?
- If demand distribution is log-normal, can treat as if market share futures trade as a geometric Brownian motion with rate r, risk premium $\delta=r-r_f$
- Result:

$$F(e^{\delta} x^*)= (p- e^{rf}c)/p \text{ or } x^*=e^{-\delta} F^{-1}((p-e^{rf}c)/p)$$

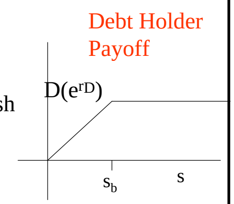
Note: time-value and risk adjustment

- Equivalent to Singhal's CAPM result

What about Financing?

- Have risk and time-value adjustment but assumes all-equity financing for cx*
- Suppose k available in equity and must borrow D to meet cash requirements
- Debt holder receives $D(e^{rD})$ if demand is greater than s_b, p s o.w. ($s_b = D(e^{rD})/p$)
- Result:

$$D\, e^{rf} = \int_0^{sb} ps\, dF_f(s) + D\, e^{rD} \int_{sb}^{\infty} dF_f(s)$$



News Vendor with Borrowing (Perfect Market)

- Optimal production x* should now maximize over $x \geq 0$

$$-ce^{rf} x + p(\int_0^x s\, dF(s) + x \int_x^{\infty} dF(s))$$

subject to:

$$c\, x \leq k + e^{-rf} (\int_0^{sb} ps\, dF_f(s) + D\, e^{rD} \int_{sb}^{\infty} dF_f(s))$$

- Result: If constraint slack, same x*; If constraint tight, separates to obtain same x*.
- Without taxes or costs of bankruptcy, production decision independent of financing

(Form of Miller-Modigliani irrelevance of capital structure)

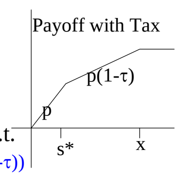
News Vendor with Corporate Tax

- Suppose all-equity and corporate tax
- Suppose corporate tax rate τ so that equity holder has returns as follows (no loss carryover):

$Y_E(x) =$ $px- \tau(p-c)x$ if $s \geq x$

$ps - \tau(ps-cx)$ if $x > s \geq s^*$

ps if $s^* > s$



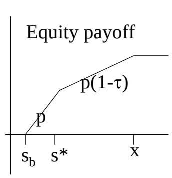
- Assume risk neutral pricing: find x* s.t.

$$F_f(x^*) = [p(1-\tau)-c(e^{rf}-\tau(1-F_f(s^*))]/(p(1-\tau)) < (p-ce^{rf})/p$$

- Tax ⇒ optimal production point ⇓

Debt and Equity Financing

- Suppose
  - financing by debt and equity
  - Interest payment produces tax shield
  - If bankruptcy (returns cannot cover debt payment), fixed recovery cost of B
- Equity holder payoff (now includes tax shield)



$Y_E(x) = px- \tau((p-c)x-(e^{rD}-1)\, D) - D(e^{rD})$ if $s \geq x$

$ps - \tau(ps-cx -(e^{rD}-1)\, D) - D(e^{rD})$ if $x > s \geq s^*$

$ps - D(e^{rD})$ if $s^* > s \geq s_b$

Debt Holder Cash Flow

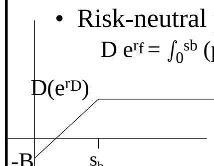
- Debt Holder Payoff

$Y_D(x) =$ $D\,(e^{rD})$ if $s \geq s_b$

$ps - B$ if $s_b > s$

- Risk-neutral pricing of debt

$$D\, e^{rf} = \int_0^{sb} (ps-B)\, dF_f(s) + D\, e^{rD} \int_{sb}^{\infty} dF_f(s)$$



Overall Value Function

- Find x^* and D^* to maximize over $x \geq 0$ and $D \leq cx$

$V(x,D) = \int_x^\infty [px - e^{rD}D - \tau (px - cx - (e^{rD}-1)D)]dF_f(s)$
$+ \int_{s^*}^x [ps - e^{rD}D - \tau (ps - cx - (e^{rD}-1)D)]dF_f(s)$
$+ \int_{sb}^{s^*}(ps - e^{rD}D)\, dF_f(s) + D e^{rD} \int_{sb}^\infty dF_f(s)$
$+ \int_0^{sb} (ps-B)\, dF_f(s) - c\, x\, (e^{rf})$

Assuming no boundary values, observe

$\partial V/\partial x = p(1-\tau)\int_x^\infty dF_f(s) + c\tau \int_{s^*}^\infty dF_f(s) - c\, e^{rf}$

i.e., marginal after-tax revenue plus marginal tax benefit equal marginal cost

Debt influence through s^* (breakeven with debt):

$\partial V/\partial D = \tau(1-F_f(s^*))(r_D + D(\partial r_D/\partial D)) - B\, \partial F(s_b)/\partial D$

i.e., expected bankruptcy cost equal expected tax shield.

Observations on Leveraged Firm

- With no bankruptcy cost (B=0),
  Solution approaches $D = cx$ (all debt)
- Production decreases as a function of financial leverage (i.e., Optimal x decreases with D, in particular, for all-equity optimum, x_e, $x^* < x_e$)
- Suppose best leverage for production of x_e is D_e, then optimal leverage, $D^* > D_e$
- General: Time value, risk, corporate tax, and financial leverage all lead to lower production than ideal case.
- Extensions: Agency effects, parameter effects.

Multiple Periods and General Production Requirements

- One-period simplifications
  - Debt must be re-paid in full
  - No decision on bankruptcy (abandonment) or continuation
  - No observation period or decision on delay
- Production/pricing simplifications
  - No general production capacity structure
  - No pricing decision

Multiperiod Cases

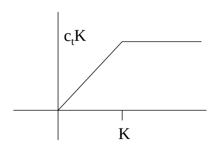
- General t-period future value of sales x_t:

  $c_t\, x_t$
- Suppose no constraints on production, assume market rate r, present value:

  $e^{-rt}\, c_t\, x_t$
- Constraints limit production to K, then price as share minus call, use risk-neutral pricing

$e^{-rf\, t} [\int_0^\infty c_t \min(s,K)\, dF_f(s)]$

Period t Payoff



Alternative Computation

- Conditions
  - Lognormal demand distribution
  - Futures on period-t demand follow geometric Brownian motion
  - $F_f(A) = F(e^{(r-r_f)(t)}A)$ for any A
- Result for sales lost (demand above K):

  Call at K = $e^{-r_f t} \int c_t(s-K)^+ dF_f(s)$

  $= e^{-(r-r_f)(t)} e^{-r_f t} \int e^{(r-r_f)(t)} c_t(s-K)^+ dF_f(s)$

  $= e^{-rt} \int c_t(s - e^{(r-r_f)(t)}K)^+ dF(s)$
- PV of production up to K:

  PV = $e^{-rt} \int (c_t s - c_t(s - e^{(r-r_f)(t)}K)^+) dF(s)$

  or max $e^{-rt} \int c_t x(s) dF(s)$

  s.t $x(s) \leq s$, $x(s) \leq e^{\delta t} K$ a.s.

General Period-t Constraints

- Assume risk neutral measure, general period t:

  max $e^{-rft} \int c_t x_t(s) dF_f(s)$

  s.t $x_t(s) \leq s$, $A_t x_t(s) \leq b$ a.s.
- Assume risk premium δ, lognormal distribution
- Substitution for d.f. F:

  $x_t'(s') = e^{\delta t} x_t(s) \leq e^{\delta t} s = s'$

  $A_t e^{-\delta t} x_t'(s') = A_t\, x_t(s) \leq b_t$ where s' has d.f. F
- Equivalent form:

  max $e^{-rt} \int c_t x_t(s) dF(s)$

  s.t $x_t(s) \leq s$, $A_t x_t(s) \leq e^{\delta t}$ b a.s.

Extreme Cases

- All production constraints slack:

$$\max e^{-rt} \int c_t x_t(s) dF(s)$$

$$\text{s.t } x_t(s) \le s,\ A_t x_t(s) \le e^{\delta t} b \text{ a.s.}$$

equivalent to: $e^{-rt} \int c_t s\, dF(s)$

(market share – correct by definition)

- All production constraints tight:

$$A_t x_t(s) = e^{\delta t} b \text{ a.s.}$$

with some basis B_t, so that $x^B_t(s) = B_t^{-1}(e^{\delta t} b)$ for all s, equivalent to:

$$e^{-rt} \int c_t B_t^{-1}(e^{\delta t} b)\, dF(s) = e^{-rt} e^{(r-rf)t} c_t B_t^{-1}(b) = e^{-rft} c_t B_t^{-1}(b)$$

(risk-free discounting – correct by definition)

- Result: method interpolates between values given by market

Multistage Model

- Private equity model (dividends either positive or negative in each period)
- One-period debt paid in each period
- Option to pay debt or abandon in each period
- Production requires cash in each period
- Corporate tax
- IPO or Sale at end of horizon
- General framework:

Maximize risk-neutral (or transformed market risk) discounted cash flows
subject to sales $\le$ demand,
sales $\le$ inventory plus production, production $\le$ capacity,
cashin – dividends – interest =cashout + productioncost +capacitycost + taxes,
capacityforward = 0 if cashout $\le$ 0.

Goals in Multi-period Model

- Consistent model for valuation
- Expandable for marketing, pricing decisions
- Observation objectives:
  - Effect of risk premium, demand volatility on production, capital structure
  - Relationship of abandonment timing to parameters
  - Equity cash flow changes under varying parameters
  - Effect of IPO/Sale timing on production, capital structure

Assumptions

- Price exogenous (fixed)
  - Set price to distributors (e.g., electricity)
  - Can relax (competitive or monopoly)
  - For monopoly, nonconvexity in optimization
- Risk-neutral pricing (constructing a risk-free hedge)
  - How to construct a hedge?
  - If NPV>0, inconsistency
  - Process: Trade option and asset to create risk-free security

Creating Best Hedge

- Underlying asset: Max potential sales in market
- Option: Plant with given capacity
- Other marketable securities:
  - Competitors' shares
  - Overall all securities min residual volatility
  - Due to incompleteness, some volatility remains (otherwise, NPV=0)

Result of Residual Risk

- In binomial model, sales can be $H+v_1$ or $L+v_2$ where v_1 and v_2 in some range
- Hedge ratio Δ is a function of v_1 and v_2 ($\Delta(v_1,v_2)$)
- Effective discount rate (and price of option) is in a range determined by v_1 and v_2
- Analogy to general case:
  - can reduce to some range on the risk premium
  - use other criteria (competitors, expanded utility) to choose

Capacity Implication

- Can adjust capacity limits by varying discount factor with risk neutral assumptions on forecasts
- Can vary constraint multipliers with original forecast distribution
- All optimal policies for the given range are consistent (i.e., cannot be beaten all the time)

Responses on Incompleteness

- Use equilibrium and utility function approaches
- Beware of model complexity
- Critical factor: range of outcomes considered
- Other challenges:
  - Effects of pricing decisions
  - Effects of competitors
  - Distribution changes from decisions

Operational and Financial Hedging in Global Markets

- Objective: Determine capacity levels in different markets, production in each market, distribution across markets, and use of financial hedging instruments to maximize total global value
- Challenges:
  - Demand and exchange rates may change
  - Correlations among demand and exchange
  - What is enough capacity?
  - What is the optimal mix of financial and operational hedges?

Situation

- Markets in US, Europe, and Asia
- Can add capacity in each place
- Have history of exchange rates and demand in each market
- Can transport across markets
- Know capacity costs
- Goal
  - Find the best capacity numbers in each market
  - Decide what measures to use
  - How well can you do?

Conclusions

- Production decisions depend on:
  - proper consideration of risk and market effects
  - method for financing operations
- Financing decisions depend on production decisions
- Integrated models address both issues
- Relaxing assumptions leads to range of outcomes
- Financial and operational hedges can be compared with comprehensive view