Multistage Stochastic Programming

John R. Birge
University of Michigan

Models - Long and short term
- Risk inclusion
Approximations - stages and scenarios
Computation

OUTLINE

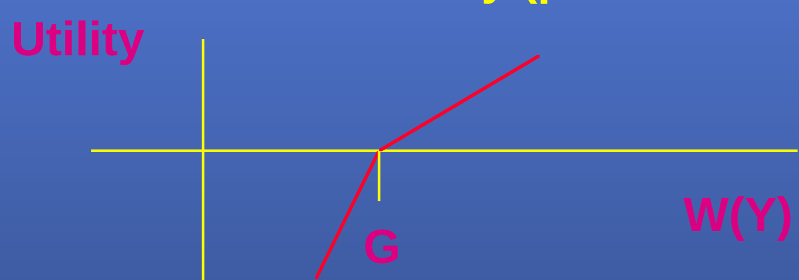
- Motivation - Short and Long Term Framework
- Long-Term: Finance/capacity decisions
  - Problems of uncertainty
  - General approach toward risk - options
- Short-Term: Production scheduling
  - Types of uncertainty
  - Results on cycles and matching up
  - Different role of risk
- General Model Approximations
- Computation
- Summary

Length of Horizon and Decisions

- LONG TERM HORIZON DECISIONS (YEARS)
  - STRATEGIES
  - OVERALL CAPACITY
  - PRODUCT MIX
  - SOURCES OF UNCERTAINTY
    - » MARKET
    - » COMPETITORS
- SHORT TO MEDIUM TERM DECISIONS (< YEAR)
  - ACTUAL PRODUCTION
  - DAILY TO MONTHLY MIX
  - VARIABLE PRODUCTIVE CAPACITY

Financial Planning

- **GOAL: Accumulate $G for tuition Y years from now (Long Term)**
- **Assume:**
  - $ W(0) - initial wealth
  - K - investments
  - concave utility (piecewise linear)



RANDOMNESS: returns r(k,t) - for k in period t
where Y ⟶ T decision periods

FORMULATION

- **SCENARIOS:** $\sigma \in \Sigma$
  - Probability, $p(\sigma)$
  - Groups, $S^t_1, \ldots, S^t_{St}$ at t
- **MULTISTAGE STOCHASTIC NLP FORM:**

$$\max \quad \Sigma_\sigma \; p(\sigma)\,(\,U(W(\sigma, T))\,)$$

s.t. (for all σ): $\Sigma_k\, x(k,1,\sigma) = W(o)$ (initial)

$$\Sigma_k\, r(k,t-1,\sigma)\, x(k,t-1,\sigma) - \Sigma_k\, x(k,t,\sigma) = 0, \quad \text{all } t > 1;$$

$$\Sigma_k\, r(k,T-1,\sigma)\, x(k,T-1,\sigma) - W(\sigma, T) = 0, \quad \text{(final)};$$

$$x(k,t,\sigma) \geq 0, \quad \text{all } k,t;$$

Nonanticipativity:

$$x(k,t,\sigma') - x(k,t,\sigma) = 0 \text{ if } \sigma', \sigma \in S^t_i \text{ for all } t, i, \sigma', \sigma$$

This says decision cannot depend on future.

DATA and SOLUTIONS

- **ASSUME:**
  - Y=15 years
  - G=$80,000
  - T=3 (5 year intervals)
  - k=2 (stock/bonds)
- **Returns (5 year):**
  - Scenario A: r(stock) = 1.25 r(bonds)= 1.14
  - Scenario B: r(stock) = 1.06 r(bonds)= 1.12
- **Solution:**

PERIOD	SCENARIO	STOCK	BONDS
1	1-8	41.5	13.5
2	1-4	65.1	2.17
2	5-8	36.7	22.4
3	1-2	83.8	0
3	3-4	0	71.4
3	5-6	0	71.4
3	7-8	64.0	0

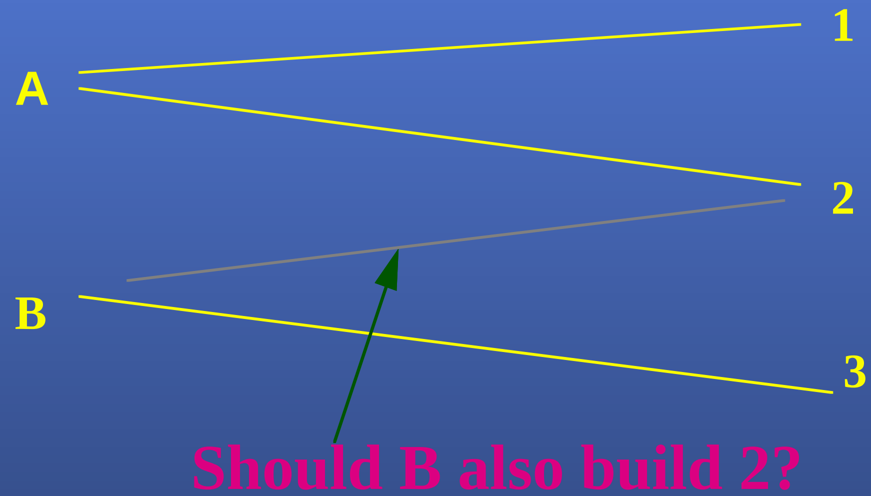
MODEL VALUES

- COMPARISON TO MEAN VALUES:
  - RP = -7 EMS=-19 (all stock investments)
    - » VSS = RP - EMS = 12
- HORIZON/PERIOD EFFECTS
  - TRUNCATION AT 10 YEARS
    - » MORE CONSERVATIVE
    - » HEAVY BOND INVESTMENT
  - LONG PERIODS
    - » MORE MEAN EFFECT - LESS DISTRIBUTION
    - » HEAVY STOCK INVESTMENT
- RESULT
  - NEED THREE PERIODS FOR HEDGING SOLUTION

CAPACITY DECISIONS

- What to produce?
- Where to produce? (When?)
- How much to produce?

EXAMPLE: Models 1,2, 3 ; Plants A,B



GOALS

- ADD AS MUCH VALUE AS POSSIBLE
- **But**: how do you measure value?

  - Net Present Values?
  - Discounted Cash Flows?
  - Net Profit?
  - Payback? IRR?

Traditional Approach

- Incremental Decision
  - Add Capacity at B for Model 2?
- Analysis
  - Find expected demand for 2?
  - Use expected demand for 1,3
  - => Discounted cash flows
- Result: No model 2 at B
  - Why?

ROLE OF UNCERTAINTY

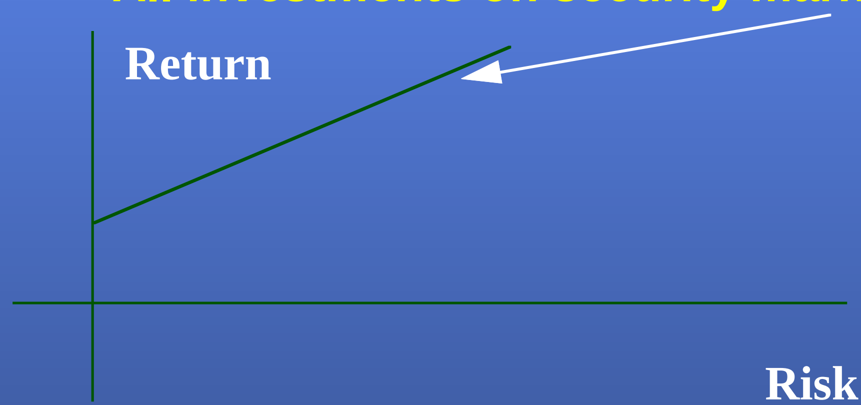
- **Problem: we do not know:**
  - what the demand will be
  - how much we really can produce in:
    - 1 day, 1 week, 1 month, 1 year
  - costs of inputs
  - competitor reaction
- **Result: Capacity for 2 at B may be useful if:**
  - demand for 2 higher than expected
  - demand for 3 lower than expected, demand for 1 higher
  - costs of 1 or 3 higher than expected, costs of 2 lower
  - short run capacity limit on 3
- **Effect: New capacity may add value**

MEASURING VALUE

- **SUPPOSE RISK NEUTRAL: (expected cost) objective**
  - **RESULT: Does not correspond to decision maker preference**
  - **Difficult to assess real value this way**
  -
- **RESOLUTION: use economic/financial theory:**
  - **Capital Asset Pricing Model**
  - **Efficient Market Theory**
  -
- **CONSEQUENCE: For financial objectives**
  - **Know how to assess based on risk**

BASICS OF CAPM

- RISK/RETURN TRADEOFF:
  - Investors can diversify
  - Firms need not diversity
  - All investments on security market line



NEED: Symmetric Risk

IMPLICATIONS FOR CAPACITY DECISIONS

- VALIDITY OF SYMMETRY:
  - Unlikely:
    - » Constrained resources
    - » Correlations among demands
- ALTERNATIVES?
  - Option Theory
    - » Allows for non-symmetric risk
    - » Explicitly considers constraints -
    - » Sell at a given price
-

USE OF OPTIONS

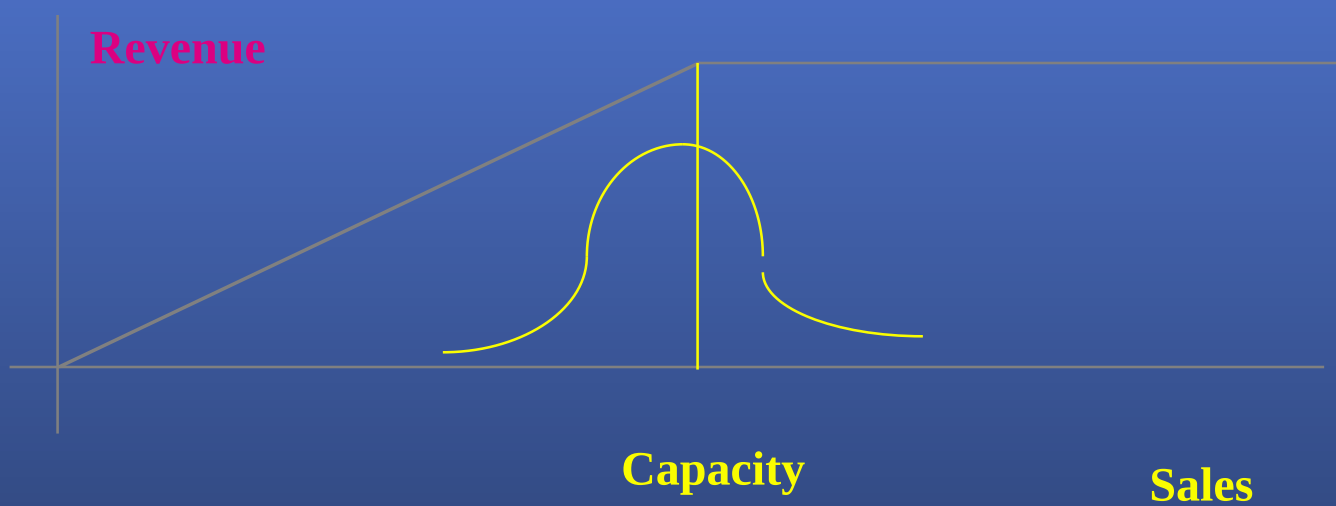
- CAPACITY LIMITS CUT OFF POTENTIAL REVENUE LIKE SELLING OPTION TO COMPETITOR
- VALUES ASYMMETRIC RISK

RESULTS FROM FINANCE:

- Assumption: risk free hedge
  - Can evaluate as if risk neutral
  - As in Black-Scholes model
- Steps with capacity evaluation:
  - Adjust revenue to risk-free equivalent
  - Discount at riskless rate

EVALUATING THE OPTION

- CANNOT USE EXPECTATIONS (SINGLE FORECASTS) ALONE BECAUSE OF:
  - Correlated Demand
    - Models 1,2,3 similar
  - Capacity Limit - cuts off revenue growth
    - => Asymmetric payoff



USE WITH A MODEL- Stochastic Programming

- Key: Maximize the Added Value with Installed Capacity
  - Must choose best mix of models assigned to plants
  - Maximize Expected Value[Σi Profit (i) Production(i)]
  - subject to: MaxSales(i) >= Σ j Production(i at j)
  - Σ i Production(i at j) <= Capacity (i)
  - Production(i at j) <= Capacity (i at j)
  - Production(i at j) >= 0
- Need MaxSales(i) - uncertain
  - Capacity(i at j) - Decision in First Stage (now)
- FIRST: Construct sales scenarios

Sales Scenarios

- **Difficulty:**
  - Many models
  - Correlations
  - High Variance
- **Simplification**
  - Graves, Jordan
  - Method for calculation with known distribution
- **Simulation**
  - Still need distribution
- **But *unknown* distribution**
  - **=> Use bounding approximations**

RESULTS OF OPTION-STOCHASTIC PROGRAMMING MODEL

- GIVES VALUE MEASURE
- INCORPORATES UNCERTAINTY AND ANY AVAILABLE INFORMATION
- CAN BE USED FOR VARYING MODEL LIFETIMES/PRODUCTION PERIODS
- INTEGRATES CAPACITY DECISIONS ACROSS FIRM (NOT JUST WITHIN 1 PLANT)
- CAN USE FOR UTILIZATION/LOST SALES/ OTHER WHAT-IF ANALYSES

GENERALIZATIONS FOR OTHER LONG-TERM DECISION

- **START: Eliminate constraints on production**
  - **Demand uncertainty remains - assume that is symmetric**
  - **Can value unconstrained revenue with market rate, r:**

$$1/(1+r)^t \; c_t \, x_t$$

IMPLICATIONS OF RISK NEUTRAL HEDGE:

Can model as if investors are risk neutral

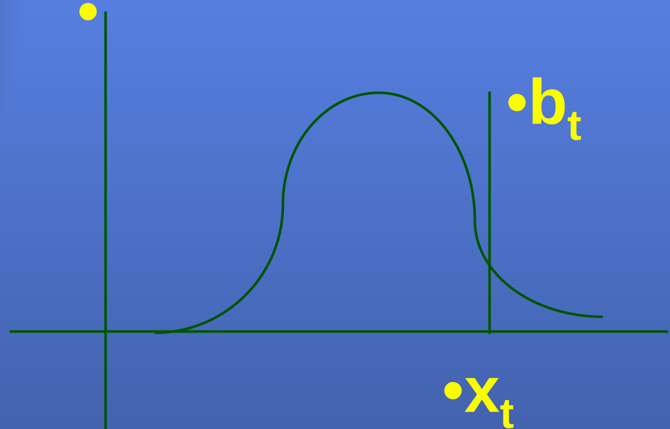
=> value grows at riskfree rate, r_f

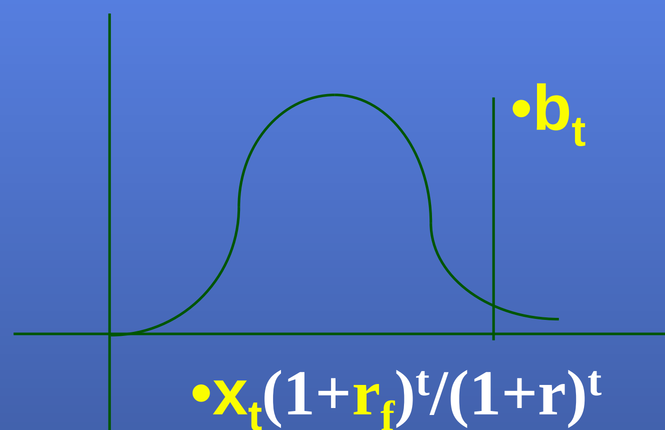
Future value: $[1/(1+r)^t \; c_t \, (1+r_f)^t \, x_t]$

BUT: This new quantity is constrained

CONSTRAINT MODIFICATION

- FORMER CONSTRAINTS: $A_t x_t \le b_t$
- NOW: $A_t x_t (1+r_f)^t/(1+r)^t \le b_t$
-





NEW PERIOD t PROBLEM

- **WANT TO FIND (present value):**

$$1/(1+r_f)^t \int \text{MAX}\,[\, c_t\, x_t\, (1+r_f)^t/(1+r)^t \mid A_t\, x_t\, (1+r_f)^t/(1+r)^t \le b]$$

EQUIVALENT TO:

$$1/(1+r)^t \int \text{MAX}\,[\, c_t\, x \mid A_t\, x \le b\, (1+r)^t/(1+r_f)^t]$$

MEANING: To compensate for lower risk with constraints, constraints expand and risky discount is used

EXTREME CASES

- **ALL SLACK CONSTRAINTS:**

$$1/(1+r)^t \int \mathrm{MAX}\,[\, c_t\, x \mid A_t\, x \le b\,(1+r)^t/(1+r_f)^t]$$

becomes equivalent to:

$$1/(1+r)^t \int \mathrm{MAX}\,[\, c_t\, x \mid A_t\, x \le b]$$

i.e. same as if unconstrained - risky rate

NO SLACK:

becomes equivalent to:

$$1/(1+r)^t \int [c_t\, x = B^{-1} b\,(1+r)^t/(1+r_f)^t] = c_t\, B^{-1} b/(1+r_f)^t$$

i.e. same as if deterministic- riskfree rate

OVERALL RESULTS - LONG-TERM

- CAN ADAPT OBJECTIVE TO RISK
- USE RATE FROM FIRM AS WHOLE
  - SYMMETRIC RISK
  - ASSUMES INVEST LIKE WHOLE FIRM
- ADJUST ALL CONSTRAINTS ON REVENUE GENERATORS BY RATE RATIOS
- END RESULT SHOULD REFLECT INVESTOR ATTITUDE TOWARD INVESTMENT

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SHORT-TERM UNCERTAINTIES

- EFFECTIVE CAPACITY LIMITED BY
  - UNCERTAIN YIELDS - QUALITY LOSS
  - MACHINE BREAKDOWNS
  - VARIABLE PRODUCTION RATES
  - UNFORESEEN ORDERS
  - LACK OF MATERIAL/SUPPLIES
  - LOGISTICAL PROBLEMS
- GENERAL FRAMEWORK
  - BASIC OPTIMIZATION PROBLEM
  - MUST DEFINE OBJECTIVES
  - LOOK AT STRUCTURE

Short Term Model

- **Risk**
  - Unique to situation (not market)
  - Solved many times
  - Focus on expectation (all unique risk - diversifiable)
- **Solution time**
  - Must implement decisions
  - Real-time framework
  - Need for efficiency
- **Coordination**
  - Maintain consistency with long-term goals

GENERAL MULTISTAGE MODEL

- **FORMULATION:**

$$\text{MIN} \quad E\left[\sum_{t=1}^{T} f_t(x_t, x_{t+1}) \right]$$

$$\text{s.t.} \quad x_t \in X_t$$

$$x_t \ \text{nonanticipative}$$

$$P[\, h_t(x_t, x_{t+1}) \le 0 \,] \ge a \ \text{(chance constraint)}$$

DEFINITIONS:

x_t - aggregate production

f_t - defines transition - only if resources available and includes subtraction of demand

DYNAMIC PROGRAMMING VIEW

- **STAGES: $t=1,\ldots,T$**
- **STATES: x_t -> $B_t x_t$(or other transformation)**
- **VALUE FUNCTION:**
  - ∠ $\Psi_t(x_t) = E[\psi_t(x_t,\xi_t)]$ where
  - ∠ ξ_t is the random element and
  - ∠ $\psi_t(x_t,\xi_t) = \min f_t(x_t,x_{t+1},\xi_t) + \Psi_{t+1}(x_{t+1})$
  - s.t. $x_{t+1} \in X_{t+1t}(\xi_t)$ x_t given
- **ASSUMPTIONS:**
  - CONVEXITY
  - EARLY AND LATENESS PENALTIES

PRODUCTION SCHEDULING RESULTS

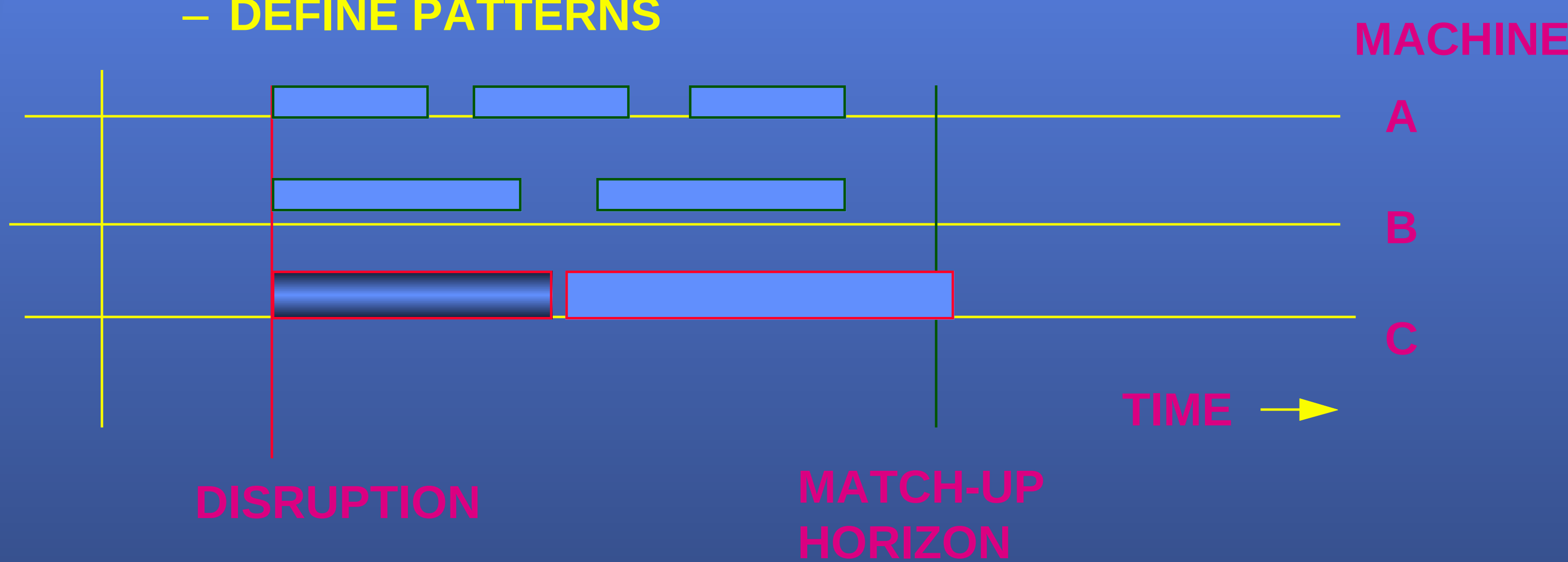
- **OPTIMALITY:**
  - CAN DEFINE OPTIMALITY CONDITIONS
  - DERIVE SUPPORTING PRICES
- **CYCLIC SCHEDULES:**
  - OPTIMAL IF STATIONARY OR CYCLIC DISTRIBUTIONS
  - MAY INDICATE KANBAN/CONWIP TYPE OPTIMALITY
- **TURNPIKE: (Birge/Dempster)**
  - FROM OTHER DISRUPTIONS:
  - RETURN TO OPTIMAL CYCLE
- **LEADS TO MATCH-UP FRAMEWORK**

MATCH-UP BASICS

- **METHOD: (Bean,Birge, Mittenthal, Noon)**
- **START: FIND a PRE-SCHEDULE (CYCLIC):**
  - FROM FORECASTS/NORMAL RANDOMNESS
- **MATCH-UP PROCESS:**
  - WHEN DISRUPTIONS OCCUR, RECOGNIZE THEM
  - TO DEVELOP RESPONSE, CONSTRUCT A PLAN TO MATCH UP WITH THE PRE-SCHEDULE IN THE FUTURE
  - OVERALL PATTERN REPRESENTS SETTING GOALS AND REACTING
  - MAY ALSO USE TO IMPROVE IN SHORT RUN

MATCH-UP PROBLEM

- **GOAL:** FIND A PERIOD OVER WHICH TO CHANGE SCHEDULE
  - DEFINE HORIZON
  - DEFINE SCENARIOS
  - DEFINE PATTERNS



HORIZON DEFINITION

- ISSUES:
  - LONG ENOUGH TO:
    - » SMOOTH OUT RESPONSE
    - » MAINTAIN LONG-TERM GOALS
    - » MAKE ECONOMIC CHOICE
  - SHORT ENOUGH TO:
    - » ALLOW RAPID RESPONSE
    - » COMPARE MANY ALTERNATIVES
    - » NOT UNDO OPTIMALITY IN PRE-SCHEDULE
- RESOLUTION
  - DAILY FOR SHORT-TERM

SCENARIO DEFINITION

- **ISSUES:**
  - NEED TO CAPTURE POSSIBLE FUTURE OUTCOMES
  - MUST MODEL
    - » DEMAND VARIATION
    - » PROCESSING INTERRUPTIONS
  - DIFFICULTIES
    - » INFINITE NUMBERS OF POSSIBILITIES
    - » LIMITED KNOWLEDGE BASES EXISTING
- **APPROACH**
  - START WITH INITIAL KNOWLEDGE
  - USE ALL INFORMATION TO ACHIEVE BEST MATCH

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Fundamental Questions

- **DP Procedure:**
  - Evaluate value from each state/stage
  - Use recursion
- **VALUE FUNCTION:**
  - ∠ $\Psi_t(x_t) = E[\psi_t(x_t,\xi_t)]$ where
  - ∠ ξ_t is the random element and
  - ∠ $\psi_t(x_t,\xi_t) = \min f_t(x_t,x_{t+1},\xi_t) + \Psi_{t+1}(x_{t+1})$
  - s.t. $x_{t+1} \in X_{t+1t}(\xi_t)$ x_t given
- **SOLVE : iterate from T to 1**
- **PROBLEM: How to find $E[\psi_t(x_t,\xi_t)]$?**
  - ∠ ξ_t may have high dimension

ALTERNATIVES FOR FINDING Ψ_t

- **DIRECT NUMERICAL INTEGRATION**
  - Possible only if very small or special structure
  - Not applicable to general, large problems
- **SIMULATION**
  - Limited convergence rate ($1/\sqrt{n}$ error for n samples)
  - Difficult estimates of confidence intervals on solutions
- **BOUNDING APPROXIMATIONS**
  - Find $\Psi_t^{l,k}$ and $\Psi_t^{u,k}$ such that:

    ∠ $\Psi_t^{l,k} \leq \Psi_t \leq \Psi_t^{u,k}$

    – $\lim_k \Psi_t^{l,k} = \Psi_t = \lim_k \Psi_t^{u,k}$

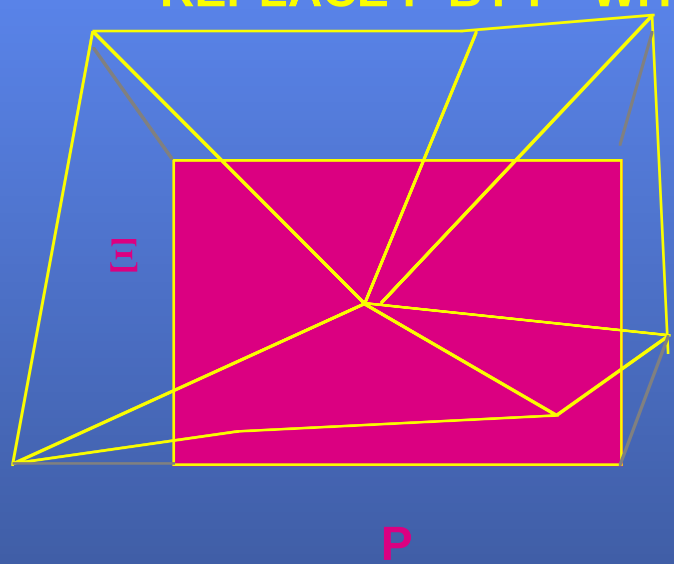
    – where limit is "epigraphical"

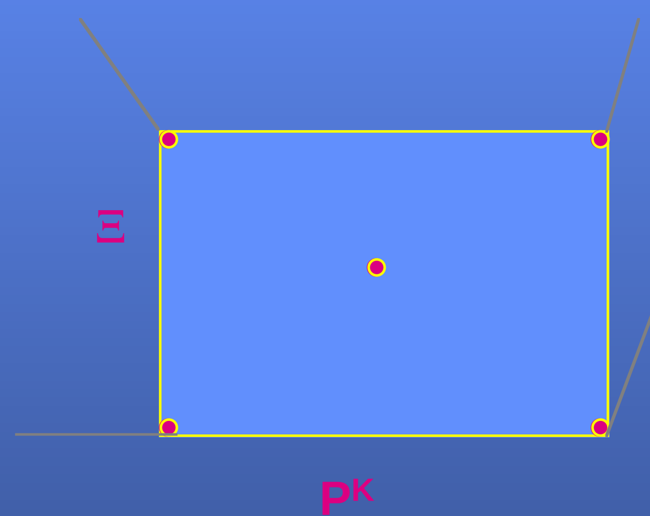
BOUNDING APPROXIMATIONS

- GOALS
  - MAINTAIN SOLVABLE SYSTEM
  - ENSURE SOLUTION VALUE WITHIN BOUNDS
  - CONVERGENCE OF BOUNDS
- BASIC IDEA
  - USE CONVEXITY/DUALITY
  - CONSTRUCT FEASIBLE:
    - » DUAL SOLUTIONS
      - LOWER BOUNDS
    - » PRIMAL SOLUTIONS
      - UPPER ROUNDS
- CONVERGENCE
  - NO DUALITY GAP
  - IMPROVING REFINEMENTS

DISCRETIZATIONS

- **SIMPLIFY THE DISTRIBUTION**
  - REPLACE P BY P^K WHICH HAS FINITE SUPPORT:





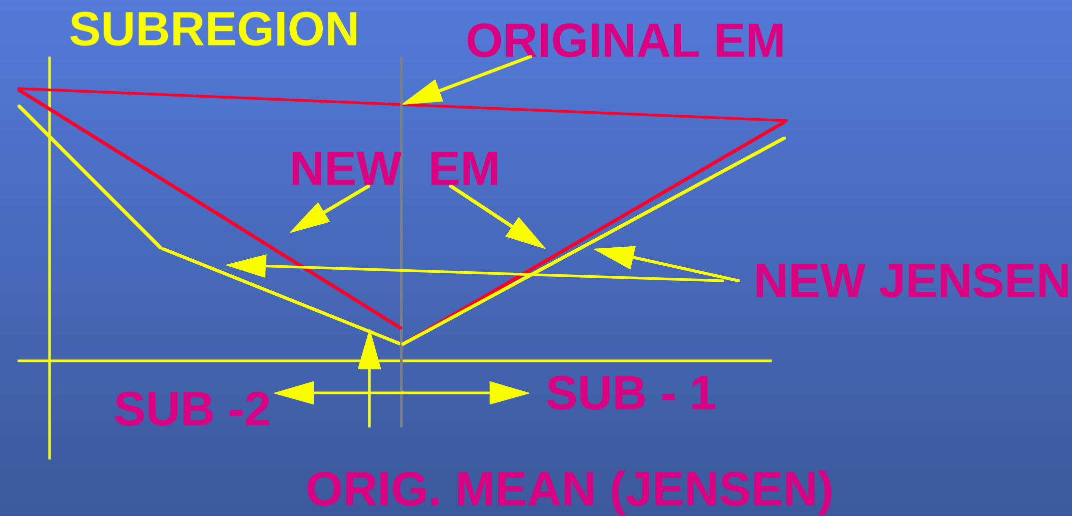
MIAIN PROCEDURES:

LOWER: JENSEN (MEAN)

UPPER: EDMUNDSON-MADANSKY (EXTREME POINTS)

BOUND IMPROVEMENTS

- **PARTITIONING**
  – SPLIT Ξ (SUPPORT OF RANDOM VECTOR) INTO SUBREGIONS
  – MAKE FUNCTION Ψ AS LINEAR AS POSSIBLE ON EACH SUBREGION



ENFORCE SEPARABILITY:

- FIND SEPARABLE RESPONSES TO ALL RANDOM PARAMETER CHANGES

Bounds across Periods

- Complications of many periods
  - Exponential growth in decision tree in no. of periods
  - End effects
- Methods:
  - Stationary/cyclic policies
    - » Just solve for the cycle length
  - Aggregation
    - » Collapse variables and constraints across periods
    - » Obtain bounds from duality/convexity
  - Response functions
    - » Find response that apply within a period
    - » Separate period effects

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SOLVING AS LARGE-SCALE MATHEMATICAL PROGRAMS

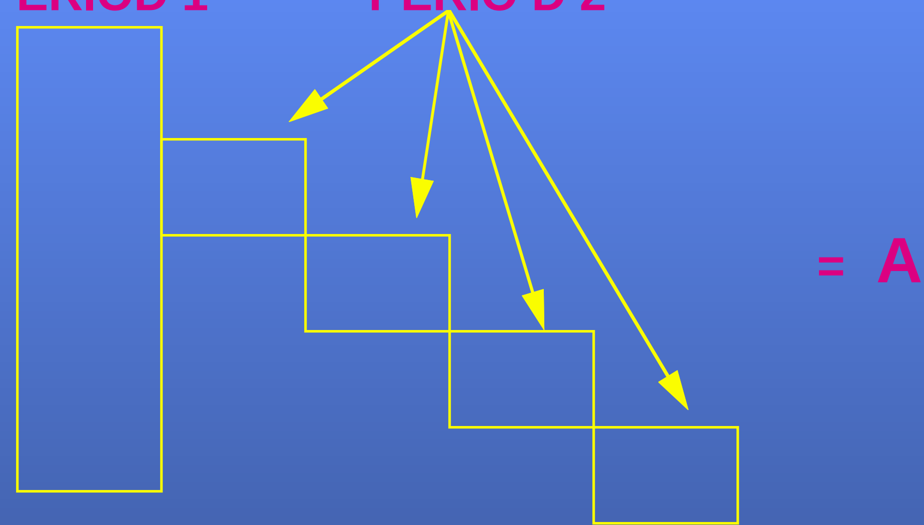
- **ORIGIN:**
  - DISCRETIZATION LEADS TO MATHEMATICAL PROGRAM BUT LARGE-SCALE
  - USE STANDARD METHODS BUT EXPLOIT STRUCTURE
- **DIRECT METHODS**
  - TAKE ADVANTAGE OF SPARSITY STRUCTURE
    - » SOME EFFICIENCIES
  - USE SIMILAR SUBPROBLEM STRUCTURE
    - » GREATER EFFICIENCY - DECOMPOSITION
- **SIZE**
  - UNLIMITED (INFINITE NUMBERS OF VARIABLES)
  - STILL SOLVABLE (CAUTION ON CLAIMS)

STANDARD APPROACHES

- PARTITIONING
- BASIS FACTORIZATION
- INTERIOR POINT FACTORIZATION
- LAGRANGIAN BASED
- MONTE CARLO APPROACHES
- DECOMPOSITION
  - BENDERS, L-SHAPED (VAN SLYKE - WETS0
  - DANTZIG-WOLFE (PRIMAL VERSION)
  - REGULARIZED (RUSZCZYNSKI)

LP-BASED METHODS

- USING BASIS STRUCTURE



- MODEST GAINS FOR SIMPLEX

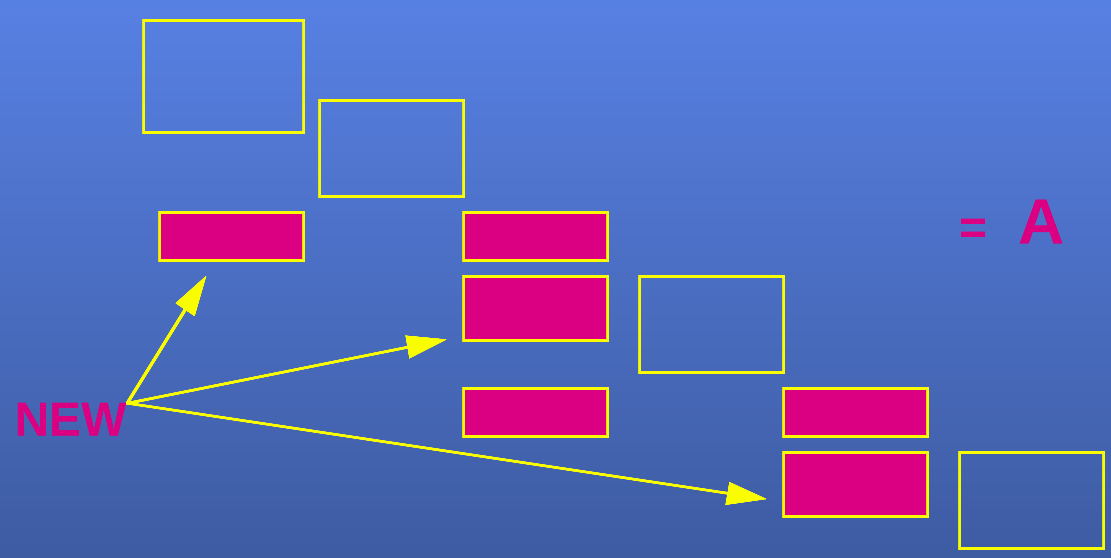
- INTERIOR POINT MATRIX STRUCTURE

$AD^2A^T=$

COMPLETE FILL-IN

ALTERNATIVES FOR INTERIOR POINTS

- **VARIABLE SPLITTING** (MULVEY ET AL.)
  - PUT IN EXPLICIT NONANTICIPATIVITY CONTRAINTS

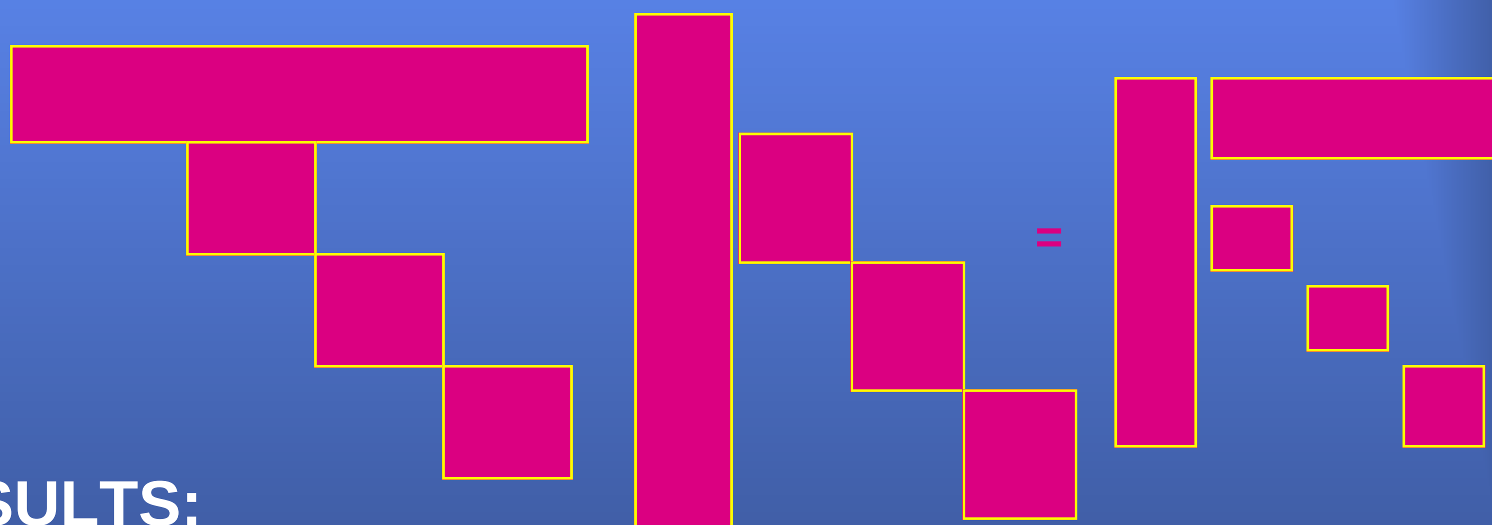


- **RESULT**
  - REDUCED FILL-IN BUT LARGER MATRIX

OTHER INTERIOR POINT APPROACHES

- USE OF DUAL FACTORIZATION OR MODIFIED SCHUR COMPLEMENT

$A^T D^2 A =$



RESULTS:

- SPEEDUPS OF 2 TO 20
- SOME INSTABILITY => INDEFINITE SYSTEM (VANDERBEI ET AL. CZYZYK ET AL.)
- MULTISTAGE IMPLEMENTATIONS USING LINKS (BERGER, MULVEY)

Lagrangian-based Approaches

- **General idea:**
  - Relax nonanticipativity
  - Place in objective
  - Separable problems

MIN $E[\sum_{t=1}^{T} f_t(x_t, x_{t+1})]$
s.t. $x_t \in X_t$
x_t nonanticipative

→

MIN $E[\sum_{t=1}^{T} f_t(x_t, x_{t+1})]$
$x_t \in X_t$
$+ E[\underline{w}_t x] + r/2 ||x - \underline{x}||^2$

Update: w_t; **Project:** x into N - nonanticipative space

Convergence: Convex problems - Progressive Hedging Alg. (Rockafellar and Wets)

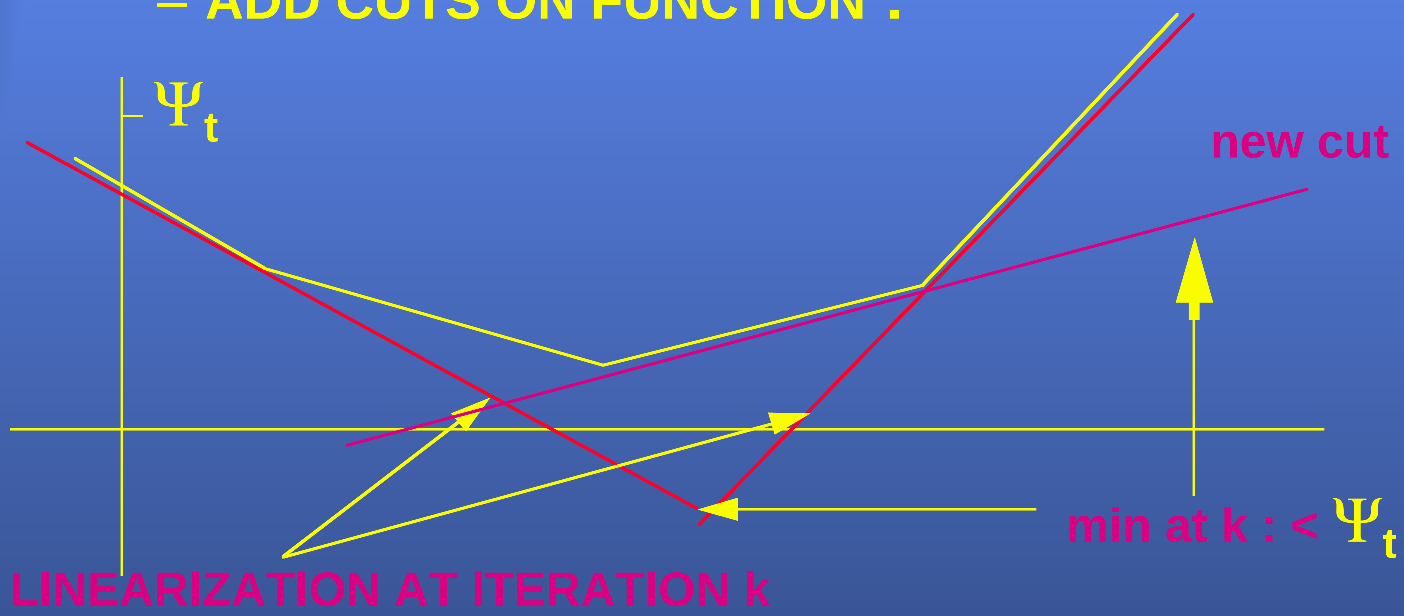
Advantage: Maintain problem structure (networks)

Lagrangian Methods and Integer Variables

- **Idea:** Lagrangian dual provides bound for primal but
  - Duality gap
  - PHA may not converge
- **Alternative:** standard augmented Lagragian
  - Convergence to dual solution
  - Less separability
  - Duality gap decreases to zero as number of scenarios increases
- **Problem structure:** Power generation problems
  - Especially efficient on parallel processors

DECOMPOSITION METHODS

- **BENDERS IDEA**
  - FORM AN OUTER LINEARIZATION OF Ψ_t
  - ADD CUTS ON FUNCTION :



USE AT EACH STAGE TO APPROXIMATE VALUE FUNCTION

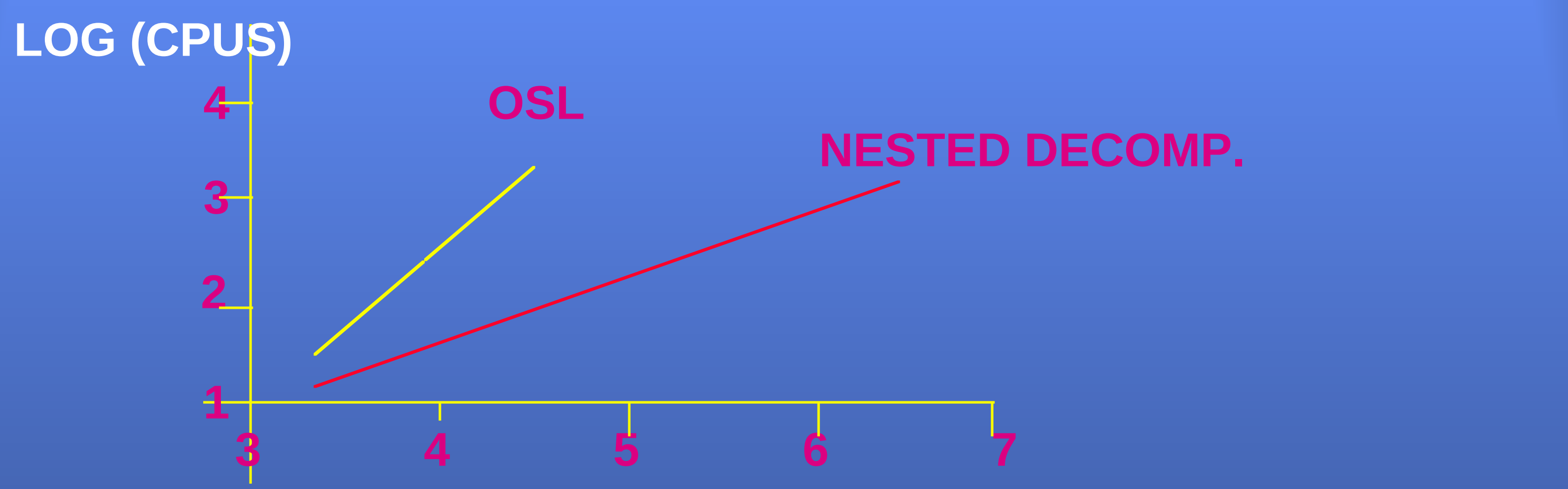
- ITERATE BETWEEN STAGES UNTIL ALL MIN = Ψ_t

DECOMPOSITION IMPLEMENTATION

- NESTED DECOMPOSITION
  - LINEARIZATION OF VALUE FUNCTION AT EACH STAGE
  - DECISIONS ON WHICH STAGE TO SOLVE, WHICH PROBLEMS AT EACH STAGE
- LINEAR PROGRAMMING SOLUTIONS
  - USE OSL FOR LINEAR SUBPROBLEMS
  - USE MINOS FOR NONLINEAR PROBLEMS
- PARALLEL IMPLEMENTATION
  - USE NETWORK OF RS6000S
  - PVM PROTOCOL

RESULTS

- SCAGR7 PROBLEM SET



PARALLEL: 60-80% EFFICIENCY IN SPEEDUP

OTHER PROBLEMS: SIMILAR RESULTS

- ONLY < ORDER OF MAGNITUDE SPEEDUP WITH STORM
  - TWO-STAGES - LITTLE COMMONALITY IN SUBPROBLEMS
  - STILL ABLE TO SOLVE ORDER OF MAGNITUDE LARGER PROBLEMS

SOME OPEN ISSUES

- MODELS
  - IM PACT ON METHODS
  - RELATION TO OTHER AREAS
- APPROXIMATIONS
  - USE WITH SAMPLING METHODS
  - COMPUTATION CONSTRAINED BOUNDS
  - SOLUTION BOUNDS
- SOLUTION METHODS
  - EXPLOIT SPECIFIC STRUCTURE
  - MASSIVELY PARALLEL ARCHITECTURES
  - LINKS TO APPROXIMATIONS

CRITICISMS

- **UNKNOWN COSTS OR DISTRIBUTIONS**
  - FIND ALL AVAILABLE INFORMATION
  - CAN CONSTRUCT BOUNDS OVER ALL DISTRIBUTIONS
    - » FITTING THE INFORMATION
  - STILL HAVE KNOWN ERRORS BUT ALTERNATIVE SOLUTIONS
- **COMPUTATIONAL DIFFICULTY**
  - FIT MODEL TO SOLUTION ABILITY
  - SIZE OF PROBLEMS INCREASING RAPIDLY (MORE THAN 10 MILLION VARIABLES)

CONCLUSIONS

- **LONG AND SHORT TERM HORIZONS**
  - LONG - NEED FOR RISK AVERSION; OPTIONS
  - SHORT - RISK MORE UNIQUE; NEED FOR EFFICIENCY
  - COORDINATION WITH LONG-TERM: MATCH-UP
- **APPROXIMATIONS**
  - STATE EXPLOSION ACROSS STAGES
  - BOUNDS ON VALUE FUNCTION
  - USES OF PROBLEM STRUCTURE
- **SOLUTIONS**
  - STRUCTURE FOR DIRECT METHODS - INTERIOR
  - VANISHING DUALITY GAPS WITH INCREASING SIZE
  - ADVANTAGES IN DECOMPOSITION
  - PROBLEM SIZES IN MILLIONS OF VARIABLES

What Next?

(A Biased Partial List)

- Integer variables - across stages
- Continuous time models
- Complexity theory
- Dynamic sampling statistics
- Path integral approaches from quantum mechanics
- Problem structure exploitation
- Deterministic sampling theory
- Real-time applications - implementations
- Incorporate learning/Bayesian type models
- Multiple agents/distributed/competition

More Information?

http://www-personal.umich.edu/~jrbirge