Real Option Valuation in Investment Planning Models

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Northwestern University

SVOR Meeting, Thun, October 2001
Outline

• Planning questions
• Problems with traditional analyses: examples
• Real-option structure
• Assumptions and differences from financial options
• Resolving inconsistencies
• Conclusions
Investment Situation: Automotive Company

• **Goal:**
  - Decide on coordinated production, distribution capacity and vendor contracts for multiple models in multiple markets (e.g., NA, Eur, LA, Asia)

• **Traditional approach**
  - Forecast demand for each model/market
  - Forecast costs
  - Obtain piece rates and proposals
  - Construct cash flows and discount

☞ **Optimize for a single-point forecast**
Planning Questions?

• Start product in production or not? When?
• What to produce in-house or outside?
• How much capacity to install?
• What contracts to make outside?
• External factors: economy, competitors, suppliers, customers, legal, political, environmental
• Where to start?
  – Build a model
Traditional Model Results

• Focus on:
  • Cost orientation (not revenue management)
  • Single program (model, product)
  • NPV
  • Piece rates

• Result: support of traditional, fixed designs, little flexibility, little ability to change, immediate investment or no investment
Trends Limiting Traditional Analysis

• Market changes
  • Former competition:
    • Cost
    • Quality
  • New competition:
    • Customization
    • Responsiveness
Limitations of Traditional Methods for New Trends

- Myopic - ignoring long-term effects
- Often missing time value of cash flow
- Excluding potential synergies
- Ignoring uncertainty effects
- Not capturing option value of delay, scalability, and agility (changing product mix)
- Mis-calculate time-value of cash flow
Outline

• Planning questions
  • Problems with traditional analyses: examples
    • Value to delay
    • Scalability
    • Reusability
    • Agility
  • Real-option structure
  • Assumptions and differences from financial options
  • Resolving inconsistencies
  • Conclusions

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Value to Delay Example

• Suppose a project may earn:
  • $100M if economy booms
  • $-50M if economy busts
• Each (boom or bust) is equally likely
• NPV = $25M (expected) - Start project
• Missing: Can we wait to observe economy?

Here, we don’t need to invest in “Bust” - Now we expect $50M
It’s worth $25M to wait.
Scale Option Example

- Scalability
- Suppose a five year program
  - Cost of fixed capacity is $100M
  - Cost of scalable capacity is $150M for same capacity
- Predicted cash flow stream:

<table>
<thead>
<tr>
<th>Year</th>
<th>Net</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
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<td>4</td>
<td>50</td>
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<td>5</td>
<td>25</td>
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</table>
Scalability Example - cont.

- Assume 15% opportunity cost of capital:
  - \( \text{NPV(Traditional)} = $50M \)
  - \( \text{NPV(Scalable)} = 0 \)

- Problem: Scalable can be configured over time:

<table>
<thead>
<tr>
<th>Year</th>
<th>Spend $50M for capacity to $25M</th>
<th>Spend $50M for cap. to $50M</th>
<th>Spend $50M for cap. to $75M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
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</table>
Scalability Result

Cash flow for Scalable:

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td>Net</td>
<td>-50</td>
<td>-25</td>
<td>0</td>
<td>75</td>
<td>50</td>
<td>25</td>
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Now, NPV(Scalable)=$75M > NPV(Fixed)

Traditional approach misses scalability advantage.
Reusability Example

- Assume:
  - Same conditions as before for fixed system
  - Two consecutive 5-year programs
  - Suppose for Reusable Manufacturing System (RMS)
    - No scalability
    - Initial cost of $125 M
    - Can reconfigure for second program at cost of $25M
Reusability Example cont.

- Traditional approach
  - Single program evaluation
  - NPV(Fixed) = $50M
  - NPV(RMS) = $25M
  - Choose Fixed

- Problem: Missing the second program
Reusability Two-Program Cash Flows

Fixed cash flow, NPV(Fixed) = $75M

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<tr>
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<th>8</th>
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<td>25</td>
<td>50</td>
<td>75</td>
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RMS Cash Flow, NPV(RMS) = $87M

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<td>50</td>
<td>75</td>
<td>50</td>
<td>25</td>
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</tbody>
</table>

Traditional method misses two-program advantage
Agility Example: Flexible Capacity Option

Difficulty: Traditional single forecast

Example: Products A, B

- Forecast demand: 100 for each; Margin: 2
- Dedicated capacity cost: 1
- Flexible capacity cost: 1.1

<table>
<thead>
<tr>
<th></th>
<th>Dedicated</th>
<th>Flexible</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Cost</td>
<td>200</td>
<td>220</td>
</tr>
<tr>
<td>Profit</td>
<td>200</td>
<td>180</td>
</tr>
</tbody>
</table>

Choose dedicated
Suppose two demand possibilities: 50 or 150 equally likely - *Four scenarios*

<table>
<thead>
<tr>
<th>Dedicated:</th>
<th>Production of A:</th>
<th>Flexible:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1: 50, 50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scenario 2: 50, 150</td>
<td></td>
<td></td>
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<tr>
<td>Scenario 3: 150, 50</td>
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<td></td>
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<tr>
<td>Scenario 4: 150, 150</td>
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</tbody>
</table>

Additional Production
## Evaluation with Scenarios

- **Four scenarios:** 50 or 150 on each
- **Dedicated**
  - Sell (50,50), (50,100), (100,50), (100, 100)
  - Expected revenue: 300
- **Flexible**
  - Sell (50,50), (50,150), (150,50), (100, 100)
  - Expected revenue: 350

<table>
<thead>
<tr>
<th>Dedicated:</th>
<th>Flexible:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. Revenue:</td>
<td>300</td>
</tr>
<tr>
<td>Cost:</td>
<td>200</td>
</tr>
<tr>
<td>Profit:</td>
<td>100</td>
</tr>
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</table>

Choose flexible
Summary of Problems

• Missing basic abilities in traditional approaches:
  – Delay option
  – Scaling option
  – Reuse option
  – Agility option

• Option evaluation:
  – Look at all possibilities
  – How to discount?
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Real Options

- Idea: Assets that are not fully used may still have option value (includes contracts, licenses)
- Value may be lost when the option is exercised (e.g., developing a new product, invoking option for second vendor)
- Traditional NPV analyses are flawed by missing the option value
- Missing parts:
  - Value to delay and learn
  - Option to scale and reuse
  - Option to change with demand variation (uncertainty)
  - Not changing discount rates for varying utilizations
Planning Questions?

• Start product in production or not? When?
• What to produce in-house or outside?
• How much capacity to install?
• What contracts to make outside?
• External factors: economy, competitors, suppliers, customers, legal, political, environmental
• Where to start?
  – Build a model
Key Steps in Building a Model

• Identify problem
• Determine objectives
• Specify decisions
• Find operating conditions
• Define metrics
  – How to measure objectives?
  – How to quantify requirements, limits?
  – How to include effect of uncertainty?
• Formulate
Utility Function Approach

• Observation:
  – Most decision makers are adverse to risk

• Assume:
  – Outcomes can be described by a utility function
  – Decision makers want to maximize expected utility

• Difficulties:
  – Is the decision maker the sole stakeholder?
  – Whose utility should be used?
  – How to define a utility?
  – How to solve?

• Alternative to decision maker - investor
Measuring Investor Value

• **RISK NEUTRAL?**
  – Expected cost objective
  – **RESULT**: Does not correspond to preference
  – Difficult to assess real value this way

• **OBSERVATIONS:**
  – Assume investors prefer lower risk
  – Investors can **diversify** away unique risk
  – Only important risk is market - contribution to portfolio

• **CONSEQUENCE**: Capital asset pricing model (CAPM)
  – With CAPM, can find a **discount rate**
Discount Rate Determination

• Traditional approach
  – Discount rate is the same for all decisions in program evaluation

• Problems
  – Program evaluation includes decisions on capacity, distribution channel, vendor contracts
  – These decisions affect correlation to market – hence, change the discount rate

• Need: discount rate to change with decisions as they are determined; How?
Discount Rate Determination

• USE CAP-M? FIND CORRELATION TO THE MARKET?
  • Can measure for known markets (beta values)
  • If capacitated, depends on decisions
    • Constrained resources - capacity
    • Correlations among demands

• ALTERNATIVES?
  • Option Theory
    • Allows for non-symmetric risk
    • Explicitly considers constraints -
      • As if selling excess to competitors at a given price

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Valuing an Option

- (European) Call Option on Share assuming:
  - Buy at $K$ at time $T$; Current time: $t$; Share price: $S_t$
  - Volatility: $\sigma$; Riskfree rate: $r_f$; No fees; Price follows Ito process
- Valuing option:
  - Assume risk neutral world (annual return=$r_f$ independent of risk)
  - Find future expected value and discount back by $r_f$

Call value at $t = C_t = e^{-r_f(T-t)}(S_T-K)^+dF_t(S_T)$

\[ \text{Share Price, } S_T \]
\[ \text{Strike, } K \]

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Relation to Real Options

• Example: What is the value of a plant with capacity K?
  • Discounted value of production up to K?

• Problems:
  • Production is limited by demand also (may be > K)
  • How to discount?

• Resolution:
  • Model as an option
  • Assume:
    • Market for demand (substitutes)
    • Forecast follows Ito process
    • No transaction costs

?? Model like share minus call

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Using Option Valuation for Capacity

• **Goal:** Production value with capacity K
  • Compute uncapacitated value based on CAPM:
    • $S_t = e^{-r(T-t)} c_T S_T dF(S_T)$
    • where $c_T$ = margin, $F$ is distribution (with risk aversion),
    • $r$ is rate from CAPM (with risk aversion)
  • Assume $S_t$ now grows at riskfree rate, $r_f$; evaluate as if risk neutral:
    • Production value = $S_t - C_t = e^{-r(T-t)} c_T \min(S_T, K) dF_f(S_T)$
    • where $F_f$ is distribution (with risk neutrality)
Generalizations for Other Long-term Decisions

• Model: period t decisions: $x_t$
• START: Eliminate constraints on production
  – Demand uncertainty remains
  – Can value unconstrained revenue with market rate, $r$: 
    $$\frac{1}{(1+r)^t} c_t x_t$$

**IMPLICATIONS OF RISK NEUTRAL HEDGE:**
Can model as if investors are risk neutral
=> value grows at riskfree rate, $r_f$

**Future value:** $[\frac{1}{(1+r)^t} c_t (1+r_f)^t x_t]$  

**BUT:** This new quantity is constrained
New Period t Problem: Linear Constraints on Production

• WANT TO FIND (present value):

$\frac{1}{(1+r_f)^t} \int \max \left[ c_t x_t \frac{(1+r_f)^t}{(1+r)^t} \mid A_t x_t \frac{(1+r_f)^t}{(1+r)^t} \leq b \right]$

EQUIVALENT TO:

$\frac{1}{(1+r)^t} \int \max \left[ c_t x \mid A_t x \leq b \frac{(1+r)^t}{(1+r_f)^t} \right]$

MEANING: To compensate for lower risk with constraints, constraints expand and risky discount is used
Constraint Modification

• FORMER CONSTRAINTS: $A_t x_t \leq b_t$
• NOW: $A_t x_t (1+r_f)^t/(1+r)^t \leq b_t$
EXTREME CASES

All slack constraints:
\[
\frac{1}{(1+r)^t} \max \left[ c_t x \mid A_t x \leq b \right] \frac{(1+r)^t}{(1+r_f)^t}
\]
becomes equivalent to:
\[
\frac{1}{(1+r)^t} \max \left[ c_t x \mid A_t x \leq b \right]
\]
i.e. same as if unconstrained - risky rate

NO SLACK:
becomes equivalent to:
\[
\frac{1}{(1+r)^t} \left[ c_t x = B^{-1}b \right] \frac{(1+r)^t}{(1+r_f)^t} = c_t B^{-1}b / (1+r_f)^t
\]
i.e. same as if deterministic - riskfree rate
Example: Capacity Planning

• What to produce?
• Where to produce? (When?)
• How much to produce?

**EXAMPLE: Models 1, 2, 3; Plants A, B**

Should B also build 2?
Result: Stochastic Linear Programming Model

- Key: Maximize the Added Value with Installed Capacity
  - Must choose best mix of models assigned to plants
  - Maximize Expected Value over $s$ of $\sum_i \sum_{t,s} e^{-rt} \text{Profit (i) Production}(i,t,s) - \text{CapCost}(i \text{ at } j,t) \text{Capacity (i at } j,t)$
  - subject to: $\text{MaxSales}(i,t,s) \geq \sum_j \text{Production}(i \text{ at } j,t,s)$
  - $\sum_i \text{Production}(i \text{ at } j,t,s) \leq e^{(r-r_f)t} \text{Capacity (i,t)}$
  - $\text{Production}(i \text{ at } j,t,s) \leq e^{(r-r_f)t} \text{Capacity (i at } j,t)$
  - $\text{Production}(i \text{ at } j,t,s) \geq 0$

- Need $\text{MaxSales}(i,t,s)$ - random
  - $\text{Capacity}(i \text{ at } j,0)$ - Decision in First Stage (now)

NOTE: Linear model that incorporates risk
Result with Option Approach

• Can include risk attitude in linear model
• Simple adjustment for the uncertainty in demand

• **Requirement 1**: correlation of all demand to market
• **Requirement 2**: assumptions of market completeness

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Assumptions

• Process of prices or sales forecasts
• No transaction fees
• Complete market (difference from financial options)
  • How to construct a hedge?
  • If NPV>0, inconsistency
  • Process: Trade option and asset to create riskfree security
Creating Best Hedge – and a Confession

• Underlying asset: Max potential sales in market
• Option: Plant with given capacity
• Other marketable securities:
  • Competitors’ shares
  • Overall all securities min residual volatility
  • Confession: Due to incompleteness, some volatility remains (otherwise, NPV=0)
Resolution

• Incompleteness gives a range of possible values
• Can adjust capacity limits by varying discount factor with risk neutral assumptions on forecasts
• Can vary constraint multipliers with original forecast distribution
• All optimal policies for the given range are consistent with the market (cannot be beaten all the time)
• Obtain a range of policies – can use other criteria
Result of Residual Risk

• In binomial model, asset price moves from $S_t$ to $uS_t + v_1$ or $dS_t + v_2$ where $v_1$ and $v_2$ vary independently and have smallest volatility

• For standard call option,

$$C_t = \frac{(S_t - dS_t + v_1)/(uS_t - dS_t + v_2)}{(uS_t - K)} = \frac{E[(S_t - K)^+]}{p(uS_t - K)}$$

• Analogous result for capacity valuation: a range of values are consistent

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Alternatives and Challenges

• Use equilibrium and utility function approaches
• Caution on complexity of models
• Critical factor: range of outcomes considered
• Other challenges:
  – Effects of pricing decisions
  – Effects of competitors
  – Distribution changes from decisions
  – Extend to financial and real options together: operational and financial hedging
Operational and Financial Hedging uses of Real Options

- **Objective:** Determine capacity levels in different markets, production in each market, distribution across markets, and use of financial hedging instruments to maximize total global value.

- **Challenges:**
  - Demand and exchange rates may change
  - Correlations among demand and exchange
  - What is enough capacity?
  - What performance metrics to use?
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Summary

• Options apply to many varied decision problems
• Can evaluate planning with proper option evaluation techniques
• Relaxed market assumptions lead to models that determine a range of policies
• Firm or investor utility can choose within range
• Questions? Comments?