Stochastic Optimization: The Present and Future of OR

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Outline

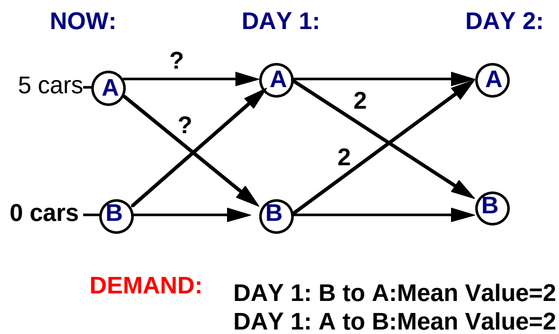
- Overview
- Examples
  - Vehicle Allocation
  - Financial planning
  - Manufacturing
- Methods
- View ahead

Overview

- Stochastic optimization
  - Traditional
    - » Small problems
    - » Impractical
  - Current
    - » Integrate with large-scale optimization (stochastic programming)
    - » Practical examples
    - » Expanding rapidly

Vehicle Allocation

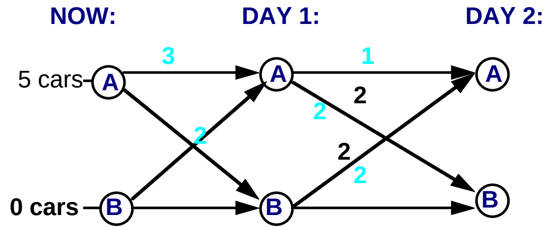
- Decision:
    - » How to position empty freight cars?



Vehicle Allocation: Mean Value Solution

Parameters: COST: 0.5 per empty car from A to B
REVENUE: 1.5 per full car from A to B

- **Maximize:** Revenue-Cost
  - » MOVE TWO EMPTY CARS FROM A to B

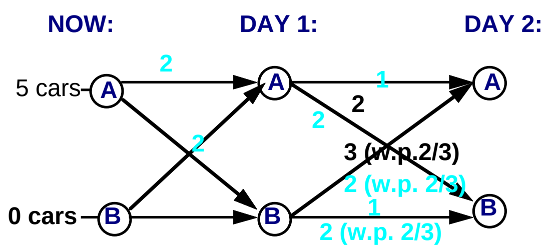


RESULT: Net 2: A to B; Net 2: B to A
TOTAL(*MV*) = 4

Expectation of Mean Value

Suppose: Demand is Random (Expectation from A to B=2)
- 0 from A to B with prob. 1/3
- 3 from A to B with prob. 2/3

- **Find: Expected** (Revenue-Cost)
  - » MOVE Two EMPTY CARS FROM A to B



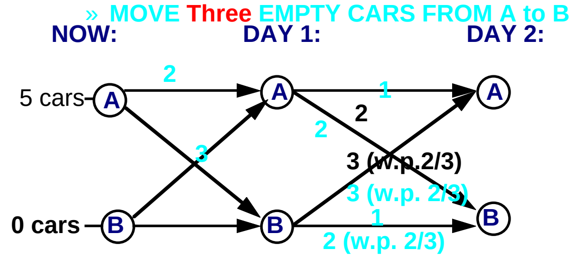
Expected Value: Net 2: A to B; Net 2: B to A (w.p. 2/3)
-1: B to A (w.p. 1/3)
TOTAL (*EMV*): 3

Stochastic Program Solution

Suppose: Demand is Random (as before)

GOAL: A solution to obtain highest expected value

- **Maximize: Expected (Revenue-Cost)**
  - » MOVE Three EMPTY CARS FROM A to B



Expected Value: Net 2: A to B; Net 3: B to A (w.p. 2/3)

-1.5 : B to A (w.p. 1/3)

TOTAL (*RP*): 3.5

***RP*=Recourse Problem**

INFORMATION and MODEL VALUE

- INFORMATION VALUE:
  - FIND Expected Value with Perfect Information or Wait-and-See (WS) solution:
    - » Know demand: if 3, send 3 from A to B If 0, send 0 from A to B:
    - » Earn: 2 (AtoB) + (2/3) (3) + (1/3)0= 4 = WS
  - Expected Value of Perfect Information (EVPI):
    - » EVPI = WS - RP = 4 - 3.5 = 0.5
    - » Value of knowing future demand precisely
- MODEL VALUE:
  - FIND EMV, RP
  - Value of the Stochastic Solution (VSS):
    - » VSS = RP - EMV=3.5 - 3 = 0.5
    - » Value of using the correct optimization model

INFORMATION/MODEL OBSERVATIONS

- EVPI and VSS:
  - ALWAYS ≥ 0 (WS ≥ RP ≥ EMV)
  - OFTEN DIFFERENT (WS=RP but RP > EMV and vice versa)
  - FIT CIRCUMSTANCES:
    - » COST TO GATHER INFORMATION
    - » COST TO BUILD MODEL AND SOLVE PROBLEM
- MEAN VALUE PROBLEMS:
  - MV IS OPTIMISTIC (MV=4 BUT EMV=3, RP=3.5)
    - » ALWAYS TRUE IF CONVEX AND RANDOM
    - » CONSTRAINT PARAMETERS
  - VSS LARGER FOR SKEWED DISTRIBUTIONS/COSTS

STOCHASTIC PROGRAM

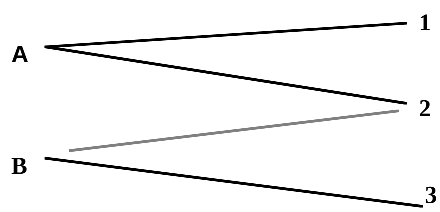
- – ASSUME: Random demand on AB and BA
- GOAL: maximize expected profits
  - (risk neutral)
- DECISIONS: x_{ij} - empty from i to j
  - $y_{ij}(s)$ - full from i to j in scenario s (RECOURSE)
  - (prob. p(s))
- FORMULATION:

$$
\begin{array}{llll}
\text{Max} & -0.5xAB + \sum_{s=s1,s2} p(s)\ (1.5\ yAB(s) + 1.5\ yBA(s)) & & \\
\text{s.t.} & xAB + xAA & = 5 & \textbf{(Initial)} \\
 & -xAB + yBA(s) & \leq 0 & \textbf{(Limit BA)} \\
 & -xAA + yAB(s) & \leq 0 & \textbf{(Limit AB)} \\
 & yBA(s) & \leq DBA(s) & \textbf{(Demand BA)} \\
 & + yAB(s) & \leq DAB(s) & \textbf{(Demand AB)} \\
 & xAA, XAB, yAA(s), yAB(s) \geq 0 & &
\end{array}
$$

- – **EXTENSIONS: Multiple stages**
  - –**Constraint/objective complexity (Powell et al.)**

Manufacturing Capacity

- **Where to Install Capacity for Different Models among Different Plants?**



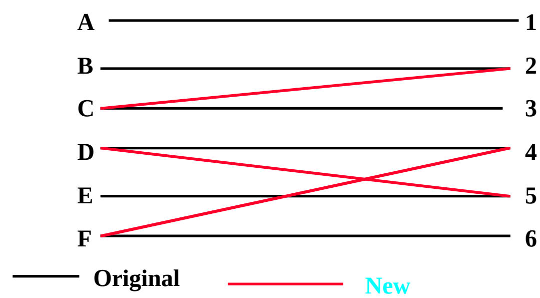
•Where to add flexibility? (multiple models)

Recourse Payoff Evaluation

- **Key: Evaluate Expected Optimal with Installed Capacity**
  - **Must choose best mix of models assigned to plants**
  - **Maximize Σi Profit (i) Production(i)**
  - **subject to: MaxSales(i) >= Σ j Production(i at j)**
  - **Σ i Production(i at j) <= Capacity (i)**
  - **Production(i at j) <= Capacity (i at j)**
  - **Production(i at j) >= 0**
- **Transportation Problem**
- **Need MaxSales(i) - random - unknown distribution**
  - **Capacity(i at j) - Decision in First Stage**

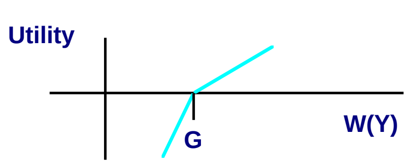
Solution Results

- **Model Data: from Graves/Jordan**
- **Vary: Model Lifetimes**
  - **Longer => More flexibility**
- **Start: 1 Year**



Financial Planning

- GOAL: Accumulate $G for tuition Y years from now
- Assume:
  - $ W(0) - initial wealth
  - K - investments
  - concave utility (piecewise linear)



RANDOMNESS: returns r(k,t) - for k in period t
where Y → T decision periods

FORMULATION

- SCENARIOS: $\sigma \in \Sigma$
  - Probability, $p(\sigma)$
  - Groups, $S^t_1, \ldots, S^t_{St}$ at t
- MULTISTAGE STOCHASTIC NLP FORM:

max $\sum_\sigma p(\sigma)\,(\,U(W(\sigma, T))\,)$

s.t. (for all σ): $\sum_k x(k,1,\sigma) = W(o)$ **(initial)**

$\sum_k r(k,t-1,\sigma)\,x(k,t-1,\sigma) - \sum_k x(k,t,\sigma) = 0$, **all t >1;**

$\sum_k r(k,T-1,\sigma)\,x(k,T-1,\sigma) - W(\sigma, T) = 0$, **(final);**

$x(k,t,\sigma) \geq 0$, **all k,t;**

Nonanticipativity:

$x(k,t,\sigma') - x(k,t,\sigma) = 0$ **if** $\sigma', \sigma \in S^t_i$ **for all t, i,** σ', σ

This says decision cannot depend on future.

DATA and SOLUTIONS

- ASSUME:
  - Y=15 years
  - G=$80,000
  - T=3 (5 year intervals)
  - k=2 (stock/bonds)
- Returns (5 year):
  - Scenario A: r(stock) = 1.25   r(bonds)= 1.14
  - Scenario B: r(stock) = 1.06   r(bonds)= 1.12
- Solution:

PERIOD	SCENARIO	STOCK	BONDS
1	1-8	41.5	13.5
2	1-4	65.1	2.17
2	5-8	36.7	22.4
3	1-2	83.8	0
3	3-4	0	71.4
3	5-6	0	71.4
3	7-8	64.0	0

GENERAL MULTISTAGE MODEL

- FORMULATION:

  MIN $E [\Sigma_{t=1}^{T} f_t(x_t,x_{t+1})]$
  s.t. $x_t \in X_t$
  x_t nonanticipative
  $P[h_t (x_t,x_{t+1}) \leq 0] \geq a$ (chance constraint)

EXAMPLES:

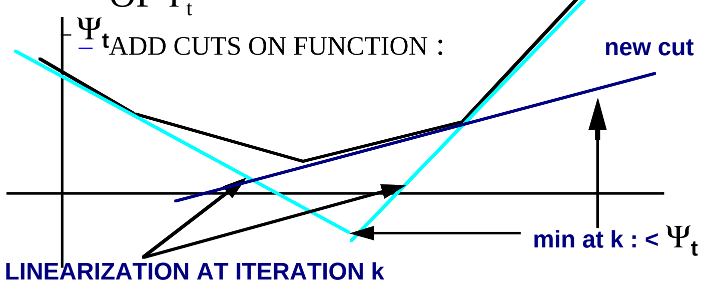
Vehicle Allocation: Linear functions, continuous or integer variables

Capacity: Linear plus integer variables

Financial Planning: Nonlinear objective, continuous variables

DECOMPOSITION METHODS

- BENDERS IDEA
  - FORM AN OUTER LINEARIZATION OF Ψ_t
  - ADD CUTS ON FUNCTION :



USE AT EACH STAGE TO APPROXIMATE VALUE FUNCTION

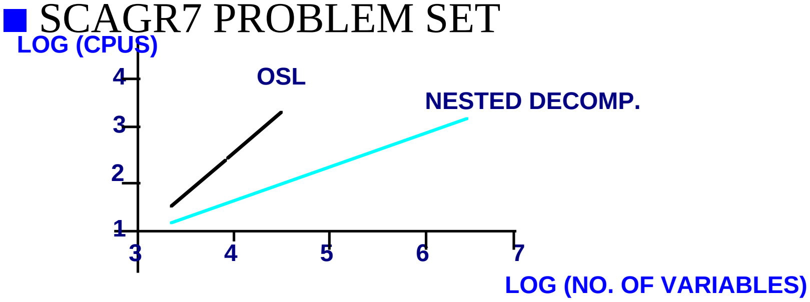
- **ITERATE BETWEEN STAGES UNTIL ALL MIN = Ψ_t**

DECOMPOSITION IMPLEMENTATION

- NESTED DECOMPOSITION
  - LINEARIZATION OF VALUE FUNCTION AT EACH STAGE
  - DECISIONS ON WHICH STAGE TO SOLVE, WHICH PROBLEMS AT EACH STAGE
- LINEAR PROGRAMMING SOLUTIONS
  - USE OSL FOR LINEAR SUBPROBLEMS
  - USE MINOS FOR NONLINEAR PROBLEMS
- PARALLEL IMPLEMENTATION
  - USE NETWORK OF RS6000S
  - PVM PROTOCOL

RESULTS

- SCAGR7 PROBLEM SET



PARALLEL: 60-80% EFFICIENCY IN SPEEDUP

OTHER PROBLEMS: SIMILAR RESULTS

- ONLY < ORDER OF MAGNITUDE SPEEDUP WITH STORM
  - TWO-STAGES - LITTLE COMMONALITY IN SUBPROBLEMS
  - STILL ABLE TO SOLVE ORDER OF MAGNITUDE LARGER PROBLEMS

View Ahead

- New Trends
  - Methods for integer variables
    - Power system implementations
    - Vehicle routing
  - Integrating simulation
    - Sampling with optimization
    - On-line optimization
    - Low-discrepancy methods

More Trends

- Modeling languages
  - Ability to build stochastic programs directly
  - Integrating across systems
- Using application structure
  - Separation of problem (dimension reduction)
  - Network properties
  - Generalized versions of convexity

Summary

- Increasing application base
- Value for solving the stochastic problem
- Efficient implementations
- Opportunities for new results