Evaluating Electricity Capacity with Option Pricing

John Birge
Northwestern University
Joint work with Steve Kou
Columbia University

Outline

• Derivatives
• Real options and electricity markets
• Asset evaluation
• Model and solution
• Challenges and extensions
• Conclusions
Energy and Weather Derivatives

- Exchange-traded options: CBOT and NYMEX
- Over-the-counter: Many weather applications
- Uses:
  - Reducing risks
  - Finding value of plant (difference in prices)

Interest in Energy Derivatives

- High Volatility
  - 10 to 100 times that of common stock
  - Prices from 0 to $10,000 per MWhr
- Difficulty in storage
  - Electricity close to un-storable
    - Difficulty substitution (liquidity)
Electricity Price Example: USA

- California Power Exchange

Electricity Price Example: Norway

- NOK Prices
Observations on Data

• Price follows:
  – Mean reversion
  – Seasonality
  – Jumps

• Need:
  – Model to capture
  – Possible evaluation

Use of Valuation Formula

• Now, can value options
• What to use for?
  – Plant is worth something when price above cost to produce
  – Suppose constant production cost:
**Constant Production Cost**

- Value is like call on production cost at all times
- What if costs vary?
  - Other commodity
  - Example: Oil or natural gas
- Can value as difference between prices
- Some traded varieties

**Modeling the Cost**

- Suppose 2 futures prices $F_1(t,T^*)$ is the future price at $T^*$ for unit of electricity at time $t$; $F_2(t,T^*)$ is the same for fuel
- $K$ is the conversion factor from fuel to electricity
- So, plant has value if $F_1(t,T^*)-F_2(t,T^*)>0$
- For $S_i(T)=F_i(T,T)$, the plant is worth
  \[ A' \left( \int_0^T E^* [P[0,t] (S_i(t)-KS_2(t))] dt \right) \]
  where $A'$ relates to plant capacity, $P[0,t]$ is present value of zero-coupon bond maturing at $t$, $E^*$ is expectation under a risk neutral measure.
The Generation Option (Spark Spread)

Under mean-reverting processes with same rates $\beta$ of mean reversion with $S(t) = J(t)K(t)$ where $J$ is a jump process with Poisson rates $\lambda \zeta$ and lognormal jump sizes with log-mean $\gamma$, st.dev. $\delta$, correlation $\rho$, $K$ is O-U process with mean rate $\alpha_i$, and covariance $\Sigma$

$$E^*(P(0,T)\{S_1(T) - S_2(T)\})^+ = P(0,T)\sum_{n=1}^{\infty} e^{-\lambda T}\frac{[(\lambda T)^n/n!]^*}{\psi(F(1,n)(0,T*),F(2,n)(0,T*),T,(\sigma^2(T,T*)+n\delta^2)/T)}$$

where.....

Parameter Definitions

$$\psi(x_1,x_2,T,a^2) = x_1 \Phi((\log(x_1/x_2)+ (a^2/2)T)/(aT^{0.5})) - x_2 \Phi( (\log( x_1/x_2) - (a^2/2)T)/(aT^{0.5} )),$$

$$F(i,n)(0,T) = c_i(T)e^{-\lambda (1+\zeta i)T(1+\zeta i)^n},$$

$$c_i(T) = \exp \left\{ e^{-\beta t} \left\{ \log(S_i(0)) + \alpha_i \int_0^t e^{\beta u} du \right\} \right\} \exp \left\{ (1/2)e^{-2\beta t} \sigma_i^2 \int_0^t e^{2\beta u} du \right\}$$

$$\delta^2 = \delta_1^2 + \delta_2^2 - 2\rho \delta_1 \delta_2,$$

$$\sigma^2(t,T) = \sigma_1^2 \int_0^t e^{-2\beta (T-u)} du,$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2.$$
Additional Complications

- Real costs:
  - Cost to start operating
  - Cost to stop operating
- Need:
  - Policy for operating the plant
  - Price to start and price to stop

Example
Valuation

- Must value integral over transition time from start to stop point
- Count the number of cycles in a period
- Evaluate average cycle value
- Include correction for end effects

Conclusions

- Electricity and other real options large part of market
- Important for valuation of assets
- Treat as Brownian motion and jumps
- Can price by traditional techniques
- Difficulties with fixed costs and imperfect markets – uses of utilities and equilibria