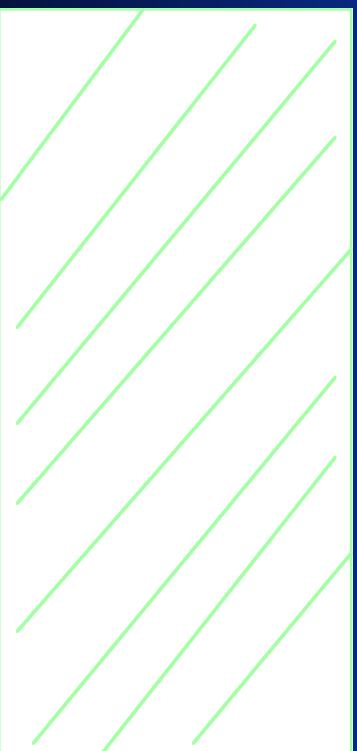


Quasi-Monte Carlo Applications in Option Pricing

- **John R. Birge**
- **University of Michigan**
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- **Outline**
 - **Basic analysis**
 - **Error results**
 - **Option Model**
 - **Results**
 - **Other applications**

Motivation

- **Error result for standard Monte-Carlo**
- **Validity of pseudo-random generators**
- **Really random? Collinear patterns possible**



Error Results for Quasi-Monte Carlo

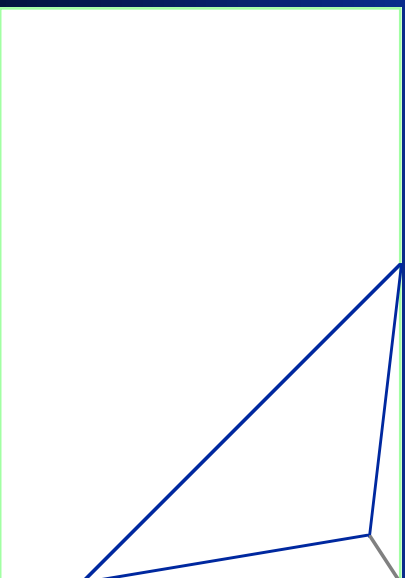
- **General result - Irrationals**
- **Result for Halton sequence**

Expected Error Analysis

- **Wozniakowski, Traub**
- **Function distribution model**
 - Mean zero/Form of multi-dimensional Wiener process
 - Expected error:

Implications

- **Form of functions**
 - Bounded variation
 - Unit hypercube
 - “Randomly” generated
- **Validity?**
 - In applications, unlikely to hold
 - BUT - can you use a specific function form?

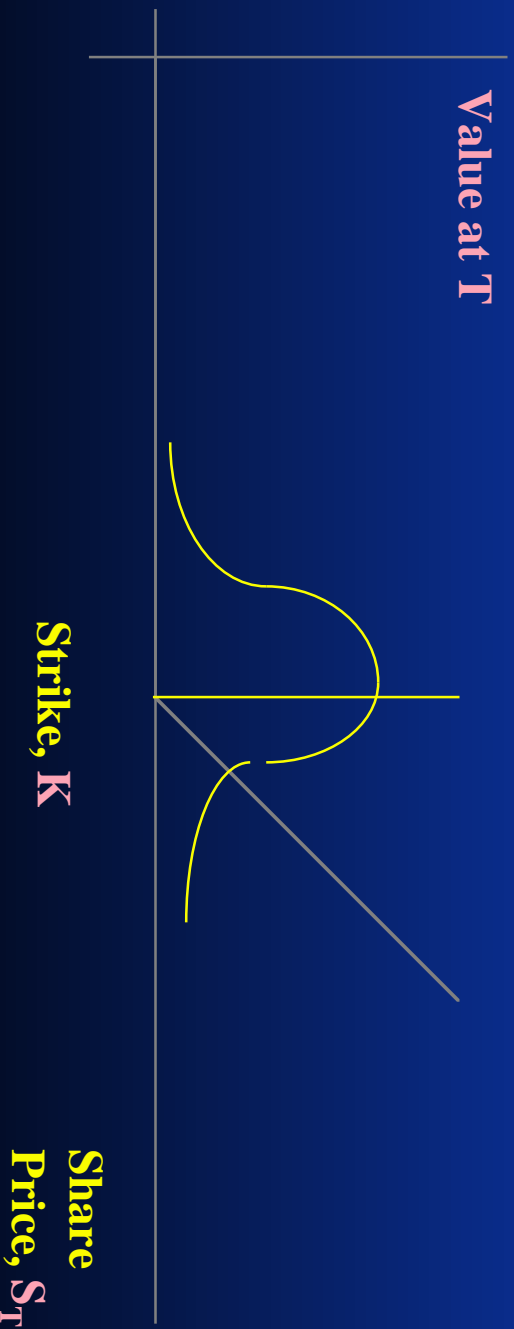


Option Models

- **“Derivative” securities**
 - Call: Buy a share at a given price at specific time (European)
 - » If by a specific time - American
 - Put: Sell; Straddle: Buy or sell
- **Why?**
 - Reduce risk (hedge)
 - Speculate
 - Arbitrage
- **Original analysis - L. Bachelier (1900 - Brownian motion)**

Valuing an Option

- **(European) Call Option on Share assuming:**
 - Buy at K at time T ; Current time: t ; Share price: S_t
 - Volatility: σ ; Riskfree rate: r_f ; No fees; Price follows Ito process
 - **Valuing option (Black/Scholes):**
 - Assume risk neutral world (annual return= r_f independent of risk)
 - Find future expected value and discount back by r_f
- Call value at $t = C_t = e^{-r_f(T-t)} \int (S_T - K)^+ dF_r(S_T)$



Black-Scholes Difficulty

- **American options**
 - Decision at all t - exercise or not?
 - Analysis difficult
- **Alternatives?**
 - Sample paths (Monte Carlo)



Monte Carlo Method for Option Valuation

- **General form**
- **Procedure**
- **Typical dimensions**
 - 30-180 stocks
 - 365+ for mortgages (each holder has option to exercise)

Methods Considered

- **Pseudo**
 - Schrage
- **Quasi**
 - Irrationals
 -
 - Halton
 -
 - Faure
 -
 - Sobol
 -

Examples

- **Exercise (strike) price - 30**
- **Current price -**
- **Volatility -**
- **Expiration date -**

ex1

ex2

ex3

ex4

ex5

ex6

Sample times

- **Pseudo**
- **Irrationals**
- **Halton**
- **Faure**
- **Sobol**

Stopping Rules

-
-

Stopping Rule Results

Recommendations

Other applications

Conclusions