Using Option Values in Location and Capacity Decisions

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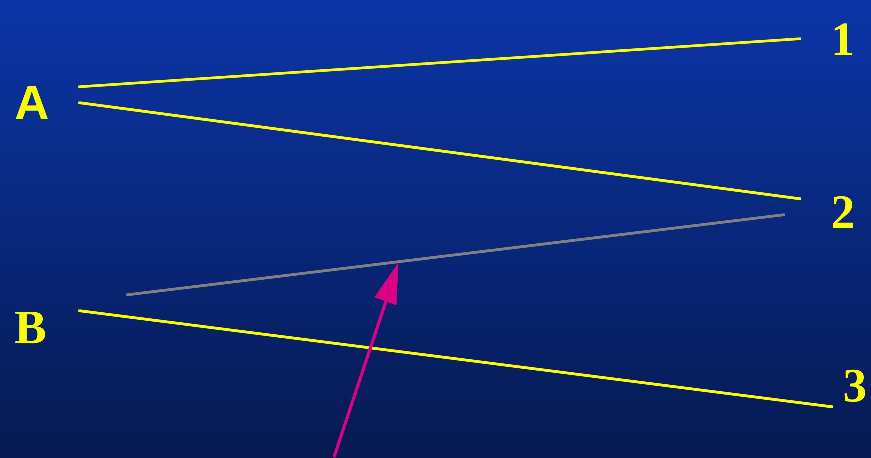
- Capacity decisions and value measures
- Observations from finance
- Option basics
- Application
- General constraints

Example: Capacity Planning

- What to produce?
- Where to produce? (When?)
- How much to produce?

EXAMPLE: Models 1,2, 3 ; Plants A,B



Should B also build 2?

GOALS

- **ADD AS MUCH VALUE AS POSSIBLE**
- **But: how do you measure value?**
  - **Net Present Values?**
  - **Discounted Cash Flows?**
  - **Net Profit?**
  - **Payback? IRR?**

Traditional Approach

- Incremental Decision
  - Add Capacity at B for Model 2?
- Analysis
  - Find expected demand for 2?
  - Use expected demand for 1,3
  - => Discounted cash flows
- Result: No model 2 at B
  - Why?

Role of Uncertainty

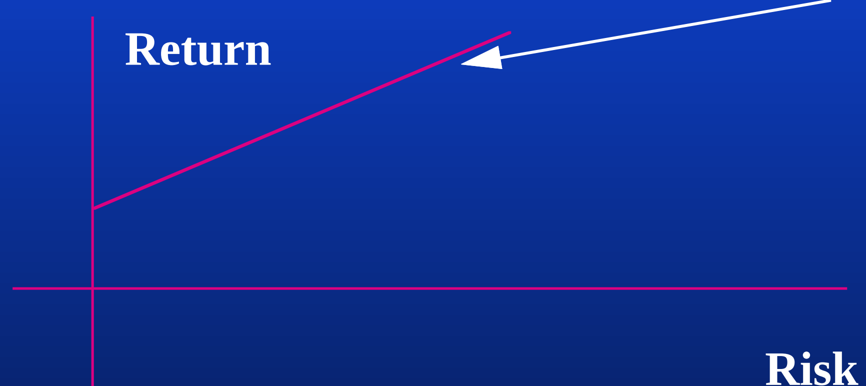
- **Problem: we do not know:**
  - what the demand will be
  - how much we really can produce in:
    - 1 day, 1 week, 1 month, 1 year
  - costs of inputs
  - competitor reaction
- **Result: Capacity for 2 at B may be useful if:**
  - demand for 2 higher than expected
  - demand for 3 lower than expected, demand for 1 higher
  - costs of 1 or 3 higher than expected, costs of 2 lower
  - short run capacity limit on 3
- **Effect: New capacity may add value**

Measuring Investor Value

- SUPPOSE RISK NEUTRAL?
- (expected cost) objective
  - RESULT: Does not correspond to preference
  - Difficult to assess real value this way
- RESOLUTION:
  - Assume investors prefer lower risk
  - Investors can diversify away unique risk
  - Only important risk is market - contribution to portfolio
- CONSEQUENCE: Capital asset pricing model (CAPM)

Basics of CAPM

- **RISK/RETURN TRADEOFF:**
  - Investors can diversify
  - Firms need not diversity
  - All investments on security market line



NEED:Portfolio contribution - symmetric risk

How to determine?

Determing Risk Contribution

- **USE CORRELATION?**
  - Can measure for known markets (beta values)
  - If capacitated, depends on decisions
    - » Constrained resources
    - » Correlations among demands
- **ALTERNATIVES?**
  - Option Theory
    - » Allows for non-symmetric risk
    - » Explicitly considers constraints -
    - » As if selling excess to competitors at a given price

Use of Options

- Capacity limits potential sales
- View: option sold to competitor

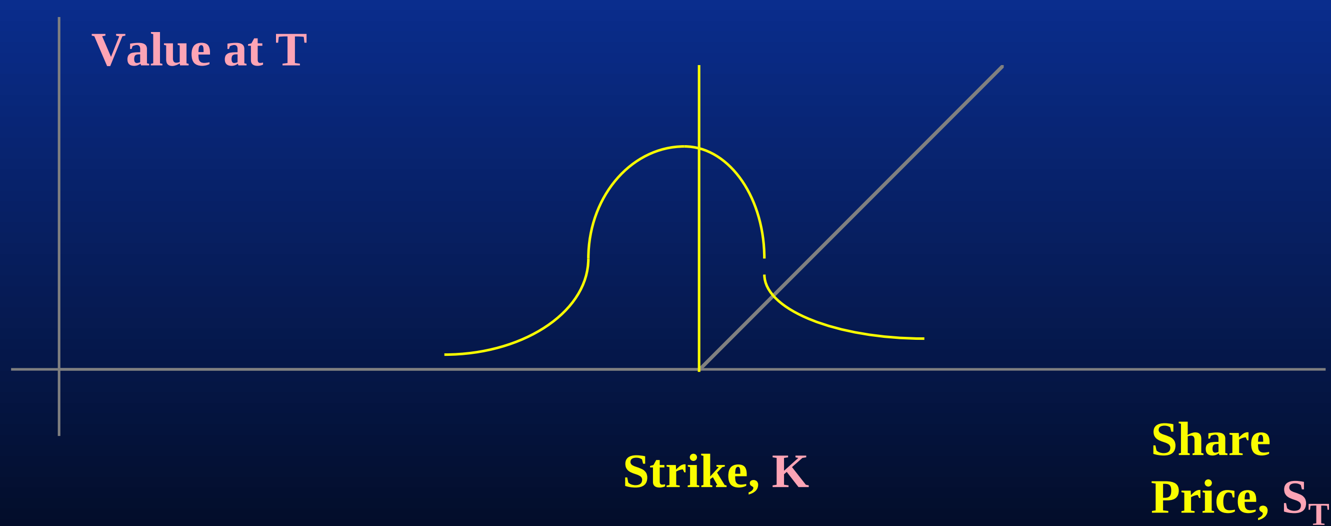
RESULTS FROM FINANCE:

- Assumption: risk free hedge
  - Can evaluate as if risk neutral
  - As in Black-Scholes model
- Steps in modeling:
  - Adjust revenue to risk-free equivalent
  - Discount at riskless rate

Valuing an Option

- (European) Call Option on Share assuming:
  - Buy at K at time T;Current time: t; Share price: S_t
  - Volatility: σ; Riskfree rate: r_f; No fees; Price follows Ito process
- Valuing option:
  - Assume risk neutral world (annual return=r_f independent of risk)
  - Find future expected value and discount back by r_f

Call value at t = $C_t = e^{-r_f(T-t)}\int(S_T-K)^+ dF_f(S_T)$



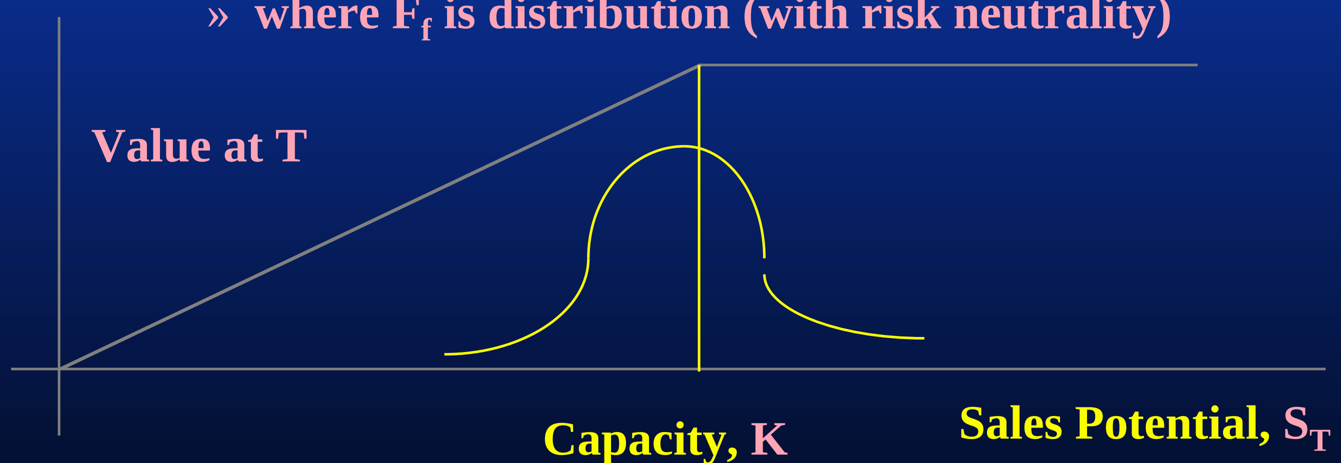
Relation to Capacity Evaluation

- **What is the value of a plant with capacity K?**
  - Discounted value of production up to K?
- **Problems:**
  - Production is limited by demand also (may be > K)
  - How to discount?
- **Resolution:**
  - Model as an option
  - Assume:
    - » Market for demand (substitutes)
    - » Forecast follows Ito process
    - » No transaction costs

∞ => **Model like share minus call**

Computing Capacity Value

- **Goal: Production value with capacity K**
  - Compute uncapacitated value based on CAPM:
    - » $S_t = e^{-r(T-t)} \int c_T S_T \, dF(S_T)$
    - » where c_T=margin,F is distribution (with risk aversion),
    - » r is rate from CAPM (with risk aversion)
  - Assume S_t now grows at riskfree rate, r_f ; evaluate as if risk neutral:
    - » Production value = $S_t - C_t = e^{-r_f(T-t)} \int c_T \min(S_T, K) \, dF_f(S_T)$
    - » where F_f is distribution (with risk neutrality)

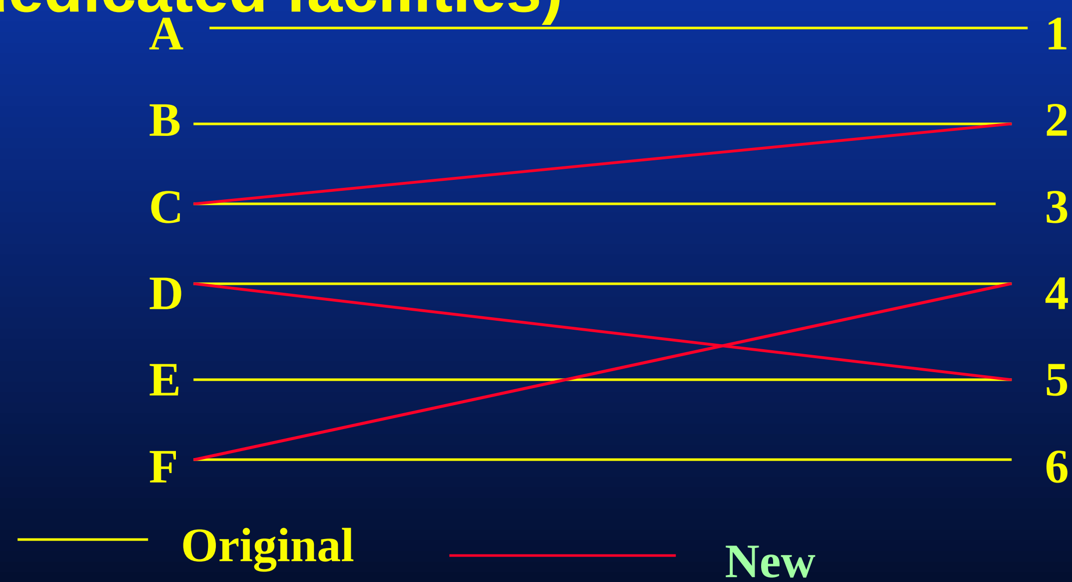


Alternative Computation

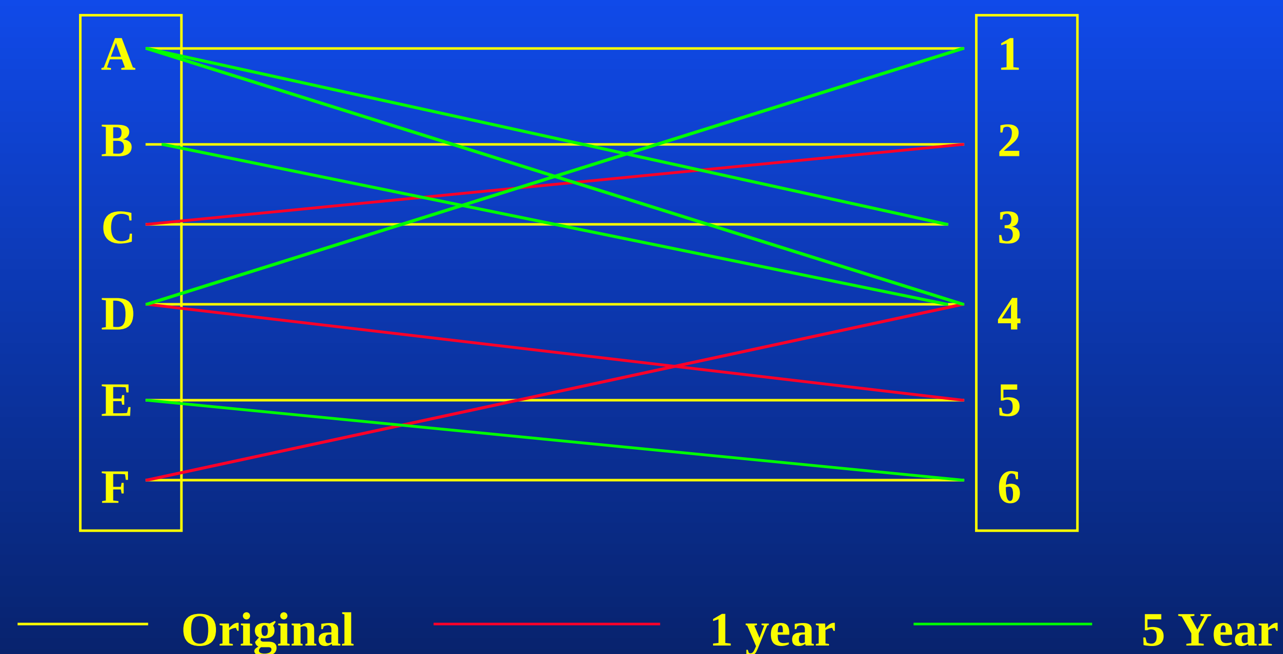
- **Approach:**
  - Shift bounds instead of distribution
  - Replace F_f by F (riskfree to risk averse)
  - $F_f(A) = F(e^{(r-rf)(T-t)}A)$ for any A
- **Result:**
  - $C_t= e^{-rf(T-t)}\int c_t(S_T-K)^+dF_f(S_T)$
  - $= e^{-(r-rf)(T-t)}e^{-rf(T-t)}\int e^{(r-rf)(T-t)}c_t(S_T-K)^+dF_f(S_T)$
  - $= e^{-r(T-t)}\int c_t(S_T-e^{(r-rf)(T-t)}K)^+dF(S_T)$
- **Advantages:**
  - No forecast changes
  - Extends to general models

EXAMPLE: Flexible Capacity- where?

- Find new capacity for next model year
- Model Data: from Graves/Jordan
- Vary: Model Lifetimes
  - Longer => More flexibility
- Start: 1 Year for all models (given all dedicated facilities)



Five Year Lifetime



- Note: new additions for 5 year
- Additional model years => more flexibility

Conclusions

- Utilility Modeling for Financial Objectives
  - Use investors' preference
  - Problems with constraints
- Incorporating Constraints
  - Use risk neutral method from option theory
  - Effect:
    - » Discount objective with market rate
    - » Adjust unique linear constraints with discount factor ratio
    - » Maintain linear model with risk aversion
- Natural capacity planning interpretation
- Need for interpretation in other areas