Using Option Values in Location and Capacity Decisions

- General constraints
- Application
- Option basics
- Observations from finance
- Capacity decisions and value measures

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and Capacity Decisions
Using Option Values in Location
Example: Capacity Planning

• What to produce?
• Where to produce? (When?)
• How much to produce?
• When to produce?

EXAMPLE: Models 1, 2, 3; Plants A, B

Should B also build 2?
GOALS

• Add as much value as possible

But: how do you measure value?

- Payback?
- IRR?
- Net Profit?
- Discounted Cash Flow?
- Net Present Value?
Traditional Approach

- Incremental Decision
  - Add Capacity at B for Model 2?
  - Find expected demand for 2?
  - Use expected demand for 1, 3
  - $\Rightarrow$ Discounted cash flows
  - Why?

- Analysis
  - Result: No model 2 at B
Role of Uncertainty

Problem: we do not know:
- what the demand will be
- how much we really can produce in: 1 day, 1 week, 1 month, 1 year
- costs of inputs
- competitor reaction
- demand for 2 or 3 higher than expected, costs of 2 lower
- demand for 3 lower than expected, demand for 1 higher
- demand for 2 higher than expected

Result: Capacity for 2 at B may be useful if:
- demand for 2 higher than expected
- demand for 3 lower than expected, demand for 1 higher
- costs of 1 or 3 higher than expected, costs of 2 lower
- short run capacity limit on 3

Effect: New capacity may add value
- competitor reaction
- costs of inputs
- 1 day, 1 week, 1 month, 1 year
- how much we really can produce
- what the demand will be
Measuring Investor Value

- **SUPPOSE RISK NEUTRAL?**
  - *(expected cost) objective objective does not correspond to preference*
  - Difficult to assess real value this way

- **RESULT: Does not correspond to preference**

- **RESOLUTION:**
  - Assume investors prefer lower risk
  - Investors can diversify away unique risk
  - Only important risk is market contribution to portfolio

- **CONSEQUENCE:**
  - Capital asset pricing model (CAPM)
Basics of CAPM

Risk/Return Tradeoff:

- All investments on security market line
- Firms need not diversify
- Investors can diversify

RISK/RETURN TRADEOFF:

How to determine?

NEED: Portfolio contribution - symmetric risk
Determining Risk Contribution

• USE CORRELATION?

• ALTERNATIVES?
  - Option Theory
    - Allows for non-symmetric risk
    - Explicitly considers constraints
    - Can measure for known markets (beta values)
    - If capacity limited, depends on decisions

• USE CORRELATIONS?
  - As if selling excess at a given price
  - Constrained amongst demands
  - Constrained resources
  - Can measure for known markets (beta values)
Use of Options

- Capacity limits potential sales
- View: option sold to competitor
- Assumption: risk-free hedge
  - Can evaluate as if risk neutral
  - As in Black-Scholes model

Steps in modeling:
- Adjust revenue to risk-free equivalent
- Discount at riskless rate

RESULTS FROM FINANCE:

- View: option sold to competitor
- Capacity limits potential sales
Valuing an Option

• (European) Call Option on Share assuming:
  – Buy at $K$ at time $T$; Current time: $t$; Share price: $S_t$
  – Volatility: $\sigma$; Risk-free rate: $r$; No fees; Price follows Ito process

Valuing Option:

- Volatility: $\sigma$; Risk-free rate: $r$; No fees; Price follows Ito process
- Valuation: Assume a risk neutral world (annual return $= r$, independent of risk)
  - Find future expected value and discount back by $r$

Call Value at $t = C_t = e^{-r(T-t)}\mathbb{E}_{\mathbb{Q}}\left[S_T^+ - K\right]$

Share Price, $S_T$
Strike, $K$
Value at $T$
Relation to Capacity Evaluation

• What is the value of a plant with capacity $K$?
  - Discounted value of production up to $K$?

• Problems:
  - Production is limited by demand also (may be $< K$)
  - How to discount?
  - Discounted value of production up to $K$?

• Resolution:
  - Model as an option
  - Assume:
    - Market for demand (substitutes)
    - Forecast follows Ito process
    - No transaction costs

Model like share minus call

$\infty$ \Rightarrow $\infty$

Relation to Capacity Evaluation
Computing Capacity Value

Goal: Production value with capacity K

- Compute uncapacitated value based on CAPM:
  \[ S^* = e^{-rf(T-t)} \int_{cT}^{\text{min}(S^*,K)} dF_f(S^*) \]

- Assume \( S^* \) now grows at riskfree rate, \( r_f \); evaluate as if risk neutral:
  \[ \text{Production value} = S^* - C^* = e^{-r_f(T-t)} \int_{cT}^{\text{min}(S^*,K)} dF_f(S^*) \]

\[ \text{where } F_f \text{ is distribution (with risk neutrality)} \]

\[ \text{Value at } T \]

Sales Potential, \( S^* \) Capacity, \( K \)
Alternative Computation

Approach:

- Shift bounds instead of distribution
- Replace $F_f$ by $F$ (riskfree to risk averse)

$$F_f(A) = F((1-\Delta)A)$$

$$C_t = e^{-r(T-t)}\int e^{(r-r_f)(T-t)}ct(ST-K)+dF_f(ST)$$

Result:

- Extends to general models
- No forecast changes

Advantages:

- No forecast changes
- Extends to general models
- Replace $F_f$ by $F$ (riskfree to risk averse)
- Shift bounds instead of distribution

Approach:
Find new capacity for next model year

**Example:** Flexible Capacity

- Where?

**Model Data:** from Graves/Jordan

- Start: 1 Year for all models (given all dedicated facilities)

- Longer => More flexibility

- Vary: Model Lifetimes

- Find new capacity for next model year

- Vary: Model Lifetimes
Five Year Lifetime

- Additional model years => more flexibility
- Note: new additions for 5 year
Conclusions

Utility Modeling for Financial Objectives

- Use investors' preference
- Problems with constraints

Incorporating Constraints

- Use risk neutral method from option theory
- Problems with constraints
- Use investors' preference

Natural Capacity Planning: Interpreting Interception

Maintain linear model with risk aversion

Adjust unique linear constraints with discount factor

Discount objective with market rate

Effect:
- Use risk neutral method from option theory

Need for interpretation in other areas

Utility Modeling for Financial Objectives

Conclusions