Jump Diffusion Models in Pricing Energy Options

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Outline

- Pricing Observations
- Alternative Models
- Regime-Switching Model
- Spike-Jump Model
- Implications for Pricing Options

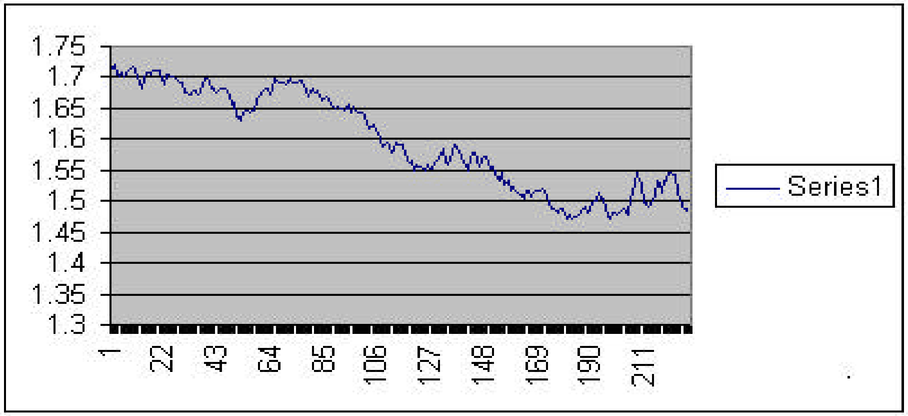
Price Observations

- Energy prices seem to follow mean-reversion pattern
- Jumps occur in varying intervals - two forms
  - Spike to high price then quick return to mean
  - Switch to new mean (regime switch)

Spot Price Process-Regime Switching

Parameters:

- **Poisson process** N_t **with rate** λ
- B_t **is a standard Brownian motion**
- $\alpha\left(S_{t-}, t\right)$ **is an adapted process modeling the drift**
- σ **is a positive constant symbolizing the volatility**
- $Y_t \geq 0$ **is a random variable with** $\mu = E\left(Y_t - 1\right)$
- Δ **means** $\Delta(Y(t)) = Y(t) - Y(t-)$, **for any process** $Y(t)$.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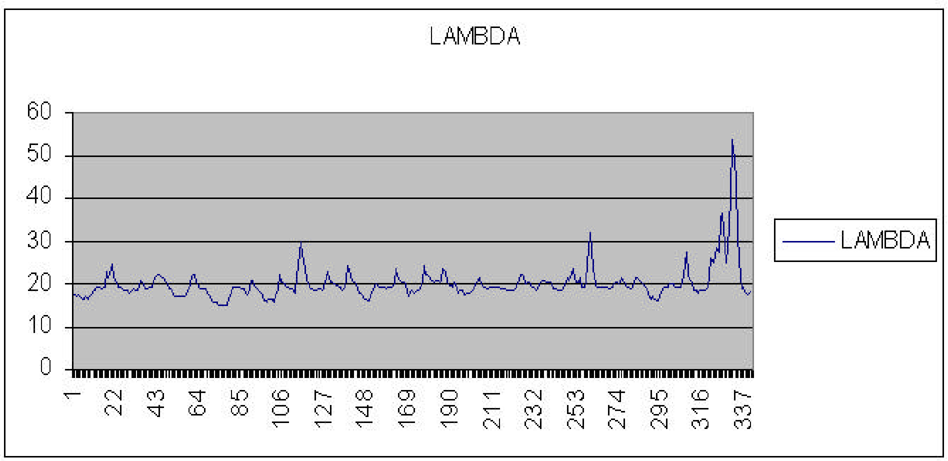
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337
LAMBDA

Spot Price Process-Regime Switching

Model:

$$S(t) = J(t)K(t), \quad S(0) = K(0)$$

with continuous mean reversion part $K(t)$**:**

$$d\log(K_i(t)) = (\alpha_i - \beta_i \log(K_i(t)))dt + \sigma_i dB_i(t), \tag{1}$$

and the jump part:

$$\frac{dJ_i(t)}{J_i(t-)} = -\lambda\zeta_i t + \Delta\left(\sum_{j=1}^{N(t)}(V_j^{(i)} - 1)\right).$$

Solutions

Results:

$\log K_i(t)$ **follows O-U process:**

$$K_i(t) = \exp\{e^{-\beta_i t}\log(K_i(0)) + \alpha_i e^{-\beta_i t}\int_0^t e^{\beta_i u}du + \sigma_i e^{-\beta_i t}\int_0^t e^{\beta_i u}dB_i(u)\}.$$

For the jump part,

$$J_i(t) = \exp(-\lambda\zeta_i t)\prod_{j=1}^{N(t)} V_j^{(i)}, \quad J_i(0) = 1,\ i = 1, 2.$$

Futures Prices

Futures Price:

$$F_{t,T} = e^{r(T-t)} S_t; \tag{2}$$

For maturity T^* and $\le t \le T^*$,

$$\frac{dF(t,T^*)}{F(t-,T^*)} = e^{-\beta(T^*-u)} \sigma dB(u) - \lambda \zeta t + \Delta \left(\sum_{j=1}^{N(t)} (V_j - 1) \right),$$

or equivalently,

$$F(t,T^*) = F(0,T^*) \exp \left\{ e^{-\beta T^*} \sigma \int_0^t e^{\beta u} dB(u) - \frac{1}{2} e^{-2\beta T^*} \sigma^2 \int_0^t e^{2\beta u} du \right\} e^{-\lambda \zeta t} \prod_{j=1}^{N(t)} V_j.$$

Option Valuation

Assuming lognormal jumps, two process with futures F_1 and F_2, same jump rates β, and $P(0,T)$ price of riskless bond maturing at T, an exchange option on future contracts has a price:

$$\begin{aligned}\psi_F(0) &= \mathsf{E}^*(P(0,T)\{F_1(T,T^*) - F_2(T,T^*)\}^+) \\ &= P(0,T)\sum_{n=1}^{\infty} e^{-\lambda T}\frac{(\lambda T)^n}{n!}\psi\left(F^{(1,n)}(0,T^*), F^{(2,n)}(0,T^*), T, \frac{\sigma^2(T,T^*)+n\delta^2}{T}\right),\end{aligned}$$

where

$$\psi\left(x_1,x_2,T,a^2\right) = x_1\Phi\left(\frac{\log\left(x_1/x_2\right)+\frac{a^2}{2}T}{a\sqrt{T}}\right) - x_2\Phi\left(\frac{\log\left(x_1/x_2\right)-\frac{a^2}{2}T}{a\sqrt{T}}\right),$$

and

$$\begin{aligned}F^{(i,n)}(0,T^*) &= F_i(0,T^*)e^{-\lambda(1+\zeta_i)T}(1+\zeta_i)^n, \quad i=1,2, \\ \delta^2 &= \delta_1^2+\delta_2^2-2\rho_x\delta_1\delta_2, \\ \sigma^2(t,T^*) &= \sigma^2\int_0^t e^{-2\beta(T^*-u)}dt, \sigma^2 = \sigma_1^2+\sigma_2^2-2\rho_b\sigma_1\sigma_2.\end{aligned}$$

Observations and Spike Model

- **Jumps accumulate here and reflect switches of regimes**
- **Mean for reversion changes at jumps**
- **Need for model to include spike and return to former mean**

Spike jump model:

$$d\log S(t) = (\alpha - \beta \log S(t))dt + \sigma dW(t) + \Delta(\sum_{j=1}^{N(t)} V_j) - \lambda\zeta t,$$

where $\zeta = E(V)$.

Spike Model Solution

Results:

Let $H(t) = \beta t$, and $\tilde{S}(t) = \log(S(t))$. By Ito's formula

$$d(e^{H(t)}\tilde{S}(t)) = e^{H(t)}\{\alpha dt - \beta\tilde{S}(t)dt + \sigma dW(t) + \beta\tilde{S}(t)dt\} + \Delta(\sum_{j=1}^{N(t)} V_j)e^{H(t)} - \lambda\zeta dt$$

$$= e^{H(t)}\{\alpha dt + \sigma dW(t)\} + \Delta(\sum_{j=1}^{N(t)} V_j)e^{H(t)} - \lambda\zeta dt$$

Therefore,

$$e^{H(t)}\tilde{S}(t) = \tilde{S}(0) + \int_0^t e^{H(u)}\alpha du + \int_0^t e^{H(u)}\sigma dW(u) - \lambda\zeta\int_0^t e^{H(u)}du + \sum_{j=1}^{N(t)} V_j e^{H(\tau_j)}, \text{ or}$$

$$\log S(t) = e^{-\beta t}\log S(0) + \alpha\int_0^t e^{-\beta(t-u)}du + \sigma\int_0^t e^{-\beta(t-u)}dW(u) - \lambda\zeta\int_0^t e^{-\beta(t-u)}du + \sum_{j=1}^{N(t)} V_j e^{-\beta(t-\tau_j)},$$

where τ_j are inter-arrival times of the Poisson process. Notice that the jump effect will diminish with β is large enough. Need distribution but can solve using mgf and taking Laplace transforms.

Observations on Spike Model

- **Can solve for prices using Laplace transforms**
- **Include exchange options**
- **Problem: How to include both the regime switching and spike jump models simultaneously? (Example: electricity and gas)**

Conclusions

- **Two-forms of jump diffusion models**
- **Regime switching may be more appropriate for gas prices**
- **Spike model may be more appropriate for electricity**
- **Solutions to most pricing problems but...**
  - **No analytical process for both models together**