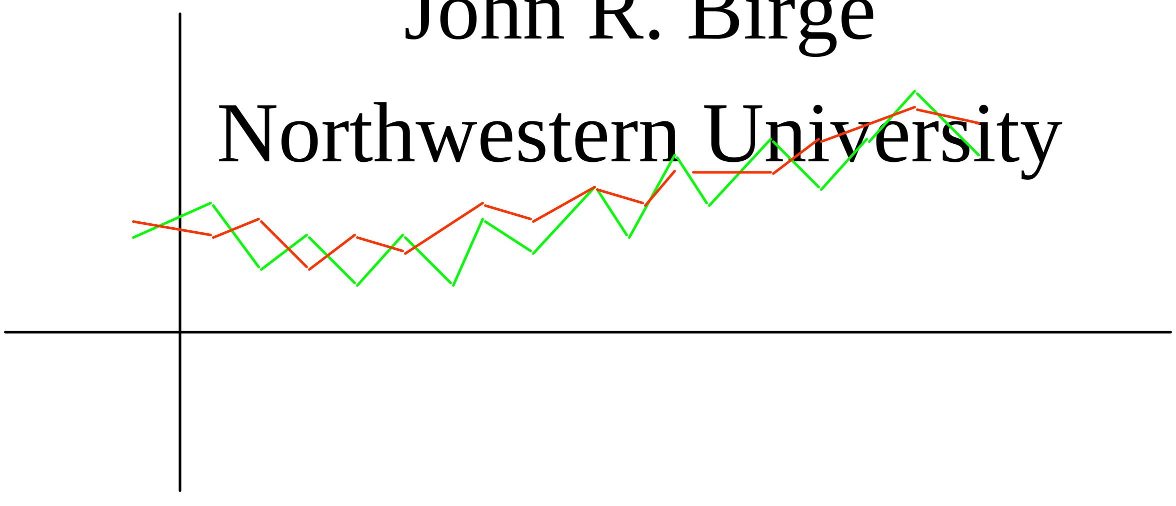
Optimal Policy Structure in Dynamic Asset-Liability Management

John R. Birge

Northwestern University

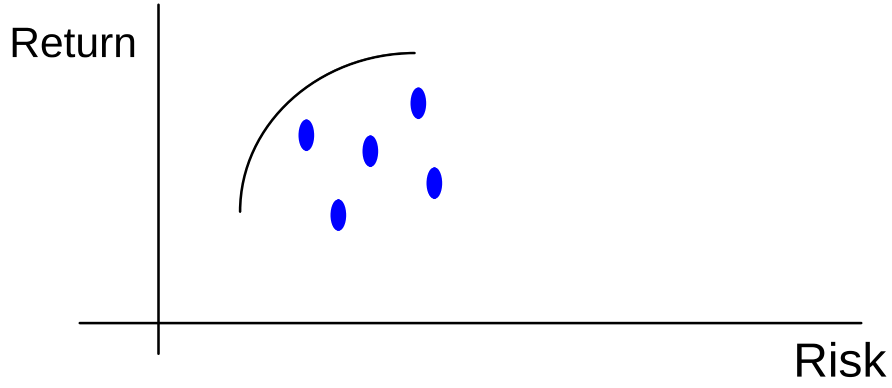


OUTLINE

- Mean-variance versus other utility functions
- Mean-Variance in dynamic portfolios
- Discrete time, piecewise linear utility
- Policy structure
- Enhanced models

Static Portfolio Model

- **Markowitz model**
  - **Choose portfolio to minimize risk for a given return**
  - **Find the efficient frontier**



Markowitz Mean-Variance model

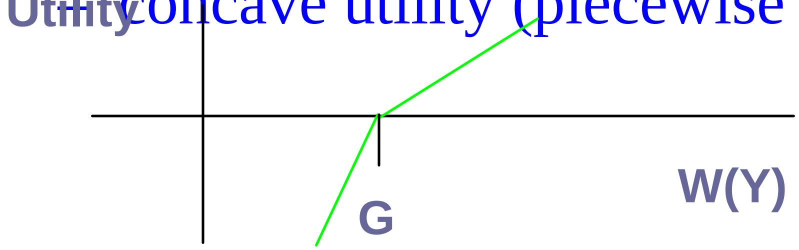
- **For a given set of assets, find**
  - **fixed percentages to invest in each asset**
  - **maintain same percentage over time**
- **Needs**
  - **rebalance as returns vary**
  - **cash to meet obligations**

Alternative Dynamic Model

- **Assume possible outcomes over time**
  - **discretize generally**
- **In each period, choose mix of assets**
- **Can include transaction costs and taxes**
- **Can include liabilities over time**
- **Can include different measures of risk aversion**

Example: Retirement Planning

- GOAL: Accumulate \$G Y years from now
- Assume:
  - \$ W(0) - initial wealth
  - K - investments
  - concave utility (piecewise linear)



RANDOMNESS: returns r(k,t) - for k in period t

where Y $\longrightarrow$ T decision periods

FORMULATION

- SCENARIOS: ? ?? ??
  - Probability, p(?)
  - Groups, S^t_1, ..., S^t_{St} at t
- MULTISTAGE STOCHASTIC NLP FORM:

max $?_?$ **p(? ???U(W(? , T))**

s.t. (for all ?): $?_k$ **x(k,1, ?) = W(o) (initial)**

$?_k$ **r(k,t-1, ?) x(k,t-1, ?) -** $?_k$ **x(k,t, ?) = 0 , all t >1;**

$?_k$ **r(k,T-1, ?) x(k,T-1, ?) - W(? , T) = 0, (final);**

x(k,t, ?) >= 0, all k,t;

Nonanticipativity:

x(k,t, ?') - x(k,t, ?) = 0 if ?', ? ?? ?S^t_i **for all t, i, ?', ?**

???????This says decision cannot depend on future.

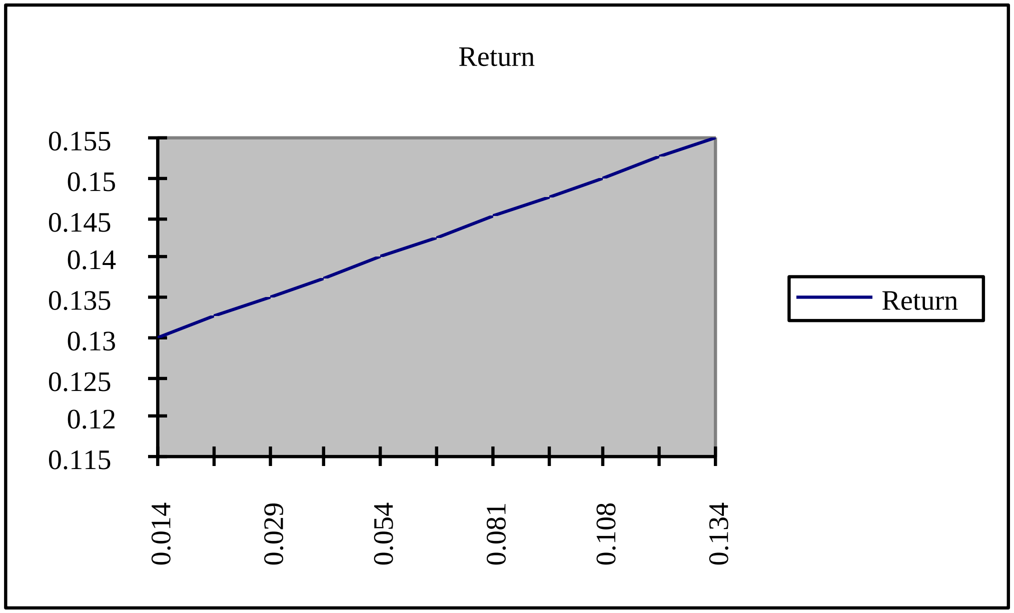
DATA and SOLUTIONS

- ASSUME:
  - Y=15 years
  - G=$80,000
  - T=3 (5 year intervals)
  - k=2 (stock/bonds)
- Returns (5 year):
  - Scenario A: r(stock) = 1.25 r(bonds)= 1.14
  - Scenario B: r(stock) = 1.06 r(bonds)= 1.12
- Solution:

PERIOD	SCENARIO	STOCK	BONDS
1	1-8	41.5	13.5
2	1-4	65.1	2.17
2	5-8	36.7	22.4
3	1-2	83.8	0
3	3-4	0	71.4
3	5-6	0	71.4
3	7-8	64.0	0

Static Markowitz Solution

- **Find efficient frontier:**



Results with Static Model

- **Fixed proportion in stock and bonds in each period**
- **80% stock for 15% return**
- **40% stock for 14% return**
- **Results: no fixed proportion achieves target better than 50% of time**
- **Dynamic achieves target 87.5% of time**

Analysis of Dynamic Model

- With discrete outcomes, p.l. utility:
  - Optimal solution has number of investments equal to number of branches in each period
  - Constrain the number of positive investments with the number of outcomes per period
- Impact of transaction fees and taxes
  - Additional constraints
  - Creates potential for more active investments in each period
  - Additional constraints can be imposed with linearization (representation other variance information)

Other Model Gains

- **Include transaction costs**
  - **Fixed proportion requires transaction costs each period just to re-balance**
  - **can accumulate**
- **Maintain consistent utility**

Current Study

- **Portfolios of major indexes**
- **Constructed efficient frontier**
- **Developed decision tree form for stochastic program**
- **Gains in basic model for stochastic program of 3-5% over 10 periods**