Optimal Policy Structure in Dynamic Asset-Liability Management

John R. Birge
Northwestern University
OUTLINE

• Mean-variance versus other utility functions
• Mean-Variance in dynamic portfolios
• Discrete time, piecewise linear utility
• Policy structure
• Enhanced models
Static Portfolio Model

Markowitz model
- Choose portfolio to minimize risk for a given return
- Find the efficient frontier
Markowitz Mean-Variance model

- For a given set of assets, find
  - fixed percentages to invest in each asset
  - maintain same percentage over time

- Needs
  - rebalance as returns vary
  - cash to meet obligations
Alternative Dynamic Model

- Assume possible outcomes over time
  - discretize generally
- In each period, choose mix of assets
- Can include transaction costs and taxes
- Can include liabilities over time
- Can include different measures of risk aversion
Example: Retirement Planning

• **GOAL:** Accumulate $G Y$ years from now

• **Assume:**
  - $W(0)$ - initial wealth
  - $K$ - investments

**Utility:** concave utility (piecewise linear)

**RANDOMNESS:** returns $r(k,t)$ - for $k$ in period $t$
where $Y$ $\rightarrow$ $T$ decision periods
FORMULATION

• **SCENARIOS:** ? ?? ?
  
  – Probability, p(?)
  
  – Groups, $S^t_1, ..., S^t_{S_t}$ at $t$

• **MULTISTAGE STOCHASTIC NLP FORM:**

\[
\text{max } \quad p(\text{???U}(W(\ ? \ , T)))
\]

\[
\text{s.t. (for all ?): } \quad \begin{align*}
?_k x(k,1, ?) &= W(o) \quad \text{(initial)} \\
?_k r(k,t-1, ?) x(k,t-1, ?) - ?_k x(k,t, ?) &= 0, \text{ all } t > 1; \\
?_k r(k,T-1, ?) x(k,T-1, ?) - W(\ ? \ , T) &= 0, \text{ (final)}; \\
x(k,t, ?) &\geq 0, \text{ all } k,t;
\end{align*}
\]

**Nonanticipativity:**

\[
x(k,t, ?') - x(k,t, ?) = 0 \text{ if } ?, ?? \in S^t_i \text{ for all } t, i, ?, ?
\]

This says decision cannot depend on future.
DATA and SOLUTIONS

• ASSUME:
  – Y=15 years
  – G=$80,000
  – T=3 (5 year intervals)
  – k=2 (stock/bonds)

• Returns (5 year):
  – Scenario A: r(stock) = 1.25   r(bonds)= 1.14
  – Scenario B: r(stock) = 1.06   r(bonds)= 1.12

• Solution:

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>SCENARIO</th>
<th>STOCK</th>
<th>BONDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-8</td>
<td>41.5</td>
<td>13.5</td>
</tr>
<tr>
<td>2</td>
<td>1-4</td>
<td>65.1</td>
<td>2.17</td>
</tr>
<tr>
<td>2</td>
<td>5-8</td>
<td>36.7</td>
<td>22.4</td>
</tr>
<tr>
<td>3</td>
<td>1-2</td>
<td>83.8</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3-4</td>
<td>0</td>
<td>71.4</td>
</tr>
<tr>
<td>3</td>
<td>5-6</td>
<td>0</td>
<td>71.4</td>
</tr>
<tr>
<td>3</td>
<td>7-8</td>
<td>64.0</td>
<td>0</td>
</tr>
</tbody>
</table>
Static Markowitz Solution

Find efficient frontier:
Results with Static Model

- Fixed proportion in stock and bonds in each period
- 80% stock for 15% return
- 40% stock for 14% return
- Results: no fixed proportion achieves target better than 50% of time
- Dynamic achieves target 87.5% of time
Analysis of Dynamic Model

• With discrete outcomes, p.l. utility:
  – Optimal solution has number of investments equal to number of branches in each period
  – Constrain the number of positive investments with the number of outcomes per period

• Impact of transaction fees and taxes
  – Additional constraints
  – Creates potential for more active investments in each period
  – Additional constraints can be imposed with linearization (representation other variance information)
Other Model Gains

- Include transaction costs
  - Fixed proportion requires transaction costs each period just to re-balance
  - can accumulate
- Maintain consistent utility
Current Study

- Portfolios of major indexes
- Constructed efficient frontier
- Developed decision tree form for stochastic program
- Gains in basic model for stochastic program of 3-5% over 10 periods