Real Options Theory and Practice

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Outline

• Planning questions
• Problems with traditional analyses: examples
• Real-option structure
• Assumptions and differences from financial options
• Resolving inconsistencies
• Conclusions
Investment Situation: Automotive Company

- **Goal:**
  - Decide on coordinated production, distribution capacity, and vendor contracts for multiple models in multiple markets (e.g., NA, Eur, LA, Asia)
- **Traditional approach**
  - Forecast demand for each model/market
  - Forecast costs
  - Obtain piece rates and proposals
  - Construct cash flows and discount

⇒ **Optimize for a single-point forecast**

Planning Questions?

- Start product in production or not? When?
- What to produce in-house or outside?
- How much capacity to install?
- What contracts to make outside?
- External factors: economy, competitors, suppliers, customers, legal, political, environmental
- Where to start?
  - Build a model
Traditional Model

• Focus on:
  • Cost orientation (not revenue management)
  • Single program (model, product)
  • NPV
  • Piece rates
• Result: support of traditional, fixed designs, little flexibility, little ability to change, immediate investment or no investment

Trends Limiting Traditional Analysis

• Market changes
  • Former competition:
    • Cost
    • Quality
  • New competition:
    • Customization
    • Responsiveness
Limitations of Traditional Methods for New Trends

- Myopic - ignoring long-term effects
- Often missing time value of cash flow
- Excluding potential synergies
- Ignoring uncertainty effects
- Not capturing option value of delay, scalability, and agility (changing product mix)
- Mis-calculate time-value of cash flow

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- Planning questions
- Problems with traditional analyses: examples
  - Value to delay
  - Scalability
  - Reusability
  - Agility
- Real-option structure
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Value to Delay Example

- Suppose a project may earn:
  - $100M if economy booms
  - $-50M if economy busts
- Each (boom or bust) is equally likely
- NPV = $25M (expected) - Start project
- Missing: Can we wait to observe economy?

Here, we don’t need to invest in “Bust” - Now we expect $50M
It’s worth $25M to wait.

Scale Option Example

- Scalability
- Suppose a five year program
  - Cost of fixed capacity is $100M
  - Cost of scalable capacity is $150M for same capacity
  - Predicted cash flow stream:

<table>
<thead>
<tr>
<th>Year</th>
<th>Net</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>50</td>
<td>25</td>
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</tbody>
</table>
Scalability Example - cont.
• Assume 15% opportunity cost of capital:
  • NPV(Traditional) = $50M
  • NPV(Scalable)= 0
• Problem: Scalable can be configured over time:

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spend</td>
<td>$50M for capacity to $25M</td>
<td>$50M for cap. to $50M</td>
<td>$50M for cap. to $75M</td>
</tr>
</tbody>
</table>

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Scalability Result

Cash flow for Scalable:

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net</td>
<td>-50</td>
<td>-25</td>
<td>0</td>
<td>75</td>
<td>50</td>
<td>25</td>
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</tbody>
</table>

Now, NPV(Scalable)=$75M > NPV(Fixed)
Traditional approach misses scalability advantage.

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Reusability Example

• Assume:
  • Same conditions as before for fixed system
  • Two consecutive 5-year programs
  • Suppose for Reusable Manufacturing System (RMS)
    • No scalability
    • Initial cost of $125 M
    • Can reconfigure for second program at cost of $25M

Reusability Example cont.

• Traditional approach
  • Single program evaluation
  • NPV(Fixed) = $50M
  • NPV(RMS) = $25M
  • Choose Fixed

• Problem: Missing the second program
Reusability Two-Program Cash Flows

Fixed cash flow, NPV(Fixed) = $75M

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<tr>
<th></th>
<th>0</th>
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<th>3</th>
<th>4</th>
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<tr>
<td></td>
<td>-100</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>50</td>
<td>-75</td>
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<tr>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
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<tr>
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<td>75</td>
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RMS Cash Flow, NPV(RMS) = $87M

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<td>50</td>
<td>75</td>
<td>50</td>
<td>25</td>
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Traditional method misses two-program advantage

Agility Example: Flexible Capacity Option

Difficulty: Traditional single forecast

Example: Products A, B

- Forecast demand: 100 for each; Margin: 2
- Dedicated capacity cost: 1
- Flexible capacity cost: 1.1

Dedicated: Flexible:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Revenue:</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Cost:</td>
<td>200</td>
<td>220</td>
</tr>
<tr>
<td>Profit:</td>
<td>200</td>
<td>180</td>
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</tbody>
</table>

Choose dedicated
Multiple Scenario Effect

Suppose two demand possibilities: 50 or 150 equally likely - *Four scenarios*

**Dedicated:**

<table>
<thead>
<tr>
<th>Scenario 1: 50, 50</th>
<th>Production of A:</th>
<th>Production of B:</th>
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<tbody>
<tr>
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</table>

<table>
<thead>
<tr>
<th>Scenario 3: 150, 50</th>
<th>Production of A:</th>
<th>Production of B:</th>
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**Flexible:**

<table>
<thead>
<tr>
<th>Scenario 2: 50, 150</th>
<th>Production of A:</th>
<th>Production of B:</th>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario 4: 150, 150</th>
<th>Production of A:</th>
<th>Production of B:</th>
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</table>

Additional Production

**Evaluation with Scenarios**

- Four scenarios: 50 or 150 on each
- Dedicated
  - Sell (50,50), (50,100), (100,50), (100, 100)
  - Expected revenue: 300
- Flexible
  - Sell (50,50), (50,150), (150,50), (100, 100)
  - Expected revenue: 350

<table>
<thead>
<tr>
<th>Dedicated:</th>
<th>Flexible:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. Revenue:</td>
<td>300</td>
</tr>
<tr>
<td>Cost:</td>
<td>200</td>
</tr>
<tr>
<td>Profit:</td>
<td>100</td>
</tr>
</tbody>
</table>

Choose flexible

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Summary of Problems

• Missing basic abilities in traditional approaches:
  – Delay option
  – Scaling option
  – Reuse option
  – Agility option

• Option evaluation:
  – Look at all possibilities
  – How to discount?

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Real Options

• Idea: Assets that are not fully used may still have option value (includes contracts, licenses)
• Value may be lost when the option is exercised (e.g., developing a new product, invoking option for second vendor)
• Traditional NPV analyses are flawed by missing the option value
• Missing parts:
  • Value to delay and learn
  • Option to scale and reuse
  • Option to change with demand variation (uncertainty)
  • Not changing discount rates for varying utilizations

How to capture in model?

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Key Steps in Building a Model

• Identify problem
• Determine objectives
• Specify decisions
• Find operating conditions
• Define metrics
  – How to measure objectives?
  – How to quantify requirements, limits?
  – How to include effect of uncertainty?
• Formulate

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Utility Function Approach

- **Observation:**
  - Most decision makers are adverse to risk
- **Assume:**
  - Outcomes can be described by a utility function
  - Decision makers want to maximize expected utility
- **Difficulties:**
  - Is the decision maker the sole stakeholder?
  - Whose utility should be used?
  - How to define a utility?
  - How to solve?
- **Alternative to decision maker - investor**

Measuring Investor Value

- **OBSERVATIONS:**
  - Investors prefer lower risk
  - Investors can diversify away unique risk
  - Only important risk is market - contribution to portfolio
- **CONSEQUENCE:** Capital asset pricing model (CAPM)
  - With CAPM, can find a discount rate
Discount Rate Determination

- Traditional approach
  - Discount rate is the same for all decisions in program evaluation
- Problems
  - Program evaluation includes decisions on capacity, distribution channel, vendor contracts
  - These decisions affect correlation to market – hence, change the discount rate
- Need: discount rate to change with decisions as they are determined; How?

Discount Rate Determination

- USE CAP-M? FIND CORRELATION TO THE MARKET?
  - Can measure for known markets (beta values)
  - If capacitated, depends on decisions
    - Constrained resources - capacity
    - Correlations among demands

- ALTERNATIVES?
  - Option Theory
    - Allows for non-symmetric risk
    - Explicitly considers constraints - As if selling excess to competitors at a given price
Valuing an Option

- (European) Call Option on Share assuming:
  - Buy at $K$ at time $T$; Current time: $t$; Share price: $S_t$
  - Volatility: $\sigma$; Riskfree rate: $r_f$; No fees; Price follows Ito process

- Valuing option:
  - Assume risk neutral world (annual return=$r_f$, independent of risk)
  - Find future expected value and discount back by $r_f$

\[
\text{Call value at } t = C_t = e^{-r_f(T-t)} \int(S_T-K)^+dF_t(S_T)
\]

Relation to Real Options

- Example: What is the value of a plant with capacity $K$?
  - Discounted value of production up to $K$?

- Problems:
  - Production is limited by demand also (may be > $K$)
  - How to discount?

- Resolution:
  - Model as an option
  - Assume:
    - Market for demand (substitutes)
    - Forecast follows Ito process
    - No transaction costs

$\Rightarrow$ Model like share minus call
Using Option Valuation for Capacity

- **Goal**: Production value with capacity K
- **Compute uncapacitated value based on CAPM**:
  - \( S_t = e^{r(T-t)} \int c_T S_t dF(S_t) \)
  - where \( c_T \) = margin, \( F \) is distribution (with risk aversion),
  - \( r \) is rate from CAPM (with risk aversion)
- **Assume \( S_t \) now grows at riskfree rate, \( r_f \); evaluate as if risk neutral**:
  - Production value = \( S_t - C_t = e^{r_f(T-t)} \int c_T \min(S_t, K) dF_f(S_t) \)
  - where \( F_f \) is distribution (with risk neutrality)

Generalizations for Other
Long-term Decisions

- **Model**: period \( t \) decisions: \( x_t \)
- **START**: Eliminate constraints on production
  - Demand uncertainty remains
  - Can value unconstrained revenue with market rate, \( r \):
    \[
    \frac{1}{1+r}^t c_t x_t
    \]

**IMPLICATIONS OF RISK NEUTRAL HEDGE**:
Can model as if investors are risk neutral
\( \Rightarrow \) value grows at riskfree rate, \( r_f \)

**Future value**: \( \frac{1}{1+r}^t c_t (1+r_f)^t x_t \)

**BUT**: This new quantity is constrained
New Period $t$ Problem: Linear Constraints on Production

• WANT TO FIND (present value):

$$\frac{1}{(1+r_p)^t} \max \left\{ c_t x_t \frac{(1+r_f)^t}{(1+r)^t} \mid A_t x_t \frac{(1+r_f)^t}{(1+r)^t} \leq b_t \right\}$$

EQUIVALENT TO:

$$\frac{1}{(1+r)^t} \max \left\{ c_t x \mid A_t x \leq b \frac{(1+r_f)^t}{(1+r)^t} \right\}$$

MEANING: To compensate for lower risk with constraints, constraints expand and risky discount is used

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Constraint Modification

• FORMER CONSTRAINTS: $A_t x_t \leq b_t$

• NOW: $A_t x_t \frac{(1+r_f)^t}{(1+r)^t} \leq b_t$

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EXTREME CASES

All slack constraints:

\[ \frac{1}{(1+r)^t} \max \{ c^T x \mid A^T x \leq b (1+r)^t / (1+r_f)^t \} \]

becomes equivalent to:

\[ \frac{1}{(1+r)^t} \max \{ c^T x \mid A^T x \leq b \} \]

i.e. same as if unconstrained - risky rate

NO SLACK:

becomes equivalent to:

\[ \frac{1}{(1+r)^t} \max \{ c^T x= b^{-1}b (1+r)^t / (1+r_f)^t \} = c^T B^{-1}b / (1+r_f)^t \]

i.e. same as if deterministic - riskfree rate

Example: Capacity Planning

- What to produce?
- Where to produce? (When?)
- How much to produce?

EXAMPLE: Models 1, 2, 3; Plants A, B

Should B also build 2?
Result: Stochastic Linear Programming Model

- Key: Maximize the Added Value with Installed Capacity
  - Must choose best mix of models assigned to plants
  - Maximize Expected Value over $\sum_i e^{-rt} \text{Profit} (i) \cdot \text{Production}(i,t,s) - \text{CapCost}(i \text{ at } j,t) \cdot \text{Capacity}(i \text{ at } j,t)$
  - subject to: $\text{MaxSales}(i,t,s) \geq \sum_j \text{Production}(i \text{ at } j,t,s)$
  - $\sum_i \text{Production}(i \text{ at } j,t,s) \leq e^{(r-f)t} \text{Capacity}(i,t)$
  - $\text{Production}(i \text{ at } j,t,s) \leq e^{(r-f)t} \text{Capacity}(i \text{ at } j,t)$
  - $\text{Production}(i \text{ at } j,t,s) \geq 0$
- Need MaxSales(i,t,s) - random
- Capacity(i at j,0) - Decision in First Stage (now)

NOTE: Linear model that incorporates risk

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Result with Option Approach

- Can include risk attitude in linear model
- Simple adjustment for the uncertainty in demand
- **Requirement 1**: correlation of all demand to market
- **Requirement 2**: assumptions of market completeness

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Assumptions

• Process of prices or sales forecasts
• No transaction fees
• Complete market (difference from financial options)
  • How to construct a hedge?
  • If NPV>0, inconsistency
  • Process: Trade option and asset to create riskfree security
Creating Best Hedge – and a Confession

• Underlying asset: Max potential sales in market
• Option: Plant with given capacity
• Other marketable securities:
  • Competitors’ shares
  • Overall all securities min residual volatility
  • Confession: Due to incompleteness, some volatility remains (otherwise, NPV=0)

Resolution

• Incompleteness gives a range of possible values
• Can adjust capacity limits by varying discount factor with risk neutral assumptions on forecasts
• Can vary constraint multipliers with original forecast distribution
• All optimal policies for the given range are consistent with the market (cannot be beaten all the time)
• Obtain a range of policies – can use other criteria
Result of Residual Risk

- In binomial model, asset price moves from $S_t$ to $uS_t + v_1$ or $dS_t + v_2$ where $v_1$ and $v_2$ vary independently and have smallest volatility
- For standard call option,
  \[
  C_t = \left[ \frac{(S_t - d S_t + v_1)}{(uS_t - d S_t + v_2)} \right] (uS_t - K)
  \]
  \[
  = \left[ \frac{(S_t - d S_t + v_1)}{p(uS_t - d S_t + v_2)} \right] p (uS_t - K)
  \]
  \[
  = e^{-r(T-t)} \langle E[(S_t-K)^+] \rangle \text{ where } r \text{ is in a range determined by } [v2,v1]
  \]
- Analogous result for capacity valuation: a range of values are consistent

Alternatives and Challenges

- Use equilibrium and utility function approaches
- Caution on complexity of models
- Critical factor: range of outcomes considered
- Other challenges:
  - Effects of pricing decisions
  - Effects of competitors
  - Distribution changes from decisions
  - Extending financial and real options together: operational and financial hedging
Operational and Financial Hedging uses of Real Options

• Objective: Determine capacity levels in different markets, production in each market, distribution across markets, and use of financial hedging instruments to maximize total global value

• Challenges:
  • Demand and exchange rates may change
  • Correlations among demand and exchange
  • What is enough capacity?
  • What performance metrics to use?

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Summary

- Options apply to many varied decision problems
- Can evaluate planning with proper option evaluation techniques
- Relaxed market assumptions lead to models that determine a range of policies
- Firm or investor utility can choose within range
- Questions? Comments?