Introduction to Stochastic Optimization in Supply Chain and Logistic Optimization

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Outline

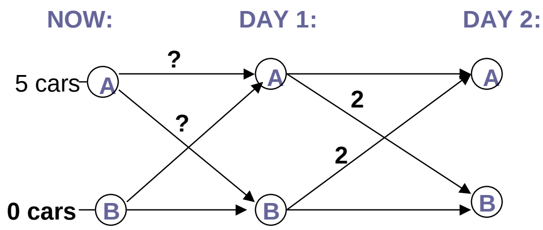
- Overview
- Part I - Models
  - Vehicle allocation (integer linear)
  - Financial plans (continuous nonlinear)
  - Manufacturing and real options (integer nonlinear)
- Part II – Optimization Methods

Overview

- Stochastic optimization
  - Traditional
    - Small problems
    - Impractical
  - Current
    - Integrate with large-scale optimization (stochastic programming)
    - Practical examples
    - Expanding rapidly
    - Integration of financial and operation considerations

Vehicle Allocation

- Decision:
  - How to position empty freight cars?

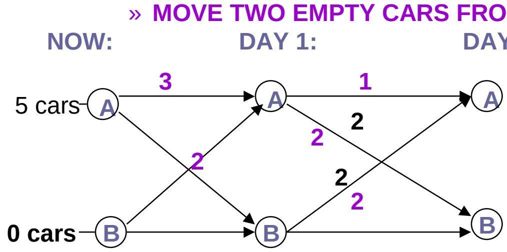


DEMAND: **DAY 1: B to A:Mean Value=2**
DAY 1: A to B:Mean Value=2

Vehicle Allocation: Mean Value Solution

Parameters: COST: 0.5 per empty car from A to B
REVENUE: 1.5 per full car from B to A, 1 from A to B

- **Maximize: Revenue-Cost**
  - » **MOVE TWO EMPTY CARS FROM A to B**

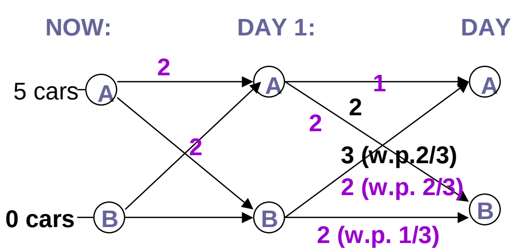


RESULT: **Net 2: A to B; Net 2: B to A**
TOTAL(*MV*) = 4

Expectation of Mean Value

Suppose: Demand is Random (Expectation from A to B=2)
- **0 from A to B with prob. 1/3**
- **3 from A to B with prob. 2/3**

- **Find: Expected (Revenue-Cost)**
  - » **MOVE Two EMPTY CARS FROM A to B**



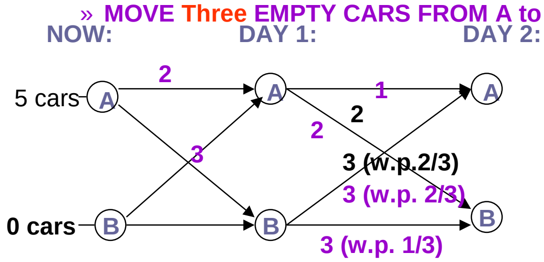
Expected Value:

Net 2: A to B;
Net 2: B to A (w.p. 2/3)
-1: B to A (w.p. 1/3)
TOTAL (*EMV*): 3

Stochastic Program Solution

Suppose: Demand is Random (as before)
GOAL: A solution to obtain highest expected value

- **Maximize: Expected (Revenue-Cost)**
  - » MOVE Three EMPTY CARS FROM A to B



Expected Value:

Net 2: A to B;
Net 3: B to A (w.p. 2/3)
-1.5 : B to A (w.p. 1/3)
TOTAL (*RP*): 3.5
***RP*=Recourse Problem**

INFORMATION and MODEL VALUE

- **INFORMATION VALUE:**
  - **FIND Expected Value with Perfect Information or Wait-and-See (WS) solution:**
    - **Know demand: if 3, send 3 from A to B; If 0, send 0 from A to B:**
    - **Earn: 2 (AtoB) + (2/3) (3) + (1/3)0= 4 = WS**
  - **Expected Value of Perfect Information (EVPI):**
    - **EVPI = WS - RP = 4 - 3.5 = 0.5**
    - **Value of knowing future demand precisely**
- **MODEL VALUE:**
  - **FIND EMV, RP**
  - **Value of the Stochastic Solution (VSS):**
    - **VSS = RP - EMV=3.5 - 3 = 0.5**
    - **Value of using the correct optimization model**

INFORMATION/MODEL OBSERVATIONS

- EVPI and VSS:
  - ALWAYS >= 0 (WS >= RP>= EMV)
  - OFTEN DIFFERENT (WS=RP but RP > EMV and vice versa)
  - FIT CIRCUMSTANCES:
    - COST TO GATHER INFORMATION
    - COST TO BUILD MODEL AND SOLVE PROBLEM
- MEAN VALUE PROBLEMS:
  - MV IS OPTIMISTIC (MV=4 BUT EMV=3, RP=3.5)
    - ALWAYS TRUE IF CONVEX AND RANDOM
    - CONSTRAINT PARAMETERS
  - VSS LARGER FOR SKEWED DISTRIBUTIONS/COSTS

STOCHASTIC PROGRAM

- ASSUME: Random demand on AB and BA
- GOAL: maximize expected profits
  - (risk neutral)
- DECISIONS: x_{ij} - empty from i to j
  - $y_{ij}(s)$ - full from i to j in scenario s (RECOURSE)
  - (prob. p(s))
- FORMULATION:

Max -0.5xAB + $\Sigma_{s=s1,s2}$ p(s) (1.5 yAB(s) + 1.5 yBA(s))

s.t. xAB +xAA = 5 (Initial)

-xAB + yBA(s) <= 0 (Limit BA)

-xAA + yAB(s) <= 0 (Limit AB)

yBA(s) <= DBA(s) (Demand BA)

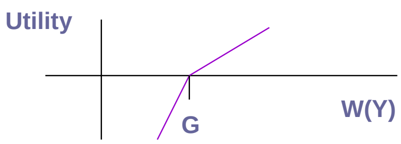
+ yAB(s)<= DAB(s) (Demand AB)

xAA, XAB, yAA(s), yAB (s)>=0

EXTENSIONS: Multiple stages;Constraint/objective complexity (Powell et al.)

Financial Planning

- **GOAL: Accumulate $G for tuition Y years from now**
- **Assume:**
  - **$ W(0) - initial wealth**
  - **K - investments**
  - **concave utility (piecewise linear)**



RANDOMNESS: returns r(k,t) - for k in period t
where Y ⟶ T decision periods

FORMULATION

- **SCENARIOS:** $\sigma \in \Sigma$
  - **Probability, p(σ)**
  - **Groups, S^t_1, ..., S^t_{St} at t**
- **MULTISTAGE STOCHASTIC NLP FORM:**

max Σ_σ p(σ) (U(W(σ , T))
s.t. (for all σ): Σ_k x(k,1, σ) = W(o) (initial)
Σ_k r(k,t-1, σ) x(k,t-1, σ) - Σ_k x(k,t, σ) = 0 , all t >1;
Σ_k r(k,T-1, σ) x(k,T-1, σ) - W(σ , T) = 0, (final);
x(k,t, σ) >= 0, all k,t;

Nonanticipativity:
x(k,t, σ') - x(k,t, σ) = 0 if σ', σ ∈ S^t_i for all t, i, σ', σ
This says decision cannot depend on future.

DATA and SOLUTIONS

- **ASSUME:**
  - **Y=15 years**
  - **G=$80,000**
  - **T=3 (5 year intervals)**
  - **k=2 (stock/bonds)**
- **Returns (5 year):**
  - **Scenario A: r(stock) = 1.25   r(bonds)= 1.14**
  - **Scenario B: r(stock) = 1.06   r(bonds)= 1.12**
- **Solution:**

PERIOD	SCENARIO	STOCK	BONDS
1	1-8	41.5	13.5
2	1-4	65.1	2.17
2	5-8	36.7	22.4
3	1-2	83.8	0
3	3-4	0	71.4
3	5-6	0	71.4
3	7-8	64.0	0

Manufacturing Planning

- Goal:
  - Decide on coordinated production, distribution capacity and vendor contracts for multiple models in multiple markets (e.g., NA, Eur, LA, Asia)
- Traditional approach
  - Forecast demand for each model/market
  - Forecast costs
  - Obtain piece rates and proposals
  - Construct cash flows and discount

➔ Optimize for a single-point forecast

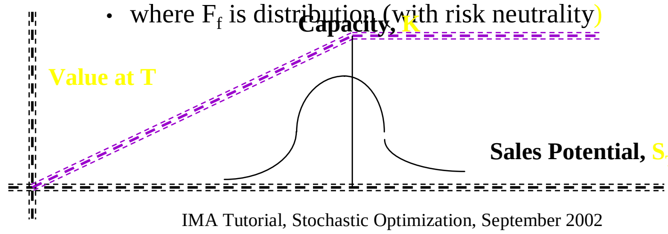
➔ Missing option value of flexible capacity

Real Options

- Idea: Assets that are not fully used may still have option value (includes contracts, licenses)
- Value may be lost when the option is exercised (e.g., developing a new product, invoking option for second vendor)
- Traditional NPV analyses are flawed by missing the option value
- Missing parts:
  - Value to delay and learn
  - Option to scale and reuse
  - Option to change with demand variation (uncertainty)
  - Not changing discount rates for varying utilizations

Real Option Valuation for Capacity

- Goal: Production value with capacity K
  - Compute uncapacitated value based on Capital Asset Pricing Model:
    - $S_t = e^{-r(T-t)} \int c_T S_T dF(S_T)$
    - where c_T=margin,F is distribution (with risk aversion),
    - r is rate from CAPM (with risk aversion)
  - Assume S_t now grows at riskfree rate, r_f; evaluate as if risk neutral:
    - Production value = $S_t - C_t = e^{-r_f(T-t)} \int c_T \min(S_T,K) dF_f(S_T)$
    - where F_f is distribution (with risk neutrality)



Generalizations for Other Long-term Decisions

- Model: period t decisions: x_t
- START: Eliminate constraints on production
  - Demand uncertainty remains
  - Can value unconstrained revenue with market rate, r:

$$1/(1+r)^t \; c_t \, x_t$$

IMPLICATIONS OF RISK NEUTRAL HEDGE:
Can model as if investors are risk neutral
=> value grows at riskfree rate, r_f

Future value: $[1/(1+r)^t \; c_t \, (1+r_f)^t \, x_t]$

BUT: This new quantity is constrained

New Period t Problem: Linear Constraints on Production

- WANT TO FIND (present value):

$$1/\,(1+r_f)^t \int \text{MAX}\,[\, c_t\, x_t\, (1+r_f)^t/(1+r)^t \mid A_t\, x_t\, (1+r_f)^t/(1+r)^t \;<=\; b]$$

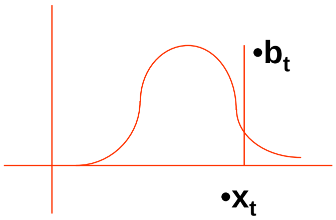
EQUIVALENT TO:

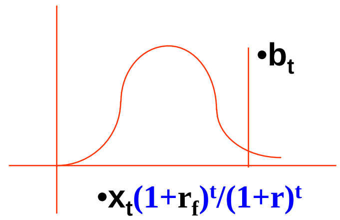
$$1/\,(1+r)^t \int \text{MAX}\,[\, c_t\, x \mid A_t\, x \;<=\; b\, (1+r)^t/(1+r_f)^t]$$

MEANING: To compensate for lower risk with constraints, constraints expand and risky discount is used

Constraint Modification

- FORMER CONSTRAINTS: $A_t\, x_t <= b_t$
- NOW: $A_t\, x_t\, (1+r_f)^t/(1+r)^t \;<= b_t$





EXTREME CASES

All slack constraints:

$$1/(1+r)^t \int \text{MAX}\,[\,c_t\, x \mid A_t\, x <= b\,(1+r)^t/(1+r_f)^t]$$

becomes equivalent to:

$$1/(1+r)^t \int \text{MAX}\,[\,c_t\, x \mid A_t\, x <= b]$$

i.e. same as if unconstrained - risky rate

NO SLACK:

becomes equivalent to:

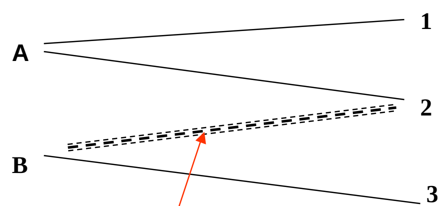
$$1/(1+r)^t \int [c_t\, x = B^{-1} b\,(1+r)^t/(1+r_f)^t] = c_t\, B^{-1} b/(1+r_f)^t$$

i.e. same as if deterministic- riskfree rate

Example: Capacity Planning

- What to produce?
- Where to produce? (When?)
- How much to produce?

EXAMPLE: Models 1,2, 3 ; Plants A,B



Should B also build 2?

Result: Stochastic Linear Programming Model

- Key: Maximize the Added Value with Installed Capacity
  - Must choose best mix of models assigned to plants
  - Maximize Expected Value over s[$\Sigma_{i,t}$ e^{-rt}Profit (i) Production(i,t,s) - CapCost(i at j,t)Capacity (i at j,t)]
  - subject to: MaxSales(i,t,s) >= Σ_j Production(i at j,t,s)
  - Σ_i Production(i at j,t,s) <= $e^{(r-rf)t}$ Capacity (i,t)
  - Production(i at j,t,s) <= $e^{(r-rf)t}$ Capacity (i at j,t)
  - Production(i at j,t,s) >= 0
- Need MaxSales(i,t,s) - random
  - Capacity(i at j,0) - Decision in First Stage (now)

NOTE: Linear model that incorporates risk

Result with Option Approach

- Can include risk attitude in linear model
- Simple adjustment for the uncertainty in demand
- Requirement 1: correlation of all demand to market
- Requirement 2: assumptions of market completeness

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General Stochastic Programming Model: Discrete Time

- Find $x=(x_1,x_2,\ldots,x_T)$ and p (unknown distribution) to

minimize $E_p [\Sigma_{t=1}^T f_t(x_t,x_{t+1},p)]$
s.t. $x_t \in X_t$, x_t nonanticipative p in P (distribution class)
$P[h_t (x_t,x_{t+1},p_t) <= 0] >= a$ (chance constraint)

General Approaches:

- **Simplify distribution (e.g., sample) and form a mathematical program:**
  - **Solve step-by-step (dynamic program)**
  - **Solve as single large-scale optimization problem**
- **Use iterative procedure of sampling and optimization steps**

Simplified Finite Sample Model

- Assume p is fixed and random variables represented by sample ξ^i_t for t=1,2,..,T, i=1,…,N_t with probabilities p^i_t ,a(i) an *ancestor* of i, then model becomes (no chance constraints):

minimize $\Sigma_{t=1}^T \Sigma_{i=1}^{N_t} p^i_t f_t(x^{a(i)}_t, x^i_{t+1}, \xi^i_t)$
s.t. $x^i_t \in X^i_t$

Observations?

- Problems for different i are similar – solving one may help to solve others
- Problems may decompose across i and across t yielding
  - smaller problems (that may scale linearly in size)
  - opportunities for parallel computation.

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  - Factorization/sparsity (interior point/barrier)
  - Decomposition
  - Lagrangian methods
- Conclusions

SOLVING AS LARGE-SCALE MATHEMATICAL PROGRAM

- PRINCIPLES:
  - DISCRETIZATION LEADS TO MATHEMATICAL PROGRAM BUT LARGE-SCALE
  - USE STANDARD METHODS BUT EXPLOIT STRUCTURE
- DIRECT METHODS
  - TAKE ADVANTAGE OF SPARSITY STRUCTURE
    - SOME EFFICIENCIES
  - USE SIMILAR SUBPROBLEM STRUCTURE
    - GREATER EFFICIENCY
- SIZE
  - UNLIMITED (INFINITE NUMBERS OF VARIABLES)
  - STILL SOLVABLE (CAUTION ON CLAIMS)

STANDARD APPROACHES

- Sparsity Structure Advantage
  - PARTITIONING
  - BASIS FACTORIZATION
  - INTERIOR POINT FACTORIZATION
- Similar/Small Problem Advantage
  - DP APPROACHES: DECOMPOSITION
    - BENDERS, L-SHAPED (VAN SLYKE – WETS)
    - DANTZIG-WOLFE (PRIMAL VERSION)
    - REGULARIZED (RUSZCZYNSKI)
    - VARIOUS SAMPLING SCHEMES (HIGLE/SEN Stochastic Decomposition, Abridge Nested Decomposition)
  - LAGRANGIAN METHODS

Sparsity Methods: Stochastic Linear Program Example

- Two-stage Linear Model:

  $X_1 = \{x_1 | A x_1 = b, x_1 >= 0\}$

  $f_0(x_0,x_1)=c x_1$

  $f_1 (x_1,x_2^i,\xi_2^i) = q x_2^i$ if $T x_1 + W x_2^i = \xi_2^i$,

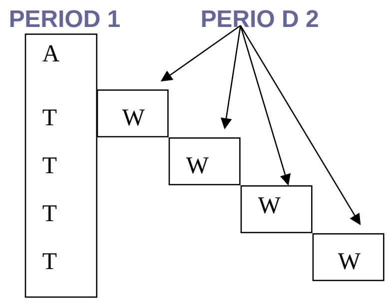
  $x_2^i >= 0$; $+ \infty$ otherwise
- Result: $\min c x_1 + \boldsymbol{\Sigma}_{\mathbf{i=1}}^{\mathbf{N1}} \mathbf{p}_2^i q x_2^i$

  s. t. $A x_1 = b, x_1 >= 0$

  $T x_1 + W x_2^i = \xi_2^i, x_2^i >= 0$

LP-BASED METHODS

- USING BASIS STRUCTURE



= A'

INTERIOR POINT MATRIX STRUCTURE

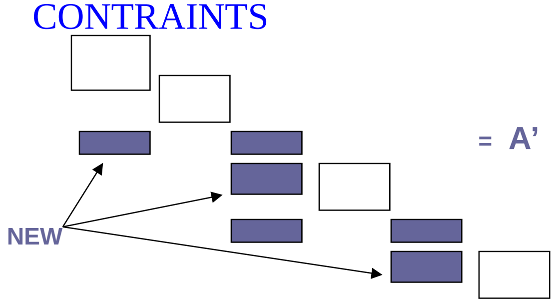
A'D²A'^T=

- **MODEST GAINS FOR SIMPLEX**

COMPLETE FILL-IN

ALTERNATIVES FOR INTERIOR POINTS

- VARIABLE SPLITTING (MULVEY ET AL.)
  - PUT IN EXPLICIT NONANTICIPATIVITY CONTRAINTS



= A'

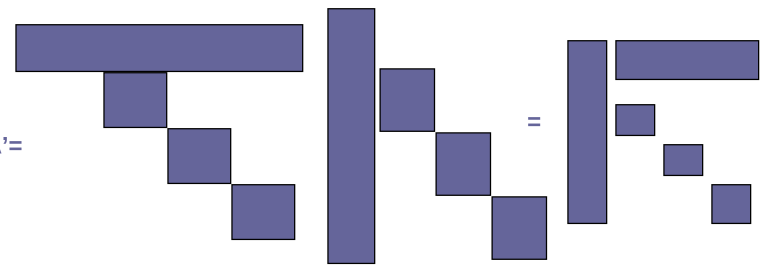
•RESULT

•REDUCED FILL-IN BUT LARGER MATRIX

OTHER INTERIOR POINT APPROACHES

- USE OF DUAL FACTORIZATION OR MODIFIED SCHUR COMPLEMENT

$A'^T D^2 A' =$



RESULTS:

- SPEEDUPS OF 2 TO 20
- SOME INSTABILITY => INDEFINITE SYSTEM (VANDERBEI ET AL. CZYZYK ET AL.)

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SIMILAR/SMALL PROBLEM STRUCTURE: DYNAMIC PROGRAMMING VIEW

- STAGES: t=1,...,T
- STATES: $x_t \to B_t x_t$(or other transformation)
- VALUE FUNCTION:

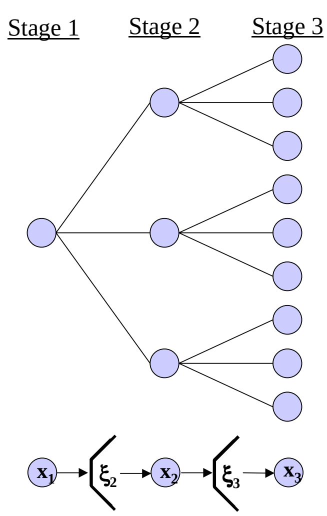
  $Q_t(x_t) = E[Q_t(x_t,\xi_t)]$ where

  ξ_t is the random element and

  $Q_t(x_t,\xi_t) = \min f_t(x_t,x_{t+1},\xi_t) + Q_{t+1}(x_{t+1})$

  s.t. $x_{t+1} \in X_{t+1t}(\xi_t)$ $\quad x_t$ given
- SOLVE : iterate from T to 1

LINEAR MODEL STRUCTURE



$$\begin{aligned} \min \quad & c_1 x_1 + Q_2(x_1) \\ s.t. \quad & W_1 x_1 = h_1 \\ & x_1 \geq 0 \end{aligned}$$

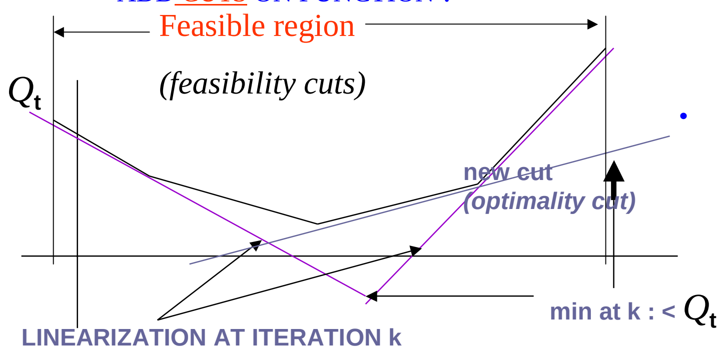
$$Q_t(x_{t-1,a(k)}) = \sum_{\xi_{t,k} \in \Xi_t} prob(\xi_{t,k}) \mathrm{Q}_{t,k}(x_{t-1,a(k)}, \xi_{t,k})$$

$$\begin{aligned} \mathrm{Q}_{t,k}(x_{t-1,a(k)}, \xi_{t,k}) = \min \quad & c_t(\xi_{t,k}) x_{t,k} + Q_{t+1}(x_{t,k}) \\ s.t. \quad & W_t x_{t,k} = h_t(\xi_{t,k}) - T_{t-1}(\xi_{t,k}) x_{t-1,a(k)} \\ & x_{t,k} \geq 0 \end{aligned}$$

- $Q_{N+1}(x_N) = 0$, for all x_N,
- $Q_{t,k}(x_{t-1,a(k)})$ is a piecewise linear, convex function of $x_{t-1,a(k)}$

DECOMPOSITION METHODS

- BENDERS IDEA
  - FORM AN OUTER LINEARIZATION OF Q_t
  - ADD *CUTS* ON FUNCTION :



- **USE AT EACH STAGE TO APPROX. VALUE FUNCTION**
- **ITERATE BETWEEN STAGES UNTIL ALL MIN = Q_t**

Nested Decomposition

- In each subproblem, replace expected recourse function $Q_{t,k}(x_{t-1,a(k)})$ with unrestricted variable $\theta_{t,k}$
  - Forward Pass:
    - Starting at the root node and proceeding forward through the scenario tree, solve each node subproblem

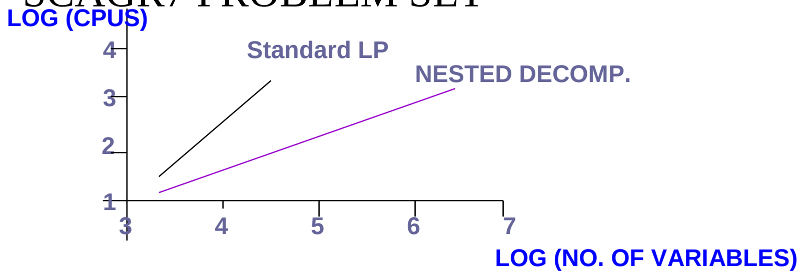
$$\begin{aligned} \hat{Q}_{t,k}\left(x_{t-1,a(k)},\xi_{t,k}\right) = \quad \min \quad & c_t\left(\xi_{t,k}\right)x_{t,k} + \theta_{t,k} \\ s.t. \quad & W_t x_{t,k} = h_t\left(\xi_{t,k}\right) - T_{t-1}\left(\xi_{t,k}\right)x_{t-1,a(k)} \\ & E_{t,k}x_{t,k} + \theta_{t,k} \geq e_{t,k} \quad (optimality \ cuts) \\ & D_{t,k}x_{t,k} \geq d_{t,k} \quad (feasibility \ cuts) \\ & x_{t,k} \geq 0 \end{aligned}$$

    - Add feasibility cuts as infeasibilities arise
  - Backward Pass
    - Starting in top node of Stage t = N-1, use optimal dual values in descendant Stage t+1 nodes to construct new optimality cut. Repeat for all nodes in Stage t, resolve all Stage t nodes, then t → t-1.
  - Convergence achieved when

$$\theta_1 = Q_2(x_1)$$

SAMPLE RESULTS

- SCAGR7 PROBLEM SET



PARALLEL: 60-80% EFFICIENCY IN SPEEDUP

OTHER PROBLEMS: SIMILAR RESULTS

- ONLY < ORDER OF MAGNITUDE SPEEDUP WITH STORM
  - TWO-STAGES - LITTLE COMMONALITY IN SUBPROBLEMS
  - STILL ABLE TO SOLVE ORDER OF MAGNITUDE LARGER PROBLEMS

Decomposition Enhancements

- Optimal basis repetition
  - Take advantage of having solved one problem to solve others
  - Use *bunching* to solve multiple problems from root basis
  - *Share* bases across levels of the scenario tree
  - Use solution of single scenario as *hot start*
- Multicuts
  - Create cuts for each descendant scenario
- Regularization
  - Add quadratic term to keep close to previous solution
- Sampling
  - Stochastic decomposition (Higle/Sen)
  - Importance sampling (Infanger/Dantzig/Glynn)
  - Multistage (Pereira/Pinto, Abridged ND)

Pereira-Pinto Method

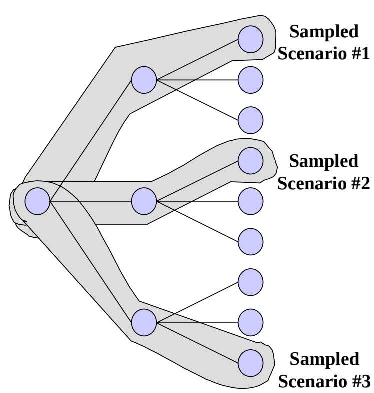
- Incorporates sampling into the general framework of the Nested Decomposition algorithm
- Assumptions:
  - relatively complete recourse
    - no feasibility cuts needed
  - serial independence
    - an optimality cut generated for any Stage t node is valid for all Stage t nodes
- Successfully applied to multistage stochastic water resource problems

Pereira-Pinto Method

1. Randomly select *H N*-Stage scenarios
2. Starting at the root, a forward pass is made through the sampled portion of the scenario tree (solving ND subproblems)
3. A statistical estimate of the first stage objective value $\bar{z}$ is calculated using the total objective value obtained in each sampled scenario

the algorithm terminates if current first stage objective value $c_1x_1 + \theta_1$ is within a specified confidence interval of $\bar{z}$

4. Starting in sampled node of Stage t = N-1, solve all Stage t+1 descendant nodes and construct new optimality cut. Repeat for all sampled nodes in Stage t, then repeat for t = t - 1



Pereira-Pinto Method

- Advantages
  - significantly reduces computation by eliminating a large portion of the scenario tree in the forward pass
- Disadvantages
  - requires a complete backward pass on all sampled scenarios
    - not well designed for bushier scenario trees

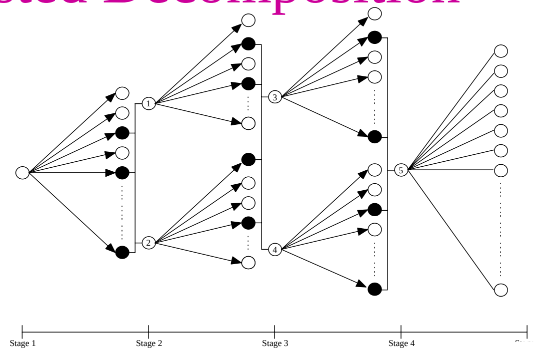
Abridged Nested Decomposition

- Also incorporates sampling into the general framework of Nested Decomposition
- Also assumes relatively complete recourse and serial independence
- Samples both the subproblems to solve and the solutions to continue from in the forward pass

Abridged Nested Decomposition

Forward Pass

1. **Solve root node subproblem**
2. **Sample Stage 2 subproblems and solve selected subset**
3. **Sample Stage 2 subproblem solutions and branch in Stage 3 only from selected subset (i.e., nodes 1 and 2)**

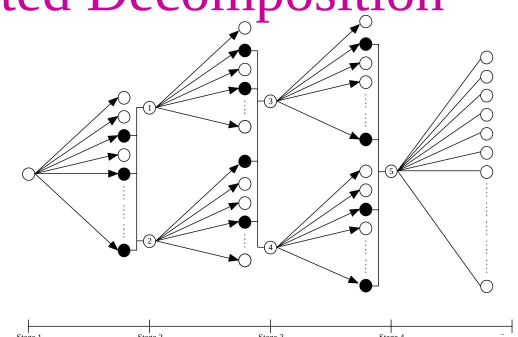


4. For each selected Stage t-1 subproblem solution, sample Stage t subproblems and solve selected subset
5. Sample Stage t subproblem solutions and branch in Stage t+1 only from selected subset

Abridged Nested Decomposition

Backward Pass

1. **Starting in first branching node of Stage t = N-1, solve all Stage t+1 descendant nodes and construct new optimality cut for all stage t subproblems. Repeat for all sampled nodes in Stage t, then repeat for t = t - 1**



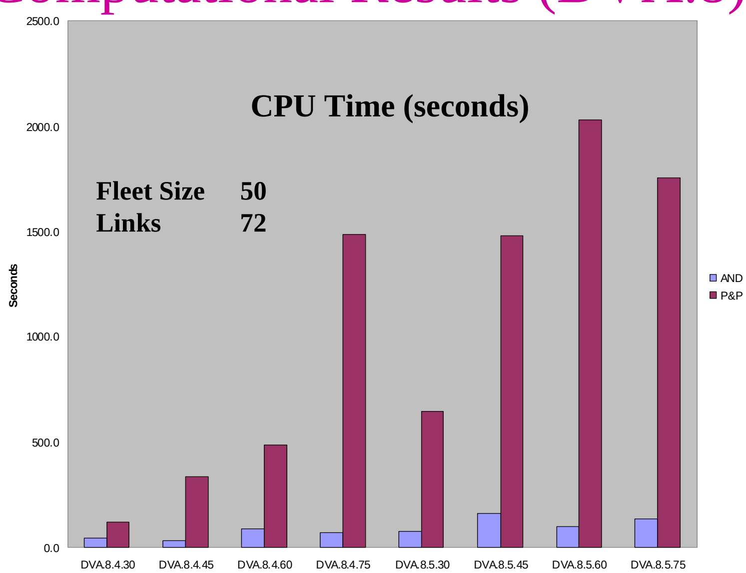
Convergence Test

1. Randomly select *H* *N*-Stage scenarios. For each sampled scenario, solve subproblems from root to leaf to obtain total objective value for scenario
2. Calculate statistical estimate of the first stage objective value $\bar{z}$
   - algorithm terminates if current first stage objective value $c_1x_1 + \theta_1$ is within a specified confidence interval of $\bar{z}$, else, a new forward pass begins

Sample Computational Results

- Test Problems
  - Dynamic Vehicle Allocation (DVA) problems of various sizes
    - set of homogeneous vehicles move full loads between set of sites
    - vehicles can move empty or loaded, remain stationary
    - demand to move load between two sites is stochastic
  - DVA.*x*.*y*.*z*
    - *x* number of sites (8, 12, 16)
    - *y* number of stages (4, 5)
    - *z* number of distinct realizations per stage (30, 45, 60, 75)
  - largest problem has > 30 million scenarios

Computational Results (DVA.8)



Outline

- Overview
- Part I - Models
- Part II – Optimization Methods
  - Factorization/sparsity (interior point/barrier)
  - Decomposition
  - Lagrangian methods
- Conclusions

Lagrangian-based Approaches

- General idea:
  - Relax nonanticipativity
  - Place in objective
  - Separable problems

$$\begin{aligned} \text{MIN} \quad & E\left[\Sigma_{t=1}^{T} f_t(x_t,x_{t+1})\right] \\ \text{s.t.} \quad & x_t \in X_t \\ & x_t \ \text{nonanticipative} \end{aligned} \quad \longrightarrow \quad \begin{aligned} \text{MIN}_{x_t \in X_t} \quad & E\left[\Sigma_{t=1}^{T} f_t(x_t,x_{t+1})\right] \\ & + E[\underline{w}_t x] + r/2\|x-\underline{x}\|^2 \end{aligned}$$

Update: w_t; **Project:** x into N - nonanticipative space as $\underline{x}$

Convergence: Convex problems - Progressive Hedging Alg. (Rockafellar and Wets)

Advantage: Maintain problem structure (networks)

Lagrangian Methods and Integer Variables

- Idea: Lagrangian dual provides bound for primal but
  - Duality gap
  - PHA may not converge
- Alternative: standard augmented Lagrangian
  - Convergence to dual solution
  - Less separability
  - May obtain simplified set for branching to integer solutions
- Problem structure: Power generation problems
  - Especially efficient on parallel processors
  - Decreasing duality gap in number of generation units

Outline

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SOME OPEN ISSUES

- MODELS
  - IM PACT ON METHODS
  - RELATION TO OTHER AREAS
- APPROXIMATIONS
  - USE WITH SAMPLING METHODS
  - COMPUTATION CONSTRAINED BOUNDS
  - SOLUTION BOUNDS
- SOLUTION METHODS
  - EXPLOIT SPECIFIC STRUCTURE
  - MASSIVELY PARALLEL ARCHITECTURES
  - LINKS TO APPROXIMATIONS

CRITICISMS

- UNKNOWN COSTS OR DISTRIBUTIONS
  - FIND ALL AVAILABLE INFORMATION
  - CAN CONSTRUCT BOUNDS OVER ALL DISTRIBUTIONS
    - FITTING THE INFORMATION
  - STILL HAVE KNOWN ERRORS BUT ALTERNATIVE SOLUTIONS
- COMPUTATIONAL DIFFICULTY
  - FIT MODEL TO SOLUTION ABILITY
  - SIZE OF PROBLEMS INCREASING RAPIDLY

View Ahead

- New Trends
  - Methods for integer variables
    - Capacity, suppliers, contracts
    - Vehicle routing
  - Integrating simulation
    - Sampling with optimization
    - On-line optimization
    - Low-discrepancy methods

More Trends

- Modeling languages
  - Ability to build stochastic programs directly
  - Integrating across systems
- Using application structure
  - Separation of problem (dimension reduction)
  - Network properties
  - Generalized versions of convexity

Summary

- Increasing application base
- Value for solving the stochastic problem
- Efficient implementations
- Opportunities for new results