Recent Results in Large-Scale Stochastic Programming Implementations

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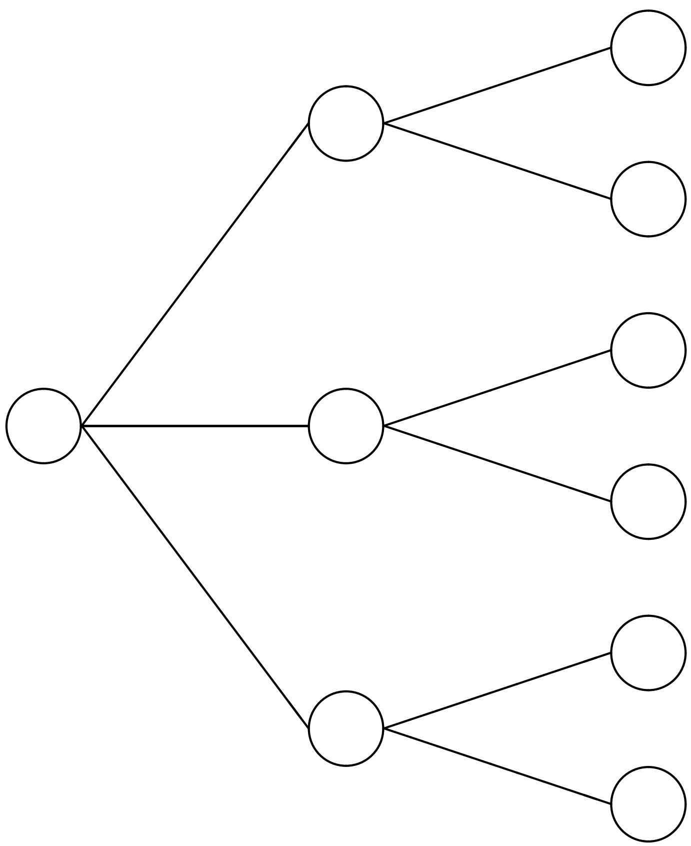
Outline

- ND-UM Base
- Basic Implementation
- Passing, Hot Starts, and Multicuts
- Abridge Version for Relatively Complete Recourse
- Abridged Results

ND-UM, GENERAL INFORMATION

- Programmed in C on RS\6000 workstations.

- Works interactively with IBM's Optimization Subroutine Library (OSL).



$$\begin{array}{lll} \min & c^1x^1 + \mathcal{Q}(x^1) & \\ s.t. & W^1x^1 & = h^1 \\ & x^1 & \geq 0 \end{array}$$

$$\mathcal{Q}(x_k^{t-1}) = \sum_{\xi^t \in \Xi^t} prob(\xi^t) Q(x_{d(k)}^t, \xi^t)$$

$$\begin{array}{llll} Q(x_k^t, \xi^t) = & \min & c^t(\xi^t)x_k^t + \mathcal{Q}(x_k^t) & \\ & s.t. & W^t x_k^t & = h^t(\xi^t) - T^{t-1}(\xi^t)x_{a(k)}^{t-1} \\ & & x_k^t & \geq 0 \end{array}$$

$$\begin{array}{lll} \min & c^t(\xi^t)x_k^t + \theta_k^t & \\ s.t. & W^t x_k^t & = h^t(\xi^t) - T^{t-1}(\xi^t)x_{a(k)}^{t-1} \\ & D_k^t x_k^t & \geq d_k^t \\ & E_k^t x_k^t + \theta_k^t & \geq e_k^t \\ & x^t & \geq 0 \end{array}$$

Nested Decomposition Algorithm

Step 1: **Set** $t = 1, \quad k = 1, \quad r_k^t = s_k^t = 0$. **Add the constraint** $\theta_k^t = 0$ **to** $1-5$ $\forall t, k$. **Let** $DIR = FORWARD$. **Go to Step 2.**

Step 2: **Solve the current problem,** $SP(t, k)$.

If infeasible, and $t = 1$**, then stop; the problem is infeasible.**

If infeasible, and $t > 1$**, then let** $r_{a(k)}^{t-1} = r_{a(k)}^{t-1} + 1$ **and use a dual basic solution,** $\pi_k^t, \rho_k^t \geq 0$ **such that** $\pi_k^t W^t + \rho_k^t D_k^t \leq 0$ **but** $\pi_k^t(h_k^t(\xi^t) - T_k^{t-1}(\xi^t)x_{a(k)}^{t-1}) + \rho_k^t d_k^t > 0$ **to create the following feasibility cut for** $SP(t-1, a(k))$**,** $D_{a(k), r_{a(k)}^{t-1}}^{t-1} = \pi_k^t T_k^{t-1}(\xi^t)$**,** $d_{a(k), r_{a(k)}^{t-1}}^{t-1} = \pi_k^t h_k^t(\xi^t) + \rho_k^t d_k^t$**. Let** $t = t-1$**,** $k = a(k)$ **and return to Step 1.**

If feasible, update the primal and dual values.

Step 3: **For all scenarios** $j = 1, \ldots, K^{t-1}$ **in Stage** $t-1$**, create the following optimality cut:**

$$E_j^{t-1} = \sum_{k \in D^t(j)} p_k^t \pi_k^t T_k^{t-1}(\xi^t), \quad e_j^{t-1} = \sum_{k \in D^t(j)} p_k[\pi_k^t h_k^t(\xi^t) + \rho_k^t d_k^t + \sigma_k^t e_k^t].$$

If $\bar{\theta}_j^{t-1} > \theta_j^{t-1}$**, then let** $s_j^{t-1} = s_j^{t-1} + 1$**, and add the new optimality cut to** $SP(t-1, j)$**. If** $t = 2$**, and a new cut is not added to** $SP(1,1)$**, then stop with the optimal solution.**

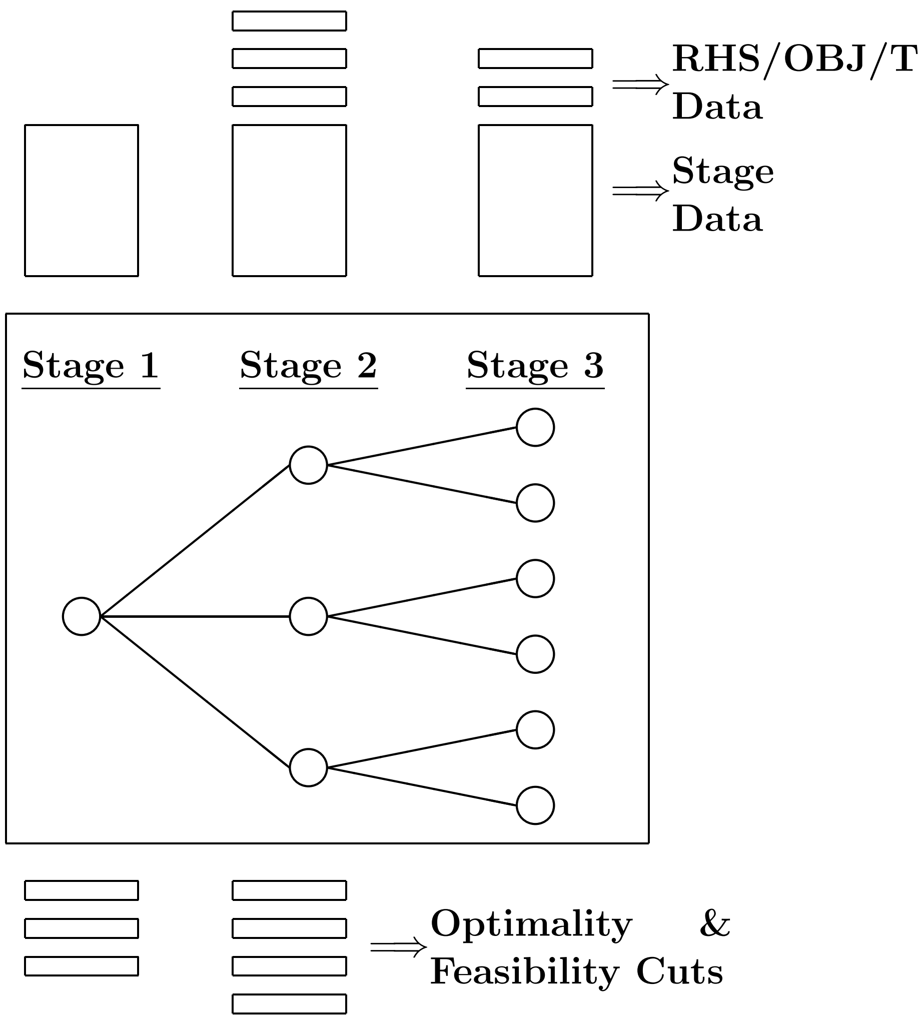
SMPS $\Longrightarrow$ ND-UM

- **Core file broken into N separate files, $t = 1, \ldots, N$:**

$$\begin{array}{lll} \min & c^t x^t + \theta & \\ s.t. & W^t x^t & = \; h^t \\ & x^t & \geq \; 0 \\ & \theta \text{ **unrestricted**} & \end{array}$$

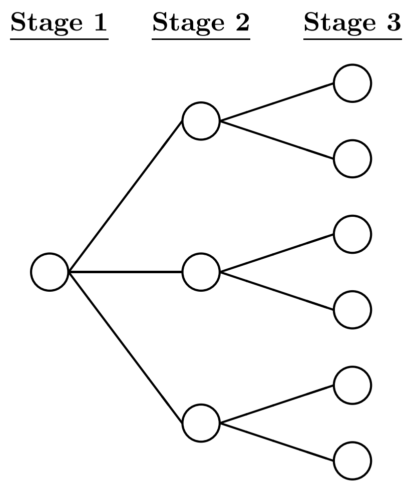
- **Different T matrices taken from Core and Scen. files (if stochastic) and stored separately.**

- **Scenario Tree built from Scen. file.**

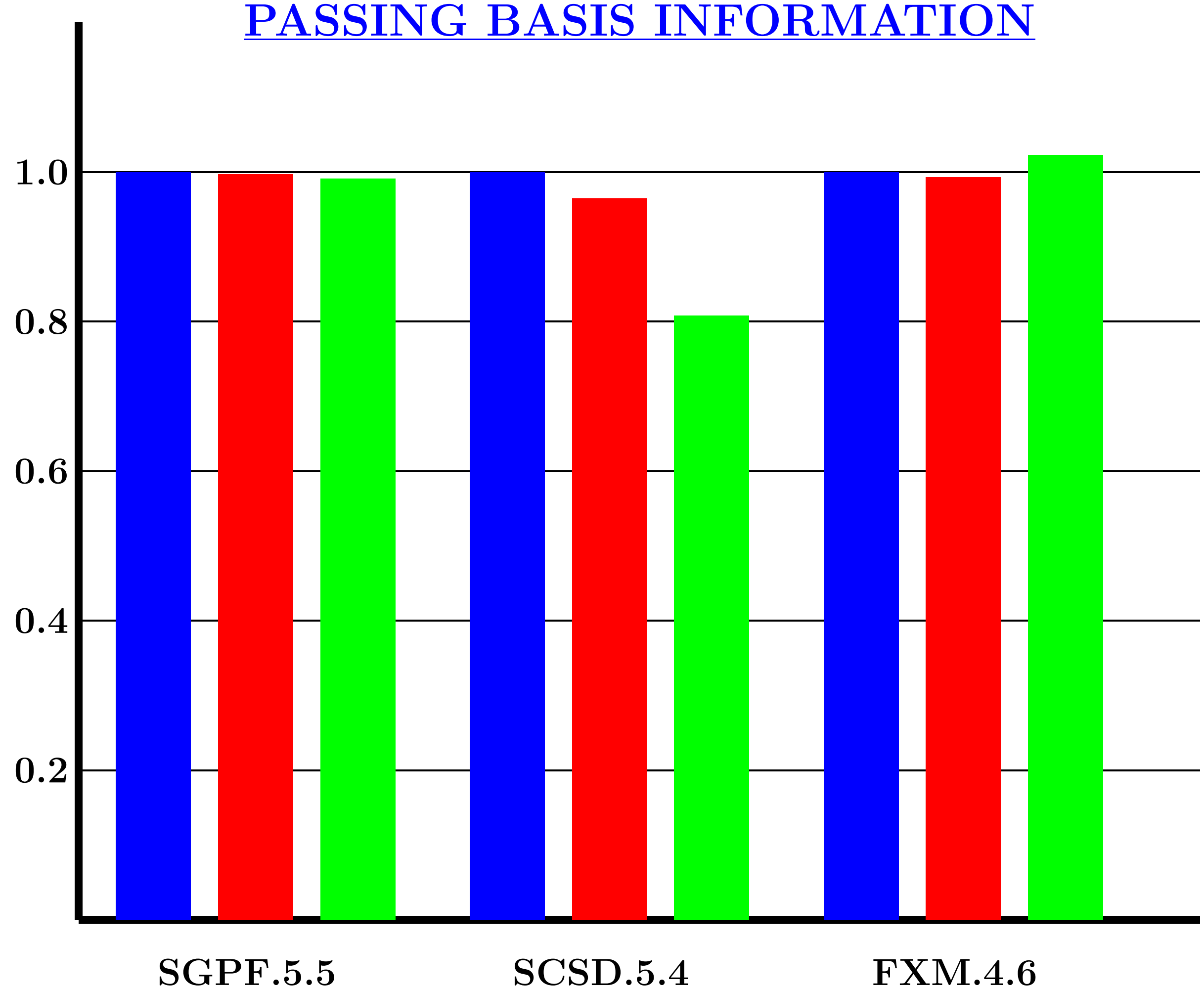


NODE INFO

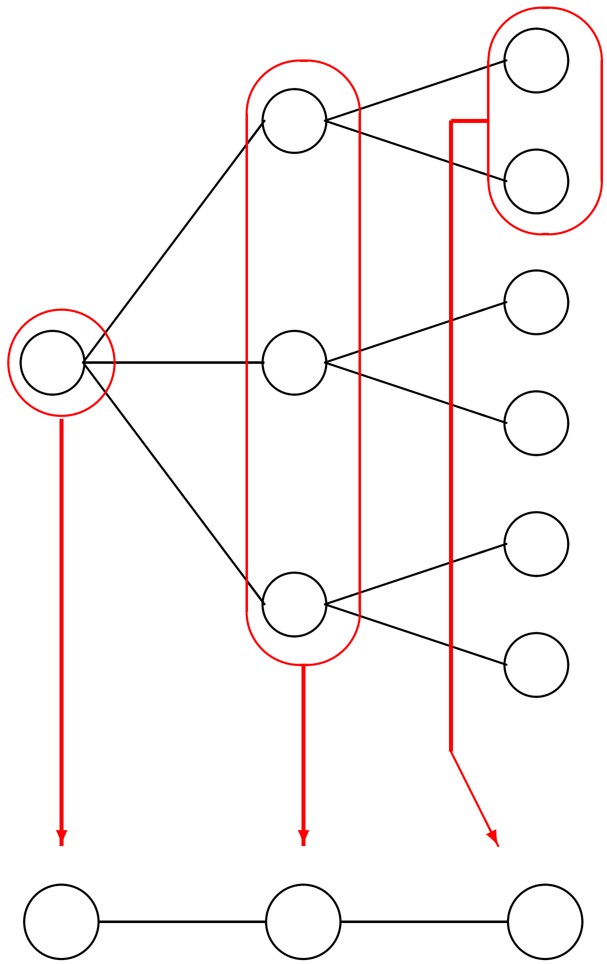
Stage
Relative Number
T-Matrix Number
Probability
Number of Children
- Primal Solution
- Dual Solution
- Optimality Cuts
- Feasibility Cuts
- Sibling
- Cousin
- Child
- Parent

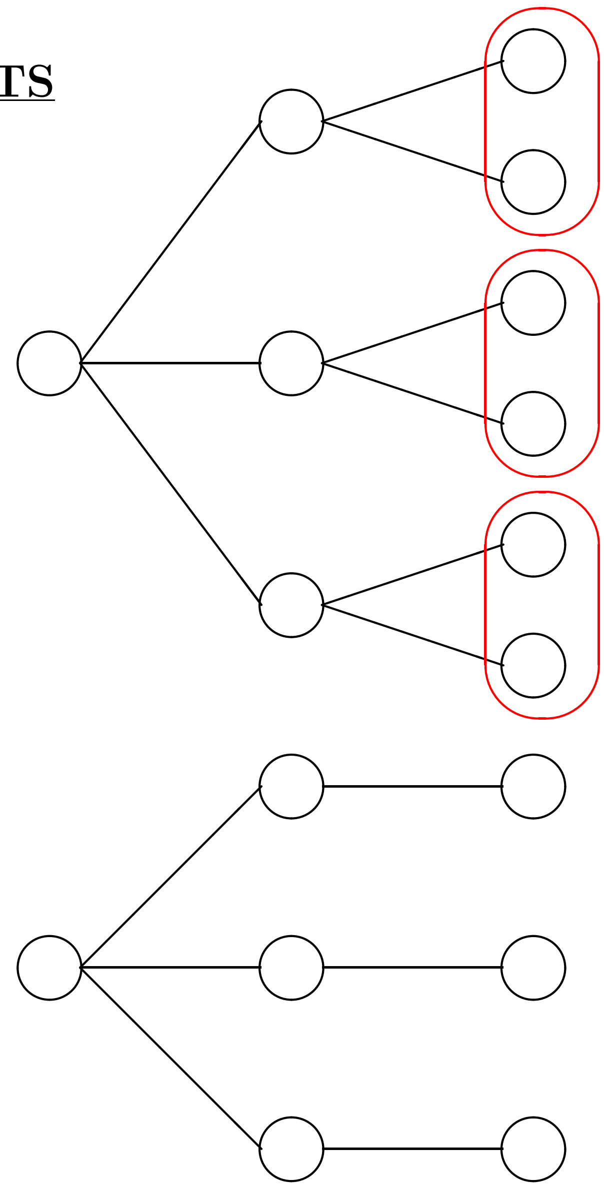


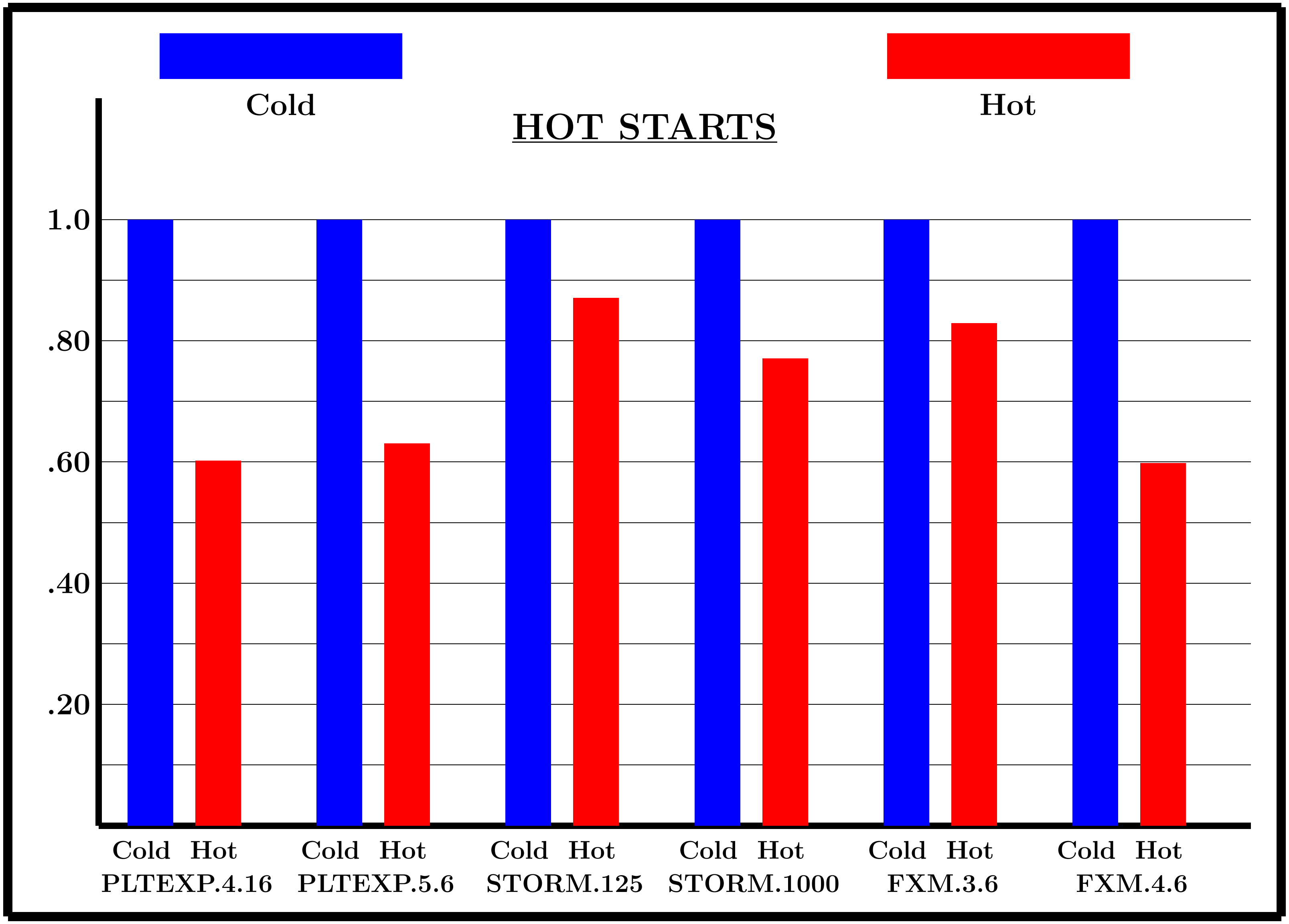
PASSING BASIS INFORMATION



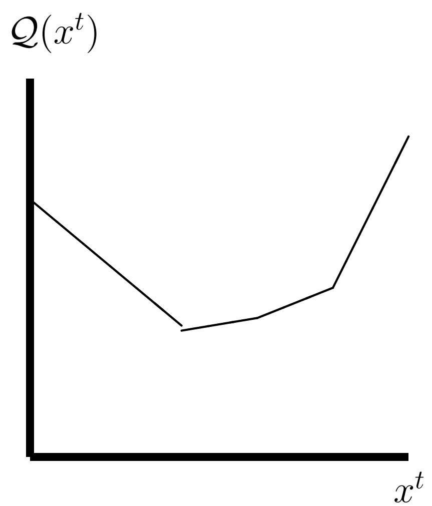
HOT STARTS

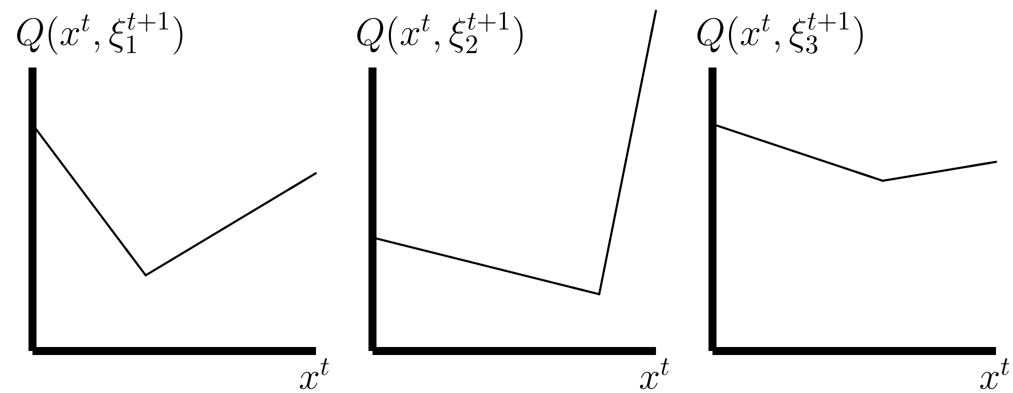





Cold
Hot
HOT STARTS
1.0
.80
.60
.40
.20
Cold Hot
PLTEXP.4.16
Cold Hot
PLTEXP.5.6
Cold Hot
STORM.125
Cold Hot
STORM.1000
Cold Hot
FXM.3.6
Cold Hot
FXM.4.6

MULTICUTS

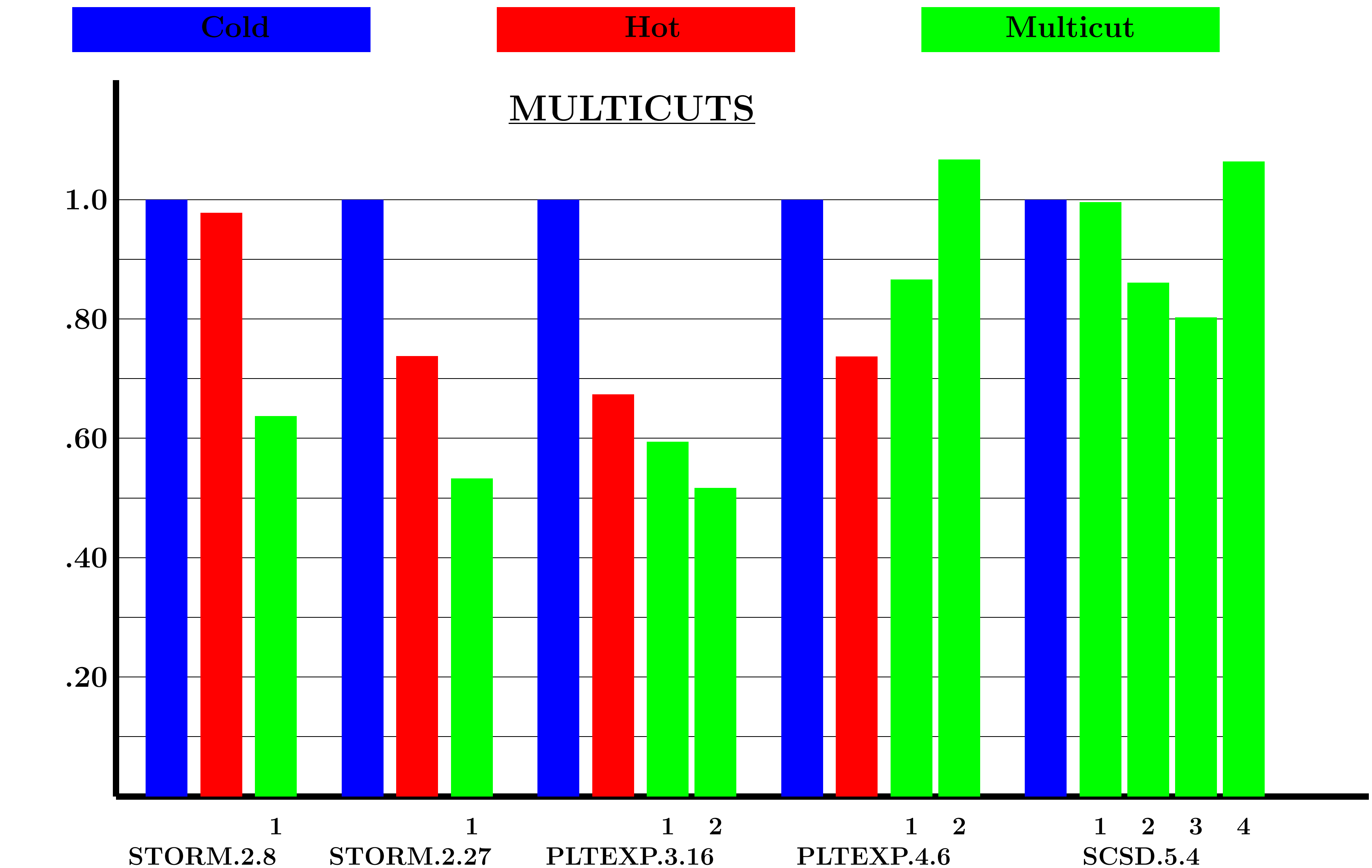




$$\begin{array}{lll} \min & c^t(\xi^t)x_k^t + \theta_k^t & \\ s.t. & W^t x_k^t & = h^t(\xi^t) - T^{t-1}(\xi^t)x_{a(k)}^{t-1} \\ & D_k^t x_k^t & \geq d_k^t \\ & E_k^t x_k^t + \theta_k^t & \geq e_k^t \\ & x^t & \geq 0 \end{array}$$

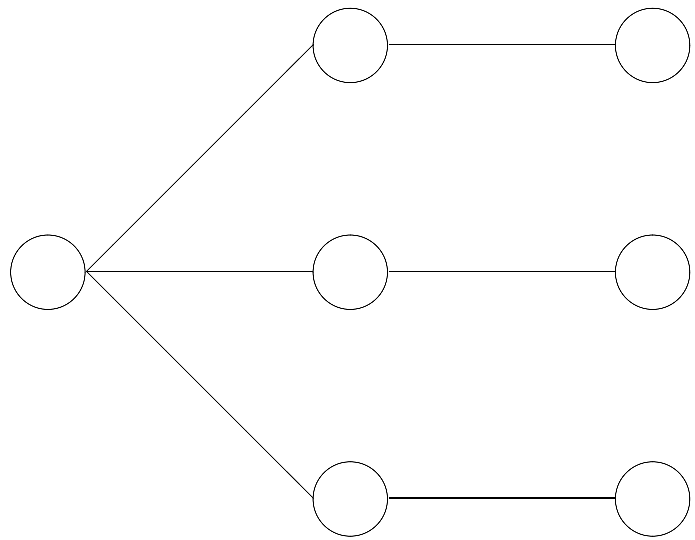
$$\begin{array}{lll} \min & c^t(\xi^t)x_k^t + \sum_{\xi^{t+1} \in \Xi^{t+1}} \theta_{\xi^{t+1}}^t & \\ s.t. & W^t x_k^t & = h^t(\xi^t) - T^{t-1}(\xi^t)x_{a(k)}^{t-1} \\ & D_k^t x_k^t & \geq d_k^t \\ & E_{\xi^{t+1}}^t x_k^t + \theta_{\xi^{t+1}}^t & \geq e_{\xi^{t+1}}^t \\ & x^t & \geq 0 \end{array}$$

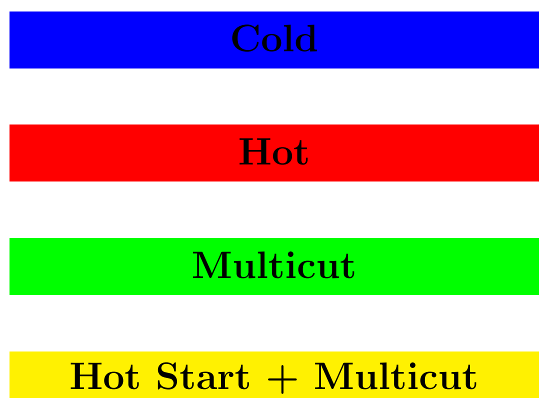
MULTICUTS

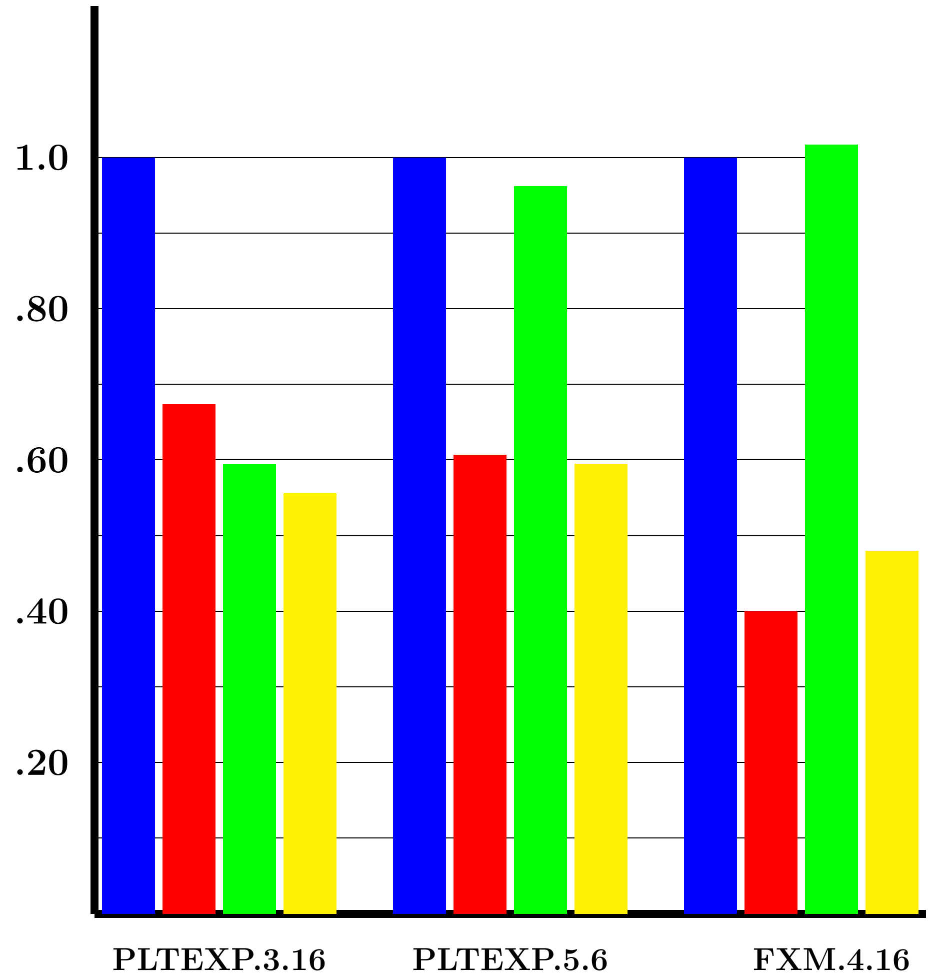


Cold
Hot
Multicut
1.0
.80
.60
.40
.20
1
STORM.2.8
1
STORM.2.27
1 2
PLTEXP.3.16
1 2
PLTEXP.4.6
1 2 3 4
SCSD.5.4

HOT START + MULTICUTS



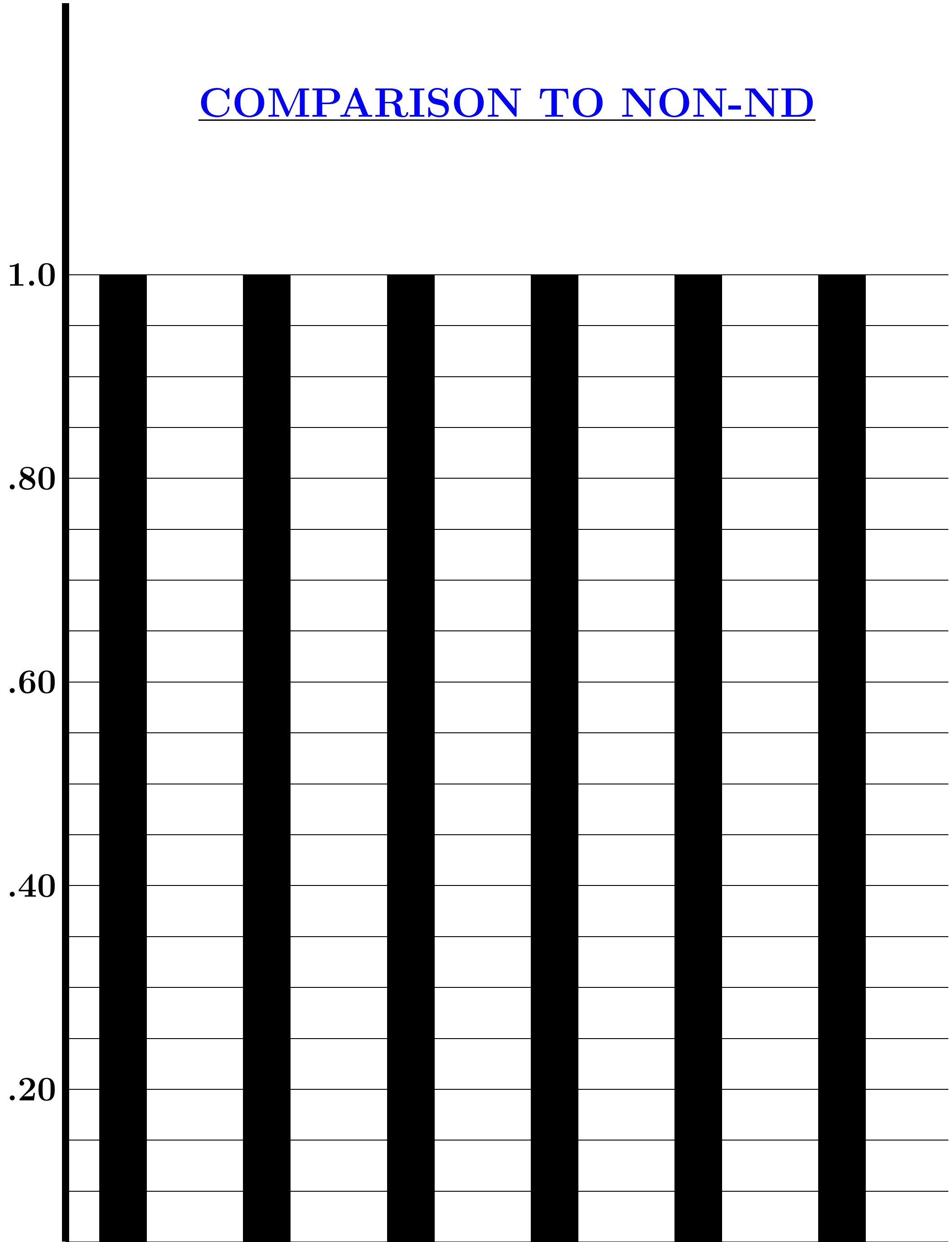




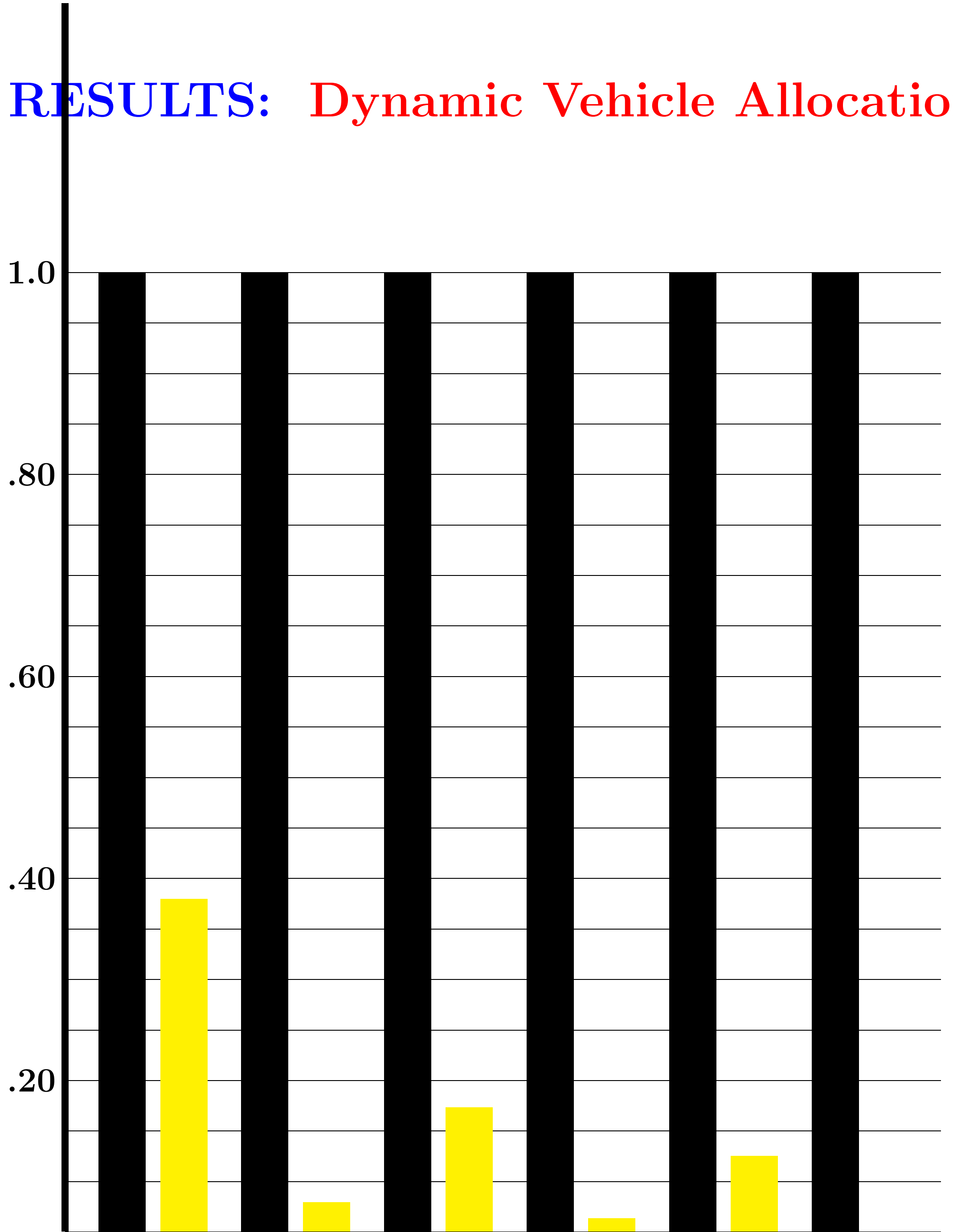
ABRIDGED VERSION

- **Idea:** Base on Pereira and Pinto
  - Use samples of tree
  - Increase sample size to gain convergence
  - Can use statistical properties
- **Enhancement:** Abridge Scenario Tree
  - Only branch from certain nodes
  - Abridge Version for Relatively Complete Recourse
  - Share Cuts with All Nodes
  - Solve with the Shared Cuts
- **Result:**
  - Can Still Obtain Convergence
  - Eliminates Many Brances

COMPARISON TO NON-ND



RESULTS: Dynamic Vehicle Allocation.



Conclusions

- Nested Decomposition
- Advantages of Hot Starts and Multicuts
- Benefits of Abridged Version
- Problems with over 2 Billion Scenarios, 500 Billion Variables