Option Pricing and Capacity Expansion

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Outline

- Capacity expansion
- Basics for option evaluation
- Modeling as stochastic program
- Assumptions
- Toward resolving inconsistencies
- Conclusions

Capacity Expansion Problem

- Goal:
  - Find an optimal sequence of decisions on where to expand or contract
- Problems:
  - How to measure objective?
  - Risk varies with decision
  - Discounted cash flow insufficient

Modeling Steps

- Identify problem
- Determine objectives
- Specify decisions
- Find operating conditions
- Define metrics
  - How to measure objectives?
  - How to quantify requirements, limits?
  - How to include effect of uncertainty?
- Formulate

Utility Function Approach

- Observation:
  - Most decision makers are adverse to risk
- Assume:
  - Outcomes can be described by a utility function
  - Decision makers want to maximize expected utility
- Difficulties:
  - Is the decision maker the sole stakeholder?
  - Whose utility should be used?
  - How to define a utility?
  - How to solve?
- Alternative to decision maker - investor

Measuring Investor Value

- SUPPOSE RISK NEUTRAL?
- (expected cost) objective
  - RESULT: Does not correspond to preference
  - Difficult to assess real value this way
- RESOLUTION:
  - Assume investors prefer lower risk
  - Investors can diversify away unique risk
  - Only important risk is market - contribution to portfolio
- CONSEQUENCE: Capital asset pricing model (CAPM)

Determing Risk Contribution

- USE CORRELATION?
  - Can measure for known markets (beta values)
  - If capacitated, depends on decisions
    - » Constrained resources
    - » Correlations among demands
- ALTERNATIVES?
  - Option Theory
    - » Allows for non-symmetric risk
    - » Explicitly considers constraints -
    - » As if selling excess to competitors at a given price

Use of Options

- Capacity limits potential sales
- View: option sold to competitor

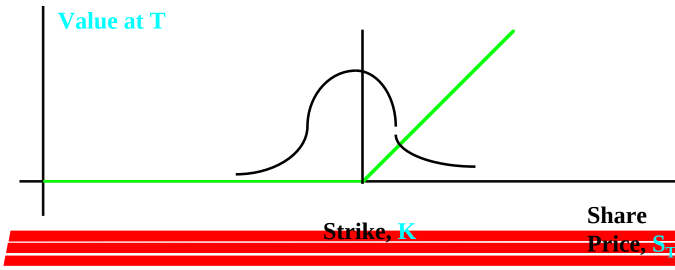
RESULTS FROM FINANCE:

- **Assumption: risk free hedge**
  - **Can evaluate as if risk neutral**
  - **As in Black-Scholes model**
- **Steps in modeling:**
  - **Adjust revenue to risk-free equivalent**
  - **Discount at riskless rate**

Valuing an Option

- (European) Call Option on Share assuming:
  - Buy at K at time T;Current time: t; Share price: S_t
  - Volatility: s ; Riskfree rate: r_f; No fees; Price follows Ito process
- Valuing option:
  - Assume risk neutral world (annual return=r_f independent of risk)
  - Find future expected value and discount back by r_f

  Call value at t = $C_t = e^{-r_f(T-t)}\int(S_T-K)^+ dF_t(S_T)$

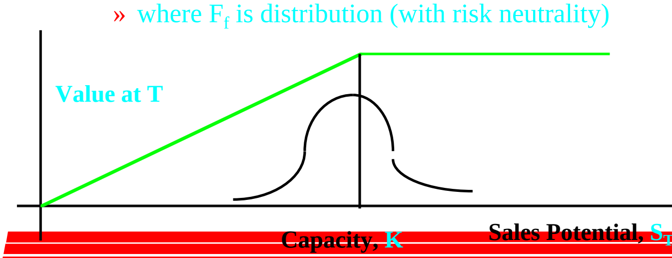


Relation to Capacity Evaluation

- What is the value of a plant with capacity K?
  - Discounted value of production up to K?
- Problems:
  - Production is limited by demand also (may be > K)
  - How to discount?
- Resolution:
  - Model as an option
  - Assume:
    - » Market for demand (substitutes)
    - » Forecast follows Ito process
    - » No transaction costs
- => Model like share minus call

Computing Capacity Value

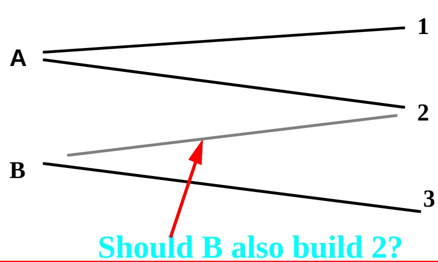
- Goal: Production value with capacity K
  - Compute uncapacitated value based on CAPM:
    - » $S_t = e^{-r(T-t)} \int c_T S_T \, dF(S_T)$
    - » where c_T=margin,F is distribution (with risk aversion),
    - » r is rate from CAPM (with risk aversion)
  - Assume S_t now grows at riskfree rate, r_f ; evaluate as if risk neutral:
    - » Production value = $S_t - C_t = e^{-r_f(T-t)} \int c_T \min(S_T,K) \, dF_f(S_T)$
    - » where F_f is distribution (with risk neutrality)



Example: Capacity Planning

- What to produce?
- Where to produce? (When?)
- How much to produce?

EXAMPLE: Models 1,2, 3 ; Plants A,B



Stochastic Programming Model

- Key: Maximize the Added Value with Installed Capacity
  - Must choose best mix of models assigned to plants
  - Maximize Expected Value[$S_{i,t}$ e^{-rt}Profit (i) Production(i,t)
  - - CapCost(i at j,t)Capacity (i at j,t)]
  - subject to: MaxSales(i,t) ≥ S_j Production(i at j,t)
  - S_i Production(i at j,t) £ $e^{(r-rf)t}$ Capacity (i,t)
  - Production(i at j,t) £ $e^{(r-rf)t}$ Capacity (i at j,t)
  - Production(i at j,t) ≥ 0
- Need MaxSales(i,t) - random
  - Capacity(i at j,0) - Decision in First Stage (now)
- FIRST: Construct sales scenarios

Assumptions

- Process of prices or sales forecasts
- No transaction fees
- Complete market
  - How to construct a hedge?
  - If NPV>0, inconsistency
  - Process: Trade option and asset to create riskfree security

Creating Best Hedge

- Underlying asset: Max potential sales in market
- Option: Plant with given capacity
- Other marketable securities:
  - Competitors' shares
  - Overall all securities min residual volatility
  - Due to incompleteness, some volatility remains (otherwise, NPV=0)

Result of Residual Risk

- In binomial model, asset price moves from S_t to $uS_t + v_1$ or $dS_t + v_2$ where v_1 and v_2 vary independently and have smallest volatility
- For standard call option,

$C_t = [(S_t - d S_t + v_1)/(uS_t - dS_t + v_2)] (uS_t - K)$

$= [(S_t - d S_t + v_1)/p(uS_t - dS_t + v_2)]p (uS_t - K)$

$= e^{-r(T-t)} (E[(S_t\text{-}K)^+])$ where r is in a range determined by [v2,v1]

Capacity Implication

- Can adjust capacity limits by varying discount factor with risk neutral assumptions on forecasts
- Can vary constraint multipliers with original forecast distribution
- All optimal policies for the given range are consistent

Summary

- Options apply to many capacity problems
- Can find simple modification with complete market assumptions
- Relaxed market assumptions lead to models with parametric constraints
- How to interpret this range of policies?