Comparison of Static and Dynamic Asset Allocation Models

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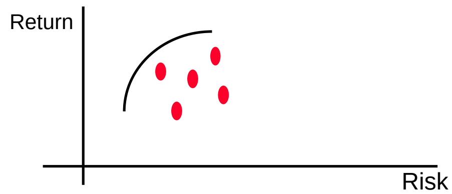
Outline

- Basic Models
  - Static Markowitz mean-variance
  - Dynamic stochastic programming
- Difficulties in static model
- Example results
- Other tests

Static Model

- Markowitz model
  - Choose portfolio to minimize risk for a given return
  - Find the efficient frontier



Markowitz model

- For a given set of assets, find
  - fixed percentages to invest in each asset
  - maintain same percentage over time
- Needs
  - rebalance as returns vary
  - cash to meet obligations

Dynamic Model

- Assume possible outcomes over time
  - discretize generally
- In each period, choose mix of assets
- Can include transaction costs
- Can include liabilities over time
- Can include different measures of risk aversion

FORMULATION

- SCENARIOS: $\sigma \in \Sigma$
  - Probability, $p(\sigma)$
  - ∠ Γρουπσ, Σ^t_1, ..., S^t_{St} at t
- MULTISTAGE STOCHASTIC NLP FORM:

max Σ_σ **p(σ) (U(W(σ , T))**

s.t. (for all σ): Σ_k **x(k,1, σ) = W(o) (initial)**

Σ_k **r(k,t-1, σ) x(k,t-1, σ) -** Σ_k **x(k,t, σ) = 0 , all t >1;**

Σ_k **r(k,T-1, σ) x(k,T-1, σ) - W(σ , T) = 0, (final);**

x(k,t, σ) ≥ 0, all k,t;

Nonanticipativity:

x(k,t, σ') - x(k,t, σ) = 0 if σ', σ ∈ S^t_i **for all t, i, σ', σ**

This says decision cannot depend on future.

GENERAL MULTISTAGE MODEL

- FORMULATION:

MIN $E[\sum_{t=1}^{T} f_t(x_t,x_{t+1})]$
s.t. $x_t \in X_t$
x_t nonanticipative
$P[h_t(x_t,x_{t+1}) \leq 0] \geq a$ (chance constraint)

EXAMPLES:

Vehicle Allocation: Linear functions, continuous or integer variables

Capacity: Linear plus integer variables

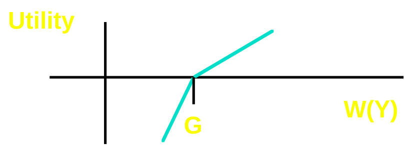
Financial Planning: Nonlinear objective, continuous variables

Problems in Static Approach

- Utility form
  - Not consistent over multiple periods
  - If near end, may be conservative
  - Different behavior at beginning
- Transaction costs
  - Missing actual needs over time - target utility

Financial Planning

- GOAL: Accumulate $G for tuition Y years from now
- Assume:
  - $ W(0) - initial wealth
  - K - investments
  - concave utility (piecewise linear)



RANDOMNESS: returns r(k,t) - for k in period t

where Y → T decision periods

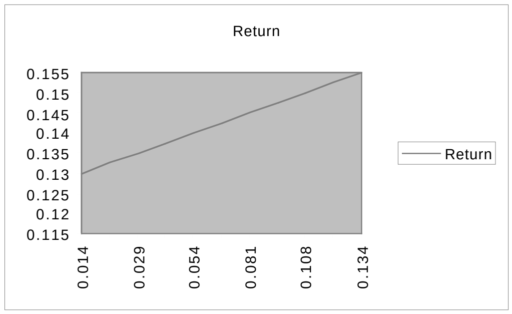
DATA and SOLUTIONS

- ASSUME:
  - Y=15 years
  - G=$80,000
  - T=3 (5 year intervals)
  - k=2 (stock/bonds)
- Returns (5 year):
  - Scenario A: r(stock) = 1.25   r(bonds)= 1.14
  - Scenario B: r(stock) = 1.06   r(bonds)= 1.12
- Solution:

PERIOD	SCENARIO	STOCK	BONDS
1	1-8	41.5	13.5
2	1-4	65.1	2.17
2	5-8	36.7	22.4
3	1-2	83.8	0
3	3-4	0	71.4
3	5-6	0	71.4
3	7-8	64.0	0

Static Markowitz Solution

- Find efficient frontier:



Results with Static Model

- Fixed proportion in stock and bonds in each period
- 80% stock for 15% return
- 40% stock for 14% return
- Results: no fixed proportion achieves target better than 50% of time
- Dynamic achieves target 87.5% of time

Other Model Gains

- Include transaction costs
  - Fixed proportion has 0.1% per period just to re-balance
  - can accumulate
- Maintain consistent utility

Current Study

- Portfolios of major indexes
- Constructured efficient frontier
- Developed decision tree form for stochastic program
- Gains in basic model for stochastic program of 3-5% over 10 periods

Summary

- Static models have real problems for dynamic problems
- Biggest gains may be in ability to change positions over time
- Large study on indices to continue