Stochastic Optimization Models with Fixed Costs

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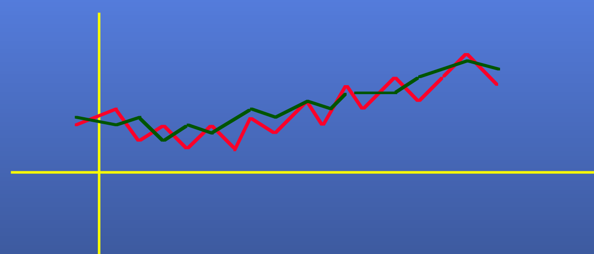

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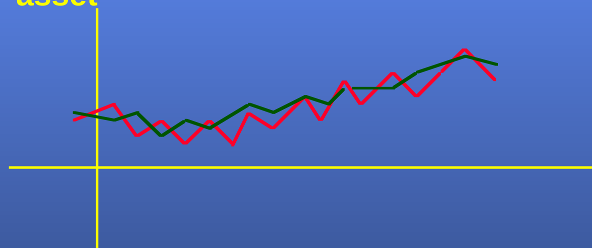
Outline

- Examples
  - Finance
  - Manufacturing
  - Power systems
- General Formulation
  - Statement
  - Optimality conditions
- Lagrangian Approach
  - Duality gap
  - Effect of increasing sample paths
- Computational Results
- Summary

Slide Number 2

Tracking a Security/Index

- **GOAL:** **Create a portfolio of assets that follows another security or index with maximum deviation above the underlying asset**



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Asset Tracking Decisions

- **Pool of Assets:**
  - TBills
  - GNMAs, Other mortgage-backed securities
  - Equity issues
- **Underlying Security:**
  - Mortgage index
  - Equity index
  - Bond index
- **Decisions:**
  - How much to hold of each asset at each point in time?

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Traditional Approach

- **Constant Proportions:**
  - Keep a fixed proportion of portfolio in each asset
  - Find the proportion in i (u(i)) that maximizes expected value for a single period
- **Formulation:**

Max $E_s[x(t,+)]$

s.t. $x(t,+) - x(t,-) = w(t-1)\Sigma_i\, u(i)(1+r(i,s)) - y(s)$

$\Sigma_i\, u(i) = \,,\ u,x \geq 0$

where w(t-1) is total value, r(i,s) is return,

y(s) is underlying price under scenario s

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Problems with Tradition

- **MODEL: variant of Markowitz model**
- **SOLUTION: Nonlinear optimization**
- **PROBLEMS:**
  - Must rebalance each period
  - Must pay transaction costs
- **RESOLUTION:**
  - Make transaction costs explicit
  - Include in dynamic model

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Model with Transaction Costs

- FORMULATION:

  Max $E_s[x(T,+)]$

s.t. $x(t,+) - x(t,-)=\Sigma_i\ u(t-1,i,s)(1+r(t-1,i,s)) - y(s)$

  $\Sigma_i\ u(t-2,i,s)(1+r(t-2,i,s))=\Sigma_i\ u(t-1,i,s)$

  $u,x \geq 0$

  U is NONANTICIPATIVE

  Decisions only depend on the past and not on the specific scenario path s

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Manufacturing to Meet Demand

- GOAL: Minimize the total cost of meeting demand, d, for products 1..n
- DECISIONS: Determine amount of each product to produce with each (limited) resource (machines)
- COSTS:
  - Inventory (overproduce)
  - Shortage (underproduce)

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Manufacturing Formulation

- **FORMULATION:**

  $\min E_s[h(t)x(t,+)+p(t)x(t,-) + J(u(t),s)]$

  s.t. $x(t,+) - x(t,-) = x(t-1,+) - x(t-1,-) + \Sigma_i u(t-1,i,s)(r(t-1,i,s)) - d(s)$

  $\Sigma_i g(t-1,i,s,j)\, u(t-1,i,s) \leq 1$ (resource limits)

  $u,x \geq 0$

  U is NONANTICIPATIVE

  (OFTEN INTEGRAL)

- **DIFFERENCES: Need inventory (memory)**
  - Discrete decisions

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Power Systems

- **GOAL: Minimize the overall cost to meet power load over a given time horizon**
- **DECISIONS: Determine the set of units to commit and their levels of operation (which plants on Automatic Generation Control)**
- **RESTRICTIONS:**
  - Must maintain load
  - Meet safety requirement
  - Ramping times, switching limits

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Power System Formulation

STOCHASTIC NONLINEAR INTEGER MODEL:

min Σ_s p(s) ($\Sigma_I \Sigma_T f_i$(x(t,i,s), u(t,i,s))

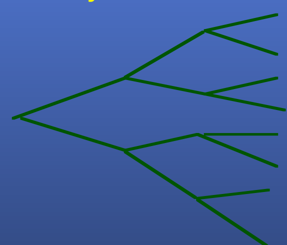
s.t. (for all s): Σ_k x(t,i, s) $\geq$ d(t),t=1..T, x(t,i, s) in X(t,i, s,u)

u(t,i,s) integer, x(t,i, s) $\geq$ 0, all i,t;

Nonanticipativity:

$E_{s'}$ x(k,t, s') - x(k,t, s) = 0 if s',s $\in$ S^t_i for all t, i, s

This says decision cannot depend on future.



S^t_i are groups at the same level of the scenario tree

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GENERAL MULTISTAGE MODEL

- **FORMULATION:**

MIN $E [\Sigma_{t=1}^{T} f_t(x_t,x_{t+1})]$
s.t. $x_t \in X_t$
x_t nonanticipative
$P[h_t (x_t,x_{t+1}) \leq 0] \geq a$ (chance constraint)

DEFINITIONS:

x_t - aggregate production

f_t - defines transition - only if resources available and includes subtraction of demand

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DYNAMIC PROGRAMMING VIEW

- **STAGES**: $t=1,\ldots,T$
- **STATES**: $x_t \rightarrow B_t x_t$ (or other transformation)
- **VALUE FUNCTION:**
  - ∠ $\Psi_t(x_t) = E[\psi_t(x_t,\xi_t)]$ where
  - ∠ ξ_t is the random element and
  - ∠ $\psi_t(x_t,\xi_t) = \min f_t(x_t,x_{t+1},\xi_t) + \Psi_{t+1}(x_{t+1})$
  - s.t. $x_{t+1} \in X_{t+1t}(,\xi_t)$ x_t given
- **FREQUENT ASSUMPTIONS:**
  - CONVEXITY
  - EARLY AND LATENESS PENALTIES

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PRODUCTION SCHEDULING RESULTS

- OPTIMALITY:
  - CAN DEFINE OPTIMALITY CONDITIONS
  - DERIVE SUPPORTING PRICES
- CYCLIC SCHEDULES:
  - OPTIMAL IF STATIONARY OR CYCLIC DISTRIBUTIONS
  - MAY INDICATE KANBAN/CONWIP TYPE OPTIMALITY
- TURNPIKE: (Birge/Dempster)
  - FROM OTHER DISRUPTIONS:
  - RETURN TO OPTIMAL CYCLE
- LEADS TO MATCH-UP FRAMEWORK

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Lagrangian-based Approaches

- **General idea:**
  - Relax nonanticipativity
  - Place in objective
  - Separable problems

MIN $E [\Sigma_{t=1}^{T} f_t(x_t,x_{t+1})]$
s.t. $x_t \in X_t$
x_t nonanticipative

→

MIN $E [\Sigma_{t=1}^{T} f_t(x_t,x_{t+1})]$
$x_t \in X_t$
$+ E[\underline{w}_t x] + r/2||x-\underline{x}||^2$

Update: w_t; Project: x into N - nonanticipative space

Convergence: Convex problems - Progressive Hedging Alg. (Rockafellar and Wets)
Advantage: Maintain problem structure (networks)

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Lagrangian Methods and Integer Variables

- **Idea:** Lagrangian dual provides bound for primal but
  - Duality gap
  - PHA may not converge
- **Alternative:** standard augmented Lagrangian
  - Convergence to dual solution
  - Less separability
  - Duality gap decreases to zero as number of scenarios increases
  - $E_{s'}$ u(k,t, s') - u(k,t, s) = 0 if s',s $\in S^t_i$ for all t, i, JUST ONE s
- **Problem structure:** Power generation problems
  - Especially efficient on parallel processors

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NEW RESULTS

- BOUND ON ERROR in POWER SYSTEMS
  - SIMILAR TO LAGRANGIAN RELAXATION (BERTSEKAS)
  - GOES TO ZERO AS PROBLEM SIZE INCREASES
- DIFFERENCE FROM LAGRANGIAN RESULTS
  - NOT IN NUMBER OF UNITS
  - JUST IN NUMBER OF SCENARIOS
- IMPLICATION
  - AS MORE SCENARIOS INCLUDED, CLOSER TO OPTIMAL
  - MORE COMPUTATION POWER IMPLIES REAL CONVERGENCE
  - NOT ARTIFICIAL IN NO. OF UNITS

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COMPUTATIONAL RESULTS

- DATA:
  - SEVERAL YEARS OF MICHIGAN DATA
  - USED SEVERAL PERIODS IN YEAR
- SCENARIOS
  - POSSIBLE YEARS (CLOSE FIT)
  - HISTORICAL SUPPLY LOSS PATTERNS
- IMPLEMENTATION
  - RS6000 WORKSTATION (PLUS PARALLEL)
  - IN C
- TIME
  - MOST SOLUTIONS FOR 60 UNITS IN 1 MINUTE

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CHALLENGES

- DIFFICULTIES
  - PUMP STORAGE FACILITY LINKING PERIODS IN WEEK
  - LARGE GENERATORS WITH FAILURES
  - DEMAND VOLATILITY
- COMPARISONS
  - COMPARED TO CURRENT PRACTICE OF "CLOSEST WEEK"
  - RESULTS:
    - » SAVINGS IN EXPECTED COST BY INCLUDING
    - » RANDOM OUTCOMES:

1% ($150,000/WEEK) TO 4% ($600,000/WEEK)

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Summary

- **MODELS:**
  - Wide variety
  - Often critical factor for discrete variables
  - Need to include dynamics/transient behavior
- **SOLUTIONS:**
  - Use of Lagrangian
  - Decreasing duality gap in sample size
- **COMPUTATION:**
  - Direct parallel implementation
  - Efficient solutions with improvement over existing methods