Discrete and Continuous Models in Stochastic Scheduling

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OUTLINE

- Motivation - Short and Long Term Framework
- Long-Term: Capacity Decisions: Flexibility
  - Problems of Uncertainty
  - Option uses
  - General approach toward risk
- Short-Term: Production scheduling
  - Types of Uncertainty
  - Results on cycles and matching up
  - Different role of risk
- Computation
- Summary

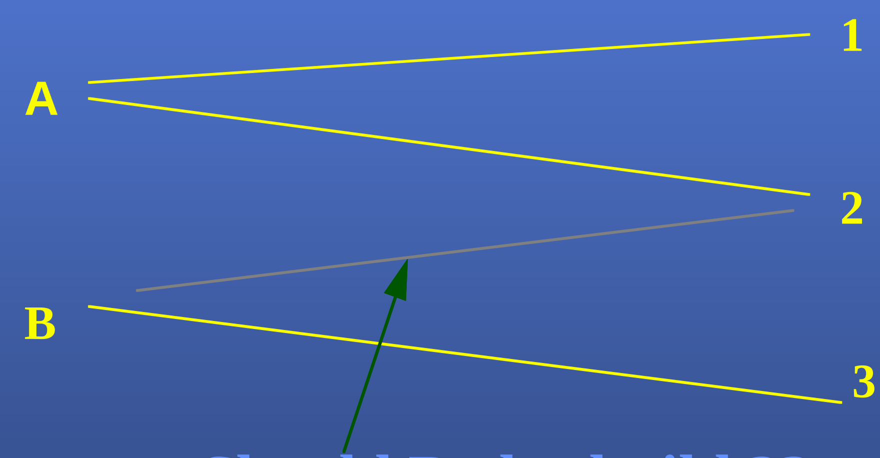
Length of Horizon and Decisions

- LONG TERM HORIZON DECISIONS (YEARS)
  - STRATEGIES
  - OVERALL CAPACITY
  - PRODUCT MIX
  - SOURCES OF UNCERTAINTY
    - » MARKET
    - » COMPETITORS
- SHORT TO MEDIUM TERM DECISIONS (< YEAR)
  - ACTUAL PRODUCTION
  - DAILY TO MONTHLY MIX
  - VARIABLE PRODUCTIVE CAPACITY

CAPACITY DECISIONS

- What to produce?
- Where to produce? (When?)
- How much to produce?

EXAMPLE: Models 1,2, 3 ; Plants A,B



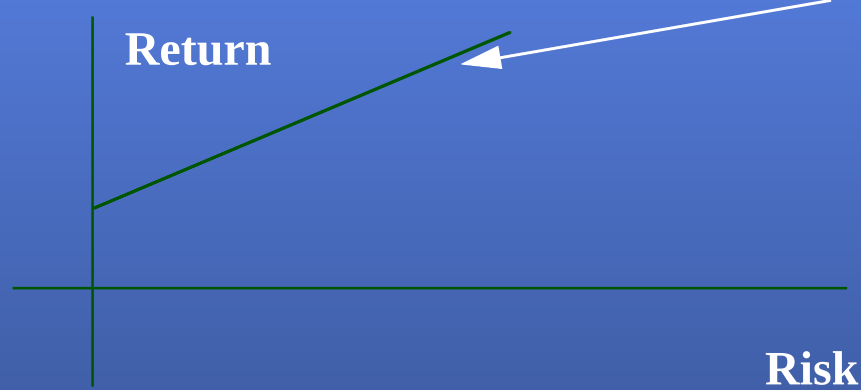
Should B also build 2?

MEASURING VALUE

- **SUPPOSE RISK NEUTRAL: (expected cost) objective**
  - **RESULT: Does not correspond to decision maker preference**
  - **Difficult to assess real value this way**
  -
- **RESOLUTION: use economic/financial theory:**
  - **Capital Asset Pricing Model**
  - **Efficient Market Theory**
  -
- **CONSEQUENCE: For financial objectives**
  - **Know how to assess based on risk**

BASICS OF CAPM

- **RISK/RETURN TRADEOFF:**
  - Investors can diversify
  - Firms need not diversity
  - All investments on security market line



NEED: Symmetric Risk

IMPLICATIONS FOR CAPACITY DECISIONS

- **VALIDITY OF SYMMETRY:**
  - Unlikely:
    - » Constrained resources
    - » Correlations among demands
- **ALTERNATIVES?**
  - Option Theory
    - » Allows for non-symmetric risk
    - » Explicitly considers constraints -
    - » Sell at a given price
-

USE OF OPTIONS

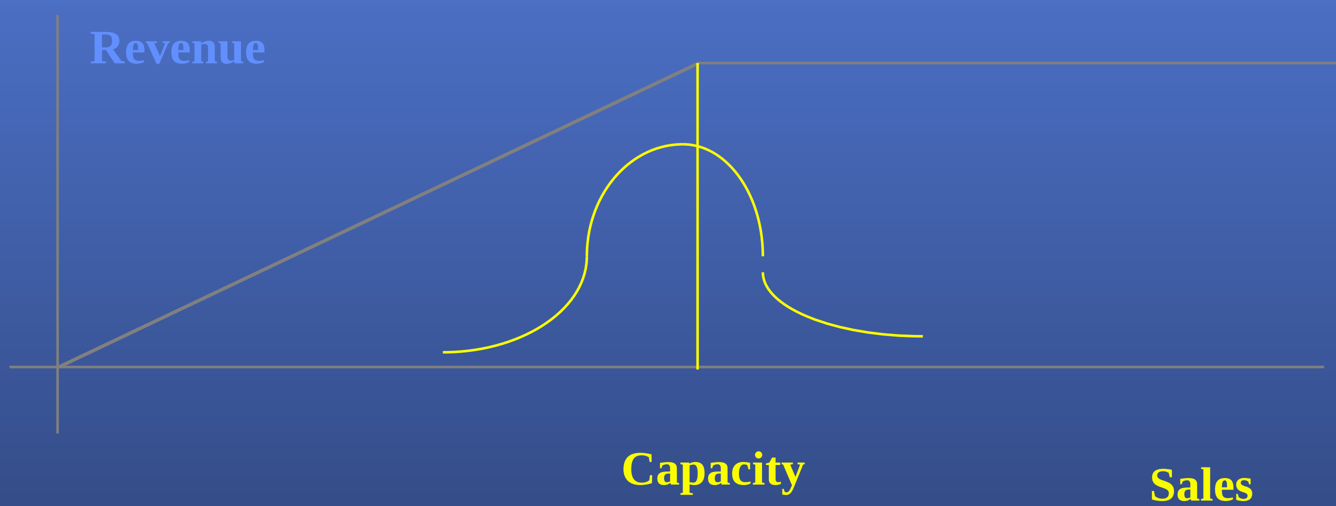
- CAPACITY LIMITS CUT OFF POTENTIAL REVENUE LIKE SELLING OPTION TO COMPETITOR
- VALUES ASYMMETRIC RISK

RESULTS FROM FINANCE:

- Assumption: risk free hedge
  - Can evaluate as if risk neutral
  - As in Black-Scholes model
- Steps with capacity evaluation:
  - Adjust revenue to risk-free equivalent
  - Discount at riskless rate

EVALUATING THE OPTION

- CANNOT USE EXPECTATIONS (SINGLE FORECASTS) ALONE BECAUSE OF:
  - Correlated Demand
    - Models 1,2,3 similar
  - Capacity Limit - cuts off revenue growth
    - => Asymmetric payoff



RESULTS OF OPTION-STOCHASTIC PROGRAMMING MODEL

- GIVES VALUE MEASURE
- INCORPORATES UNCERTAINTY AND ANY AVAILABLE INFORMATION
- CAN BE USED FOR VARYING MODEL LIFETIMES/PRODUCTION PERIODS
- INTEGRATES CAPACITY DECISIONS ACROSS FIRM (NOT JUST WITHIN 1 PLANT)
- CAN USE FOR UTILIZATION/LOST SALES/ OTHER WHAT-IF ANALYSES

GENERALIZATIONS FOR OTHER LONG-TERM DECISION

- **START: Eliminate constraints on production**
  - Demand uncertainty remains - assume that is symmetric
  - Can value unconstrained revenue with market rate, r:

$$1/(1+r)^t \; c_t \, x_t$$

IMPLICATIONS OF RISK NEUTRAL HEDGE:

Can model as if investors are risk neutral

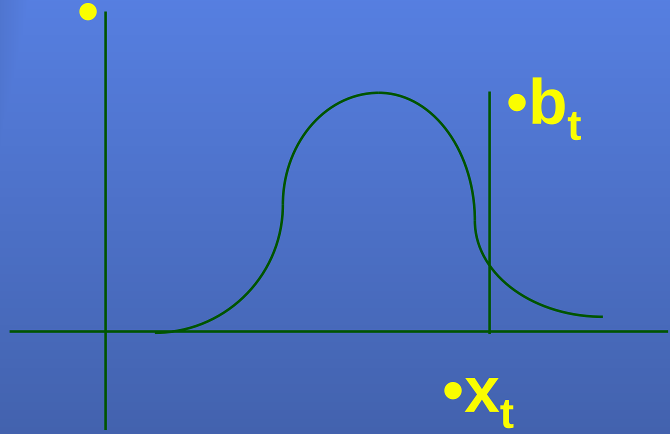
=> value grows at riskfree rate, r_f

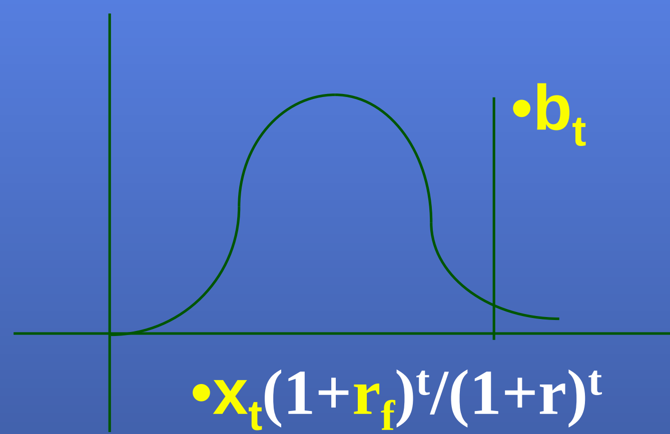
Future value: $[1/(1+r)^t \; c_t \, (1+r_f)^t \, x_t]$

BUT: This new quantity is constrained

CONSTRAINT MODIFICATION

- FORMER CONSTRAINTS: $A_t x_t \le b_t$
- NOW: $A_t x_t (1+r_f)^t/(1+r)^t \le b_t$
-





NEW PERIOD t PROBLEM

- **WANT TO FIND (present value):**

$$1/(1+r_f)^t \int \text{MAX}\left[c_t x_t (1+r_f)^t/(1+r)^t \mid A_t x_t (1+r_f)^t/(1+r)^t \leq b \right]$$

EQUIVALENT TO:

$$1/(1+r)^t \int \text{MAX}\left[c_t x \mid A_t x \leq b (1+r)^t/(1+r_f)^t \right]$$

MEANING: To compensate for lower risk with constraints, constraints expand and risky discount is used

EXTREME CASES

- **ALL SLACK CONSTRAINTS:**

$$1/(1+r)^t \int \text{MAX}\,[\, c_t x \mid A_t x \le b\,(1+r)^t/(1+r_f)^t]$$

becomes equivalent to:

$$1/(1+r)^t \int \text{MAX}\,[\, c_t x \mid A_t x \le b]$$

i.e. same as if unconstrained - risky rate

NO SLACK:

becomes equivalent to:

$$1/(1+r)^t \int [c_t x = B^{-1} b\,(1+r)^t/(1+r_f)^t] = c_t B^{-1} b/(1+r_f)^t$$

i.e. same as if deterministic- riskfree rate

OVERALL RESULTS - LONG-TERM

- CAN ADAPT OBJECTIVE TO RISK
- USE RATE FROM FIRM AS WHOLE
  - SYMMETRIC RISK
  - ASSUMES INVEST LIKE WHOLE FIRM
- ADJUST ALL CONSTRAINTS ON REVENUE GENERATORS BY RATE RATIOS
- END RESULT SHOULD REFLECT INVESTOR ATTITUDE TOWARD INVESTMENT

SHORT-TERM UNCERTAINTIES

- EFFECTIVE CAPACITY LIMITED BY
  - UNCERTAIN YIELDS - QUALITY LOSS
  - MACHINE BREAKDOWNS
  - VARIABLE PRODUCTION RATES
  - UNFORESEEN ORDERS
  - LACK OF MATERIAL/SUPPLIES
  - LOGISTICAL PROBLEMS
- GENERAL FRAMEWORK
  - BASIC OPTIMIZATION PROBLEM
  - MUST DEFINE OBJECTIVES
  - LOOK AT STRUCTURE

GENERAL MULTISTAGE MODEL

- **FORMULATION:**

$$\text{MIN} \quad E\left[\sum_{t=1}^{T} f_t(x_t, x_{t+1})\right]$$

$$\text{s.t.} \quad x_t \in X_t$$

$$x_t \ \text{nonanticipative}$$

$$P[\, h_t(x_t, x_{t+1}) \le 0 \,] \ge a \ \text{(chance constraint)}$$

DEFINITIONS:

x_t - aggregate production of all components

f_t - defines transition - only if resources available and includes subtraction of demand

DYNAMIC PROGRAMMING VIEW

- **STAGES: $t=1,\ldots,T$**
- **STATES: $x_t \to B_t x_t$ (or other transformation)**
- **VALUE FUNCTION:**
  - ∠ $\Psi_t(x_t) = E[\psi_t(x_t,\xi_t)]$ where
  - ∠ ξ_t is the random element and
  - ∠ $\psi_t(x_t,\xi_t) = \min f_t(x_t,x_{t+1},\xi_t) + \Psi_{t+1}(x_{t+1})$
  - – s.t. $x_{t+1} \in X_{t+1t}(\xi_t)$ x_t given
- **ASSUMPTIONS:**
  - – CONVEXITY
  - – EARLY AND LATENESS PENALTIES

PRODUCTION SCHEDULING RESULTS

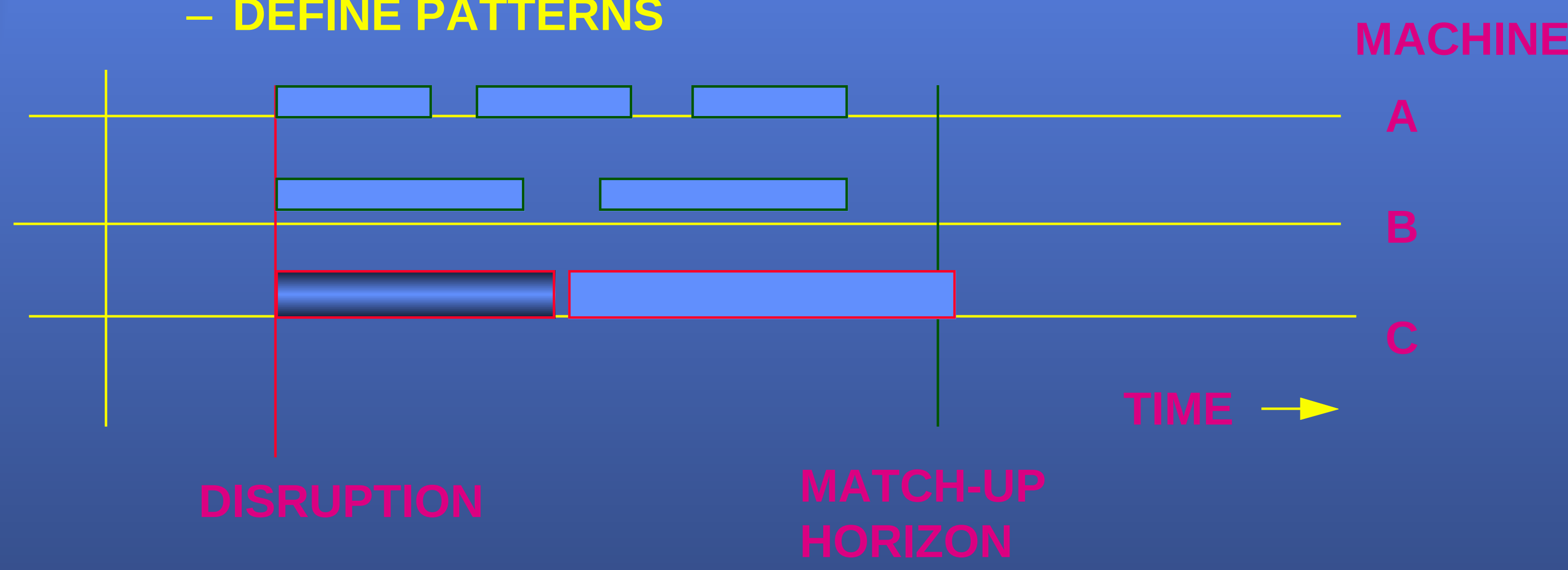
- **OPTIMALITY:**
  - CAN DEFINE OPTIMALITY CONDITIONS
  - DERIVE SUPPORTING PRICES
- **CYCLIC SCHEDULES:**
  - OPTIMAL IF STATIONARY OR CYCLIC DISTRIBUTIONS
  - MAY INDICATE KANBAN/CONWIP TYPE OPTIMALITY
- **TURNPIKE: (Birge/Dempster)**
  - FROM OTHER DISRUPTIONS:
  - RETURN TO OPTIMAL CYCLE
- **LEADS TO MATCH-UP FRAMEWORK**

MATCH-UP BASICS

- **METHOD: (Bean,Birge, Mittenthal, Noon)**
- **START: FIND a PRE-SCHEDULE (CYCLIC):**
  - FROM FORECASTS/NORMAL RANDOMNESS
- **MATCH-UP PROCESS:**
  - WHEN DISRUPTIONS OCCUR, RECOGNIZE THEM
  - TO DEVELOP RESPONSE, CONSTRUCT A PLAN TO MATCH UP WITH THE PRE-SCHEDULE IN THE FUTURE
  - OVERALL PATTERN REPRESENTS SETTING GOALS AND REACTING
  - MAY ALSO USE TO IMPROVE IN SHORT RUN

MATCH-UP PROBLEM

- **GOAL:** FIND A PERIOD OVER WHICH TO CHANGE SCHEDULE
  - DEFINE HORIZON
  - DEFINE SCENARIOS
  - DEFINE PATTERNS



HORIZON DEFINITION

- **ISSUES:**
  - LONG ENOUGH TO:
    - » SMOOTH OUT RESPONSE
    - » MAINTAIN LONG-TERM GOALS
    - » MAKE ECONOMIC CHOICE
  - SHORT ENOUGH TO:
    - » ALLOW RAPID RESPONSE
    - » COMPARE MANY ALTERNATIVES
    - » NOT UNDO OPTIMALITY IN PRE-SCHEDULE
- **RESOLUTION**
  - DAILY FOR SHORT-TERM

SCENARIO DEFINITION

- **ISSUES:**
  - NEED TO CAPTURE POSSIBLE FUTURE OUTCOMES
  - MUST MODEL
    - DEMAND VARIATION
    - PROCESSING INTERRUPTIONS
  - DIFFICULTIES
    - INFINITE NUMBERS OF POSSIBILITIES
    - LIMITED KNOWLEDGE BASES EXISTING
- **APPROACH**
  - START WITH INITIAL KNOWLEDGE
  - USE ALL INFORMATION TO ACHIEVE BEST MATCH

SOLUTION APPROACH

- **FORMULATION: BASIC MODELS**
  - ASSUME HORIZON AND SCENARIOS
  - PRIME WITH SOLUTION RESPONSES FROM KNOWLEDGE CACHE
- **SCENARIOS:** $\sigma \in \Sigma$
  - Probability, $p(\sigma)$
  - Groups, $S^t_1, \ldots, S^t_{St}$ at t
- **MULTISTAGE STOCHASTIC NONLINEAR PROGRAM**

min $\quad \Sigma_\sigma \; p(\sigma) \, (\Sigma_I \Sigma_T f_i(x(i,t,\sigma), u(i,t,\sigma))$

s.t. (for all σ): $\quad \Sigma_k \, x(i,t,\sigma) \ge d(t), t=1..T$

$u(i,t,\sigma)$ integer, $x(i,t,\sigma) \ge 0$, all i,t;

Nonanticipativity:

$x(k,t,\sigma') - x(k,t,\sigma) = 0$ if $\sigma', \sigma \in S^t_i$ for all t, i, σ', σ

This says decision cannot depend on future.

LAGRANGIAN APPROACH:

Relax nonanticip. constraints and place in constraint with multiplier updated on each iteration

STANDARD APPROACHES

- PARTITIONING
- BASIS FACTORIZATION
- INTERIOR POINT FACTORIZATION
- LAGRANGIAN BASED
- MONTE CARLO APPROACHES
- DECOMPOSITION
  - BENDERS, L-SHAPED (VAN SLYKE - WETS0
  - DANTZIG-WOLFE (PRIMAL VERSION)
  - REGULARIZED (RUSZCZYNSKI)

Summary

- PRODUCTION UNCERTAINTY IN LONG AND SHORT TERMS
- CAPACITY EXAMPLE -LONG TERM
  - USE OF OPTION FOR RISK
- SCHEDULING - SHORT TERM
  - CYCLIC OPTIMALITY
  - MATCH-UP
- COMPUTATION
  - DECOMPOSITION
  - APPROXIMATION