Flight Crew Scheduling under Uncertainty

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Outline

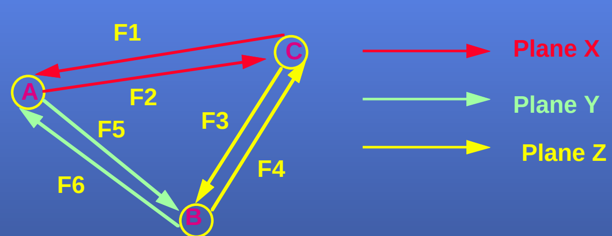
- Introduction to Problem
- Deterministic Models
- Example
- Issues and Research

Introduction

- **Must decide:**
  - routes
  - flight assignment
  - crew assignments
    - » significant cost factor
    - » losses from idle time, deadheading, delays
- **Deterministic models**
  - Difficult to capture delay effects - cost to repair
- **Stochastic model**
  - Include random delays
  - Consider overall system effects
  - Anticipate schedule repair

Crew Scheduling Example

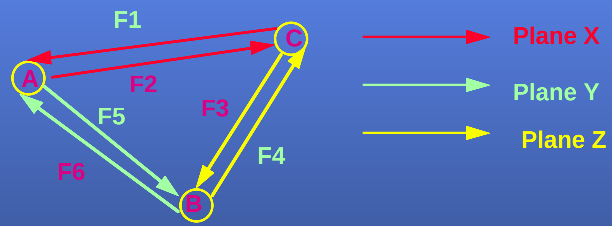
- **Decision:**
  - » How to assign two crews to Flights F1 to F6 ?



DATA: Flight Schedule Times (expectations)
Crew trips (possible)
• Include excess crew hours
Cover all flights

Mean Value Solution

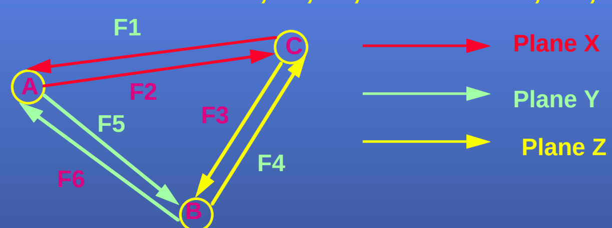
- Objective: MINIMIZE excess crew hours (ECH)
- Result: Crew 1: F2,F3,F6; Crew 2: F5, F4, F1



RESULT: 2 ECH for Crew 1
2 ECH for Crew 2
TOTAL MV = 4

Expectation of Mean Value

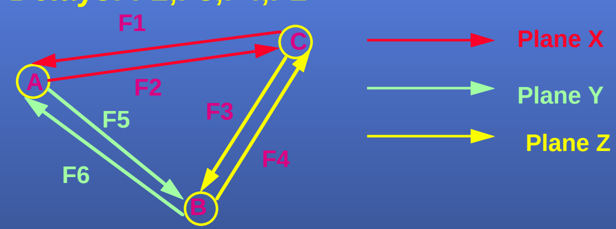
- Suppose: F2 delayed 2 hours w. p. 1/2
- Result: Delays on F2,F3,F6,F4, F1
- Use: Crew 1: F2,F3,F6; Crew 2: F5, F4, F1



RESULT: Crew 1: 2 ECH w.p. 1/2 ; 4 ECH w.p. 1/2
Crew 2: 2 ECH w.p. 1/2; 4 ECH w.p. 1/2
TOTAL EMV = 6
NOTE: Also customer costs for late flights

Stochastic Solution

- Suppose: F2 delayed 2 hours w. p. 1/2
- GOAL: Minimize EXPECTED (ECH)
- Result: Crew 1: F2,F3,F4,F1; Crew 2: F5, F6
- Delays: F2,F3,F4,F1



RESULT: Crew 1: 2 ECH w.p. 1/2 ; 4 ECH w.p. 1/2
Crew 2:2.5 ECH w.p. 1;
TOTAL RP = 5.5

EVPI and VSS

- INFORMATION VALUE:
  - Expected Value with Perfect Information or Wait-and-See (WS) solution:
    - » No Delay: ECH=4; Delay: ECH=6.5
    - » Cost: (1/2) (4) + (1/2)6.5 = 5.25 = WS
  - Expected Value of Perfect Information (EVPI):
    - » EVPI = RP - WS = 5.5 - 5.25 = 0.25
    - » Note: Minimizing -> RP-WS
- MODEL VALUE:
  - FIND EMV=6, RP=5.5
  - Value of the Stochastic Solution (VSS):
    - » VSS = EMV - RP = 6 -5.5 = 0.5
    - » NOTE: Here VSS is two times EVPI

STOCHASTIC PROGRAM

- **SCENARIOS:** s, Probability, p(s)
- **DECISIONS:** x(i) - (0,1) - use trip i
  - a(i,j) = 1 if trip i includes flight j
- **RECOURSE VALUE:** U(x,s) given s
- **FORMULATION:**

$$\min \quad \Sigma_k\, c(i)\, x(i) \;+\; \Sigma_s\; p(s)\,(\,U(x,s)\,)$$
$$\text{s.t.: } \Sigma_k\, a(i,j)\, x(i) \;\geq\; 1, \text{ For all flights } j$$
$$x(i) = 0,1$$

NOTE: Form of Recourse Value U - Nonlinear
-> Nonlinear Integer Program

Recourse Form

- **Finding U(x,s)**
  - Recourse or repair action
  - Original schedule x and delay scenario s
  - Solve a linear program for one repair
  - Another stochastic program for multiple repairs
- **Difficulties**
  - Schedule x variables appear in constraint matrix
  - Nonlinear form
- **Alternatives:**
  - Quadratic approximation
  - Subgradient (cutting planes)

Current Approach

- Exact representation of U(x,s) (polynomial)
- Frank-Wolfe form of solution with branch-and-bound for integer variables
- Results:
  - Find pairings that separate effects
  - Isolate disruptions
  - Natural following of planes with crews
- Next steps
  - General approximation of U
  - Use of special structure
  - Incorporate stochastic cutting plane method (ND-UM)

Summary

- Crew scheduling with significant costs
- Deterministic models changed at disruption
- Lose flight interaction in deterministic case
- Stochastic model avoids cascading costs
- Solvable with nonlinear stochastic program
  - Greater robustness
  - Easier repair
  - Lower overall cost