Stochastic Programming Models in Practice

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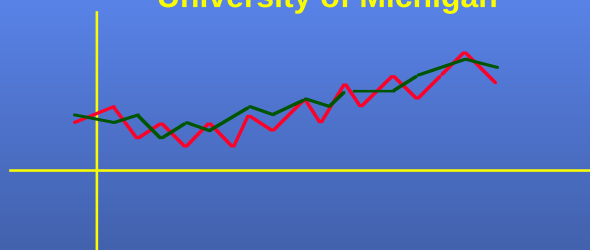

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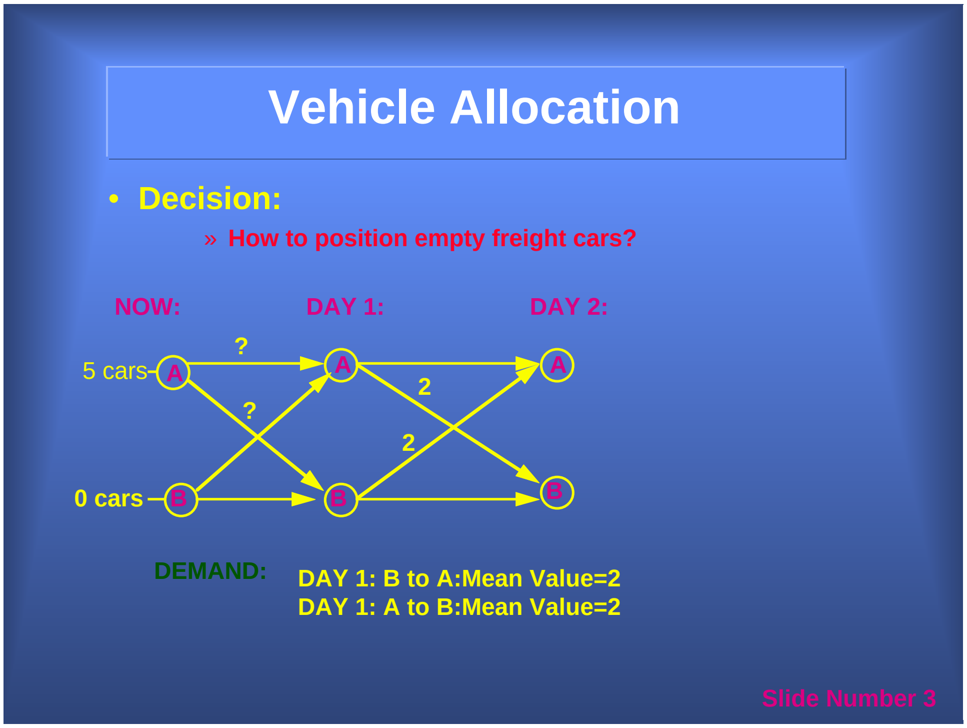
OUTLINE

- •Models
  - •Vehicle Allocation
  - •Manufacturing capacity
  - •Financial planning
- •Solutions
- •Revisions

Slide Number 2


Vehicle Allocation
• Decision:
» How to position empty freight cars?
NOW:
DAY 1:
DAY 2:
5 cars
0 cars
A
B
?
?
2
2
DEMAND:
DAY 1: B to A:Mean Value=2
DAY 1: A to B:Mean Value=2
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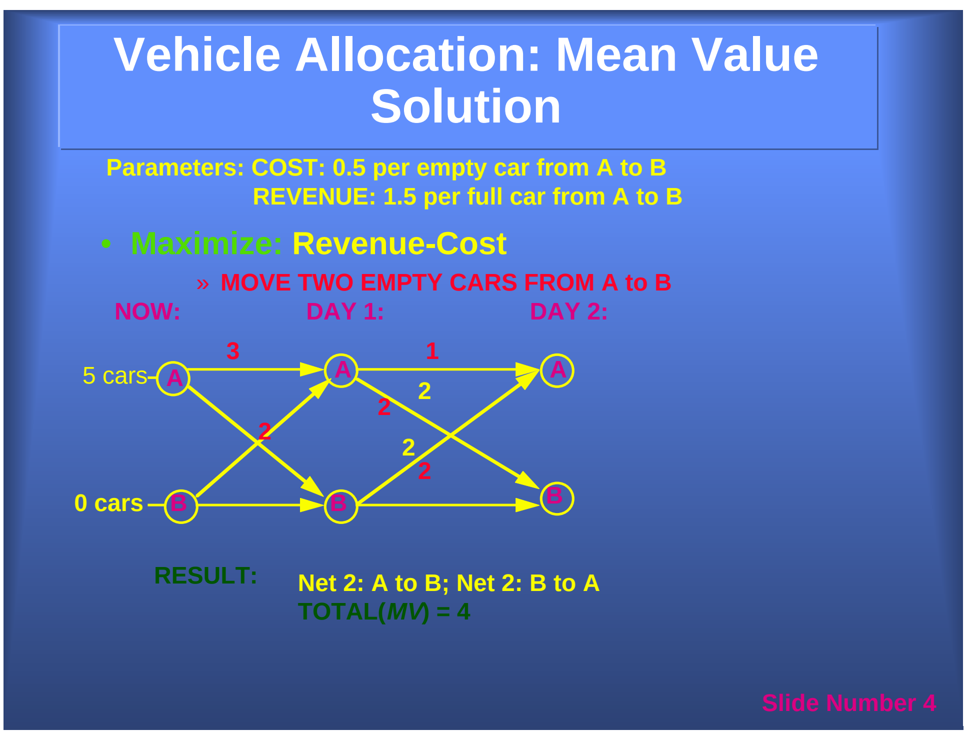
Vehicle Allocation: Mean Value Solution
Parameters: COST: 0.5 per empty car from A to B
REVENUE: 1.5 per full car from A to B
• Maximize: Revenue-Cost
» MOVE TWO EMPTY CARS FROM A to B
NOW:
DAY 1:
DAY 2:
5 cars
0 cars
A
B
3
1
2
2
2
2
2
RESULT:
Net 2: A to B; Net 2: B to A
TOTAL(MV) = 4
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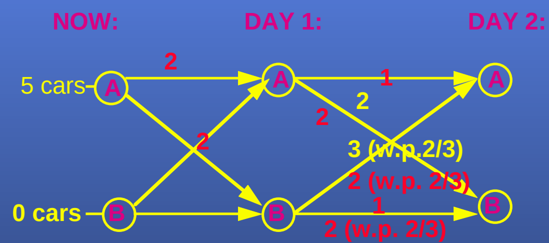
Expectation of Mean Value

Suppose: Demand is Random (Expectation from A to B=2)

- 0 from A to B with prob. 1/3
- 3 from A to B with prob. 2/3

- **Find: Expected (Revenue-Cost)**
  - » MOVE Two EMPTY CARS FROM A to B



Expected Value: Net 2: A to B; Net 2: B to A (w.p. 2/3)
-1: B to A (w.p. 1/3)
TOTAL (*EMV*): 3

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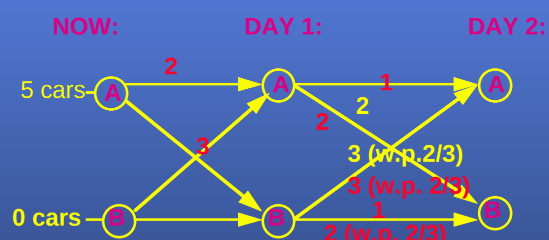
Stochastic Program Solution

Suppose: Demand is Random (as before)
GOAL: A solution to obtain highest expected value

- **Maximize: Expected (Revenue-Cost)**
  - » MOVE Three EMPTY CARS FROM A to B



Expected Value: Net 2: A to B; Net 3: B to A (w.p. 2/3)
-1.5 : B to A (w.p. 1/3)
TOTAL (*RP*): 3.5
RP=Recourse Problem

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INFORMATION and MODEL VALUE

- **INFORMATION VALUE:**
  - FIND Expected Value with Perfect Information or Wait-and-See (WS) solution:
    - » Know demand: if 3, send 3 from A to B
      If 0, send 0 from A to B:
    - » Earn: 2 (AtoB) + (2/3) (3) + (1/3)0= 4 = WS
  - Expected Value of Perfect Information (EVPI):
    - » EVPI = WS - RP = 4 - 3.5 = 0.5
    - » Value of knowing future demand precisely
- **MODEL VALUE:**
  - FIND EMV, RP
  - Value of the Stochastic Solution (VSS):
    - » VSS = RP - EMV=3.5 - 3 = 0.5
    - » Value of using the correct optimization model

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INFORMATION/MODEL OBSERVATIONS

- **EVPI and VSS:**
  - ALWAYS ≥ 0 (WS ≥ RP ≥ EMV)
  - OFTEN DIFFERENT (WS=RP but RP > EMV and vice versa)
  - FIT CIRCUMSTANCES:
    - » COST TO GATHER INFORMATION
    - » COST TO BUILD MODEL AND SOLVE PROBLEM
- **MEAN VALUE PROBLEMS:**
  - MV IS OPTIMISTIC (MV=4 BUT EMV=3, RP=3.5)
    - » ALWAYS TRUE IF CONVEX AND RANDOM
    - » CONSTRAINT PARAMETERS
  - VSS LARGER FOR SKEWED DISTRIBUTIONS/COSTS

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STOCHASTIC PROGRAM

- ASSUME: Random demand on AB and BA
- GOAL: maximize expected profits
  - (risk neutral)
- DECISIONS: x_{ij} - empty from i to j
  - $y_{ij}(s)$ - full from i to j in scenario s (RECOURSE)
  - (prob. p(s))
- FORMULATION:

$$\text{Max } -0.5xAB + \Sigma_{s=s1,s2}\, p(s)\ (1.5\, yAB(s) + 1.5\, yBA(s))$$

s.t.

xAB	$+xAA$			$= 5$	(Initial)
$-xAB$			$+ yBA(s)$	≤ 0	(Limit BA)
	$-xAA$	$+ yAB(s)$		≤ 0	(Limit AB)
			$yBA(s)$	$\leq DBA(s)$	(Demand BA)
		$+ yAB(s)$		$\leq DAB(s)$	(Demand AB)

$xAA, XAB, yAA(s), yAB(s) \geq 0$

  - EXTENSIONS: Multiple stages
    - Constraint/objective complexity (Powell et al.)

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Outline

- Models
  - Vehicle Allocation
  - Manufacturing Capacity
  - Financial Planning
  - General
- Solutions
- Revisions

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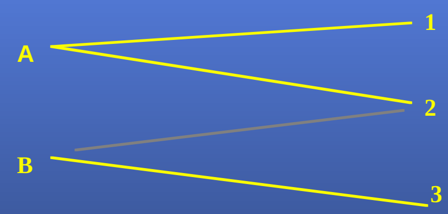
Manufacturing Capacity

- Where to Install Capacity for Different Models among Different Plants?



- Where to add flexibility? (multiple models)

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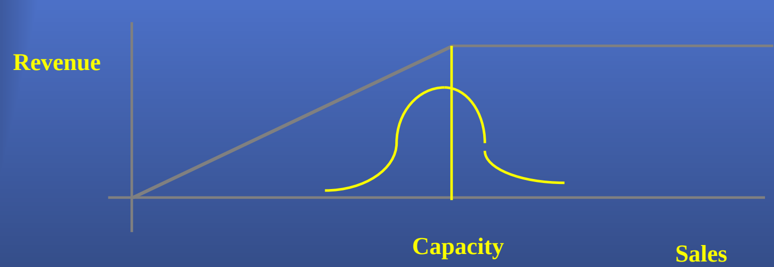
Traditional Problems

- Correlated Demand
  - Models 1,2,3 similar
- Capacity Limit
  - => Asymmetric payoff



=> OPTIONS

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Option Approaches

- Previous work:
  - S. Andreou, C. Byrd
- Assumption: risk free hedge
  - Can evaluate as if risk neutral
  - As in Black-Scholes model
- Steps
  - Adjust revenue to risk-free equivalent
  - Discount at riskless rate

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Recourse Payoff Evaluation

- Key: Evaluate Expected Optimal with Installed Capacity
  - Must choose best mix of models assigned to plants
  - Maximize Σi Profit (i) Production(i)
  - subject to: MaxSales(i) >= Σ j Production(i at j)
  - Σ i Production(i at j) <= Capacity (i)
  - Production(i at j) <= Capacity (i at j)
  - Production(i at j) >= 0
- Transportation Problem
- Need MaxSales(i) - random - unknown distribution
  - Capacity(i at j) - Decision in First Stage

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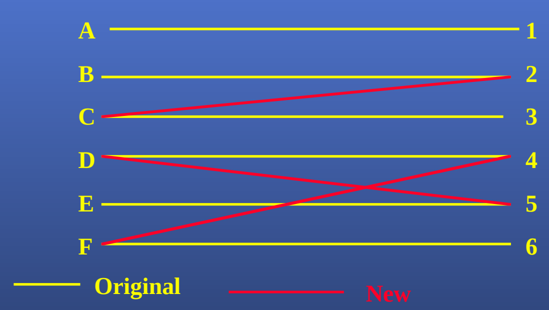
Solution Results

- Model Data: from Graves/Jordan
- Vary: Model Lifetimes
  - Longer => More flexibility
- Start: 1 Year



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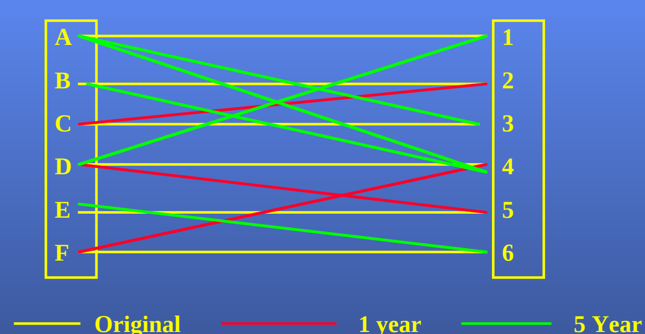
Five Year Lifetime Solution



- Note: new additions for 5 year
- Additional model years => more flexibility

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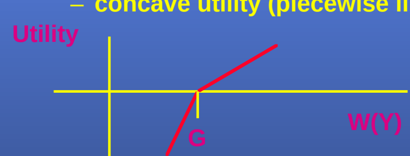
Financial Planning

- **GOAL:** Accumulate $G for tuition Y years from now
- **Assume:**
  - $ W(0) - initial wealth
  - K - investments
  - concave utility (piecewise linear)



RANDOMNESS: returns r(k,t) - for k in period t
where Y ⟶ T decision periods

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FORMULATION

- **SCENARIOS:** $\sigma \in \Sigma$
  - Probability, $p(\sigma)$
  - Groups, S^t_1, ..., S^t_{St} at t
- **MULTISTAGE STOCHASTIC NLP FORM:**

max $\sum_\sigma p(\sigma) \left(U(W(\sigma, T)) \right)$

s.t. (for all σ): $\sum_k x(k,1,\sigma) = W(o)$ (initial)

$\sum_k r(k,t-1,\sigma)\, x(k,t-1,\sigma) - \sum_k x(k,t,\sigma) = 0$, all t >1;

$\sum_k r(k,T-1,\sigma)\, x(k,T-1,\sigma) - W(\sigma, T) = 0$, (final);

$x(k,t,\sigma) \geq 0$, all k,t;

Nonanticipativity:

$x(k,t,\sigma') - x(k,t,\sigma) = 0$ if $\sigma', \sigma \in S^t_i$ for all t, i, σ', σ

This says decision cannot depend on future.

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DATA and SOLUTIONS

- **ASSUME:**
  - Y=15 years
  - G=$80,000
  - T=3 (5 year intervals)
  - k=2 (stock/bonds)
- **Returns (5 year):**
  - Scenario A: r(stock) = 1.25 r(bonds)= 1.14
  - Scenario B: r(stock) = 1.06 r(bonds)= 1.12
- **Solution:**

PERIOD	SCENARIO	STOCK	BONDS
1	1-8	41.5	13.5
2	1-4	65.1	2.17
2	5-8	36.7	22.4
3	1-2	83.8	0
3	3-4	0	71.4
3	5-6	0	71.4
3	7-8	64.0	0

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MODEL VALUES

- COMPARISON TO MEAN VALUES:
  - RP = -7 EMS=-19 (all stock investments)
    - » VSS = RP - EMS = 12
- HORIZON/PERIOD EFFECTS
  - TRUNCATION AT 10 YEARS
    - » MORE CONSERVATIVE
    - » HEAVY BOND INVESTMENT
  - LONG PERIODS
    - » MORE MEAN EFFECT - LESS DISTRIBUTION
    - » HEAVY STOCK INVESTMENT
- RESULT
  - NEED THREE PERIODS FOR HEDGING SOLUTION
  - MANY CURRENT USERS (ALM MODELING, ZIEMBA, MULVEY, ZENIOS, et al.)

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GENERAL MULTISTAGE MODEL

- **FORMULATION:**

MIN $E[\, \Sigma_{t=1}^{T} f_t(x_t, x_{t+1}) \,]$

s.t. $x_t \in X_t$

x_t nonanticipative

$P[\, h_t(x_t, x_{t+1}) \le 0 \,] \ge a$ (chance constraint)

EXAMPLES:

Vehicle Allocation: Linear functions, continuous or integer variables

Capacity: Linear plus integer variables

Financial Planning: Nonlinear objective, continuous variables

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DECOMPOSITION METHODS

- **BENDERS IDEA**
  - FORM AN OUTER LINEARIZATION OF Ψ_t
  - ADD CUTS ON FUNCTION :

USE AT EACH STAGE TO APPROXIMATE VALUE FUNCTION

- ITERATE BETWEEN STAGES UNTIL ALL MIN = Ψ_t

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DECOMPOSITION IMPLEMENTATION

- NESTED DECOMPOSITION
  - LINEARIZATION OF VALUE FUNCTION AT EACH STAGE
  - DECISIONS ON WHICH STAGE TO SOLVE, WHICH PROBLEMS AT EACH STAGE
- LINEAR PROGRAMMING SOLUTIONS
  - USE OSL FOR LINEAR SUBPROBLEMS
  - USE MINOS FOR NONLINEAR PROBLEMS
- PARALLEL IMPLEMENTATION
  - USE NETWORK OF RS6000S
  - PVM PROTOCOL

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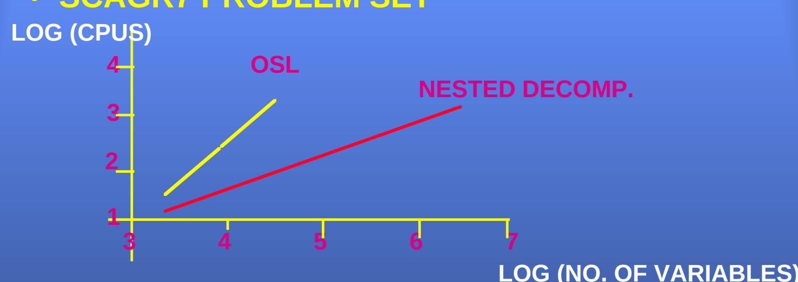
RESULTS

- SCAGR7 PROBLEM SET



PARALLEL: 60-80% EFFICIENCY IN SPEEDUP

OTHER PROBLEMS: SIMILAR RESULTS
- ONLY < ORDER OF MAGNITUDE SPEEDUP WITH STORM
  - TWO-STAGES - LITTLE COMMONALITY IN SUBPROBLEMS
  - STILL ABLE TO SOLVE ORDER OF MAGNITUDE LARGER PROBLEMS

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CONCLUSIONS

- STOCHASTIC PROGRAMS CAN BE:
  - LINEAR, NONLINEAR, INTEGER PROGRAMS
  - CONTINUOUS OR DISCRETE R.V.'S
  - OF SIGNIFICANT VALUE (VSS) OVER DETERMINISTIC MODELS
- RANDOMNESS =>
  - VALUE OF MODELING
  - DIFFICULTY IN EVALUATING OBJECTIVES
  - MOTIVATION FOR APPROXIMATION
- SOLUTIONS
  - DECOMPOSITION FOR LINEAR PROBLEMS
  - SPEEDUPS OF ORDERS OF MAGNITUDE

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