When Should One Invest in Reconfigurable Capacity?

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Abstract

Motivated by our work with machine tool manufacturers who are building "reconfigurable" machines, and auto companies who are considering buying them, we investigate the conditions under which it would be economically advantageous to invest in reconfigurable capacity. Reconfigurable machines are those which can be reconfigured to produce more than one generation of product with relatively little cost or in little time. In general, a buyer's choice is between buying dedicated equipment for every generation of product or buying reconfigurable equipment which will last more than one generation. The attractiveness of reconfigurable equipment depends on the probability that the next generation product will soon be demanded, and the relative costs of reconfigurable or new dedicated equipment. We develop several models to gain insight into the question of when it would be optimal to invest in reconfigurable capacity and present some structural results on these decisions.

1 Introduction

We consider the machine investment decisions of a manufacturer who produces a single product. Driven by increasing competition, and changing customer tastes, the manufacturer has to introduce new or updated generations of this product over time. Should this manufacturer buy new dedicated equipment each time the next generation product is introduced or would it be economically advantageous to consider equipment which is more expensive in initial capital investment, but less costly to convert to produce several generations of products? Our paper is focused on this question.

The situation described above arises in many industries. Our work is motivated by machine tool manufacturers in the auto industry who are now producing such "reconfigurable" machinery for the automobile companies and their suppliers. A typical example is in valve machining for cylinder heads. Whereas a dedicated machine would be capable of producing only one type of engine valve, a reconfigurable machine can be adjusted (reconfigured) to produce multiple types (e.g., angles) of valves. To allow for such changes, several engine manufacturers have bought reconfigurable machinery that allows them to switch from producing cylinder heads with 2 valves per cylinder to those with 3 or 4 valves per cylinder. Another example of a reconfigurable purchase is a Cummins Engine plant that bought machinery capable of producing different sizes of cyclinder heads. They initially produced 5.9 liter engine heads but eventually when the demand switched, they reconfigured the machinery to produce 6.8 liter engine heads.

Although most of our experience is with auto manufacturing, similar examples abound in other industries. For example, in semiconductor manufacturing, the average lifetime of a chip is rather short. Given the huge capital expense of semiconductor manufacturing equipment, companies are paying significant attention to machines that can be easily adjusted to produce multiple generations of chips.

Our paper is focused on determining the conditions under which it would be optimal to invest in reconfigurable machines rather than dedicated ones. As such, it is related to a large body of work that exists in the machine replacement and flexible manufacturing literature. First, unlike most papers in the flexible manufacturing literature, the reconfigurable machines we consider can only produce one product at a time. That is, although both flexible and reconfigurable machines can be set up to produce more than one product type, what differentiates them is one of time scale. Whereas a flexible machine may be set up to produce a different type of product every day or every week, the reconfigurable machines that machine tool manufacturers are producing will be set up at most once or twice in their lifetime to produce a different generation product. Whereas flexible equipment can switch very quickly between previously determined types of products, reconfigurable equipment is designed to be able to be set up less quickly but over a wider range of possible configurations that might be required by changing customer tastes (e.g, a changeover time of 5-10 minutes for flexible equipment versus 3-20 days for reconfigurable equipment). The main reason for considering these single-product-at-a-time models is that we consider high-volume products that can easily consume the capacity of one or several dedicated lines. Typical flexible equipment tends to be much slower than dedicated equipment. The reconfigurable equipment we consider is typically as fast as dedicated equipment. From these observations, our investment problem is different than those addressed in typical flexible manufacturing investment decisions.

Typical examples of papers on flexible manufacturing investment are Fine and Freund (1990), Gupta et al. (1992) and Laengle et al. (1994). For example, Laengle et al. assume that whereas a single product can be assigned to dedicated capacity during a single time period, multiple products can be assigned to flexible capacity. In our modelling framework, a single product can be assigned to both reconfigurable and dedicated capacity in a single period. However, whereas a technology or demand shift makes the dedicated equipment obsolete, the recon¯gurable equipment is able to produce the next generation product. For further perspectives on °exibility, see Buzacott and Kahyaoglu(1998), Gerwin (1993) and Milgrom and Roberts (1990). Sethi and Sethi (1990) and Gray et. al. (1993) provide surverys on manufacturing °exibility. Birge (1995) illustrates the relationship between ¯-nancial options and investment in °exibility. Dixit and Pindyck (1994) provide general approches to use option pricing theory to determine the economic value of a °exible manufacturing system. Fine and Freund (1990) and Van Mieghem (1995) formulate two-stage stochastic programs to determine optimal investment levels in °exible manufacturing systems.

There is a large literature on machine replacement. In recent years much attention has focused on machine replacement under technological change. Luss (1982) provides an extensive survey of capacity expansion problems. Pierskella and Voelker (1976) and Sherif and Smith (1981) provide reviews of replacement of deteriorating systems. The papers that are closest to our work in scope are those focusing on machine replacement under technological change. Hopp and Nair (1991) consider a machine replacement problem where the decision maker is deciding whether to replace the machine in an environment where a machine with better technology might appear sometime in the future. They assume that bene¯ts and costs of current and future technologies are known but the arrival time of the future technology is uncertain. The decision is then between keeping the old machine for another period (and observing whether the new technology becomes available) or replacing it with a new machine using the existing technology. Nair and Hopp (1992) extend the problem to include non-stationary revenue functions. Goldstein et. al. (1988) and Hopp and Nair (1994) also take machine deterioration into consideration and show the optimality of a control limit policy. Finally, whereas the previous papers by Nair and Hopp limit the analysis to at most two future technologies, Nair(1995) considers the more general problem with n future technologies. In related work, Rajagopalan et al. (1996) analyzes capacity expansion and replacement models under uncertain technology breakthroughs.

What di®erentiates the present paper from the previous work on machine replacement under uncertain technology change is that in all the previous papers, the uncertainty a®ects the timing of when the new, more advanced machine will become available for purchase. The decision maker then decides between using his present machine longer until new technology is available or not waiting and replacing the present machine with a new machine using the present technology. (An exception is a recent paper by Gardner and Buzacott (1998) where the uncertainty is captured by the probability p that the new technology will be successful and available to everyone a given number of years from now. Using a stochastic modelling framework and a case study based upon direct steelmaking, the authors discuss three di®erent planning strategies. The ¯rst two, common in industry, are plan for success and plan for failure, i.e., assuming $p = 1$ or $p = 0$ and constructing a capacity plan based on this assumption. The authors contrast this strategy with a stochastic programming approach based on the firm's estimate of p, which they call the hedging strategy and show that in one sense, this strategy is an insurance against either failure or success). In our modelling framework, the decision maker can at the present time purchase either the dedicated or the reconfigurable machine. What is uncertain is when (if ever) the reconfigurable machine will be reconfigured to produce a different product than the one being currently produced. That is, production start time for the next generation product is uncertain. This leads to a different type of model than the previous ones in the literature. For example, whereas in previous models (e.g., Hopp and Nair (1994)), the decision maker only had to choose between the two decisions (e.g., keep and replace) described above when taking the age (or deterioration level) of the machine into account, in our models, the decision maker faces three choices: 1) keep the existing dedicated machine for one more period, 2) replace the current dedicated machine by a new dedicated machine, 3) replace the current dedicated machine by a reconfigurable machine. Therefore, although the uncertainty is still one of timing, our model is significantly different than previous models in the uncertain technology-change literature. (We can view the decision to buy reconfigurable machinery as a hedging strategy (as in Gardner and Buzacott) where the firm is hedging against the probability that a new model that can not be produced by current machinery will be introduced. In that sense, our models evealuate when such hedging makes sense.)

Before we start presenting our models, we mention some modelling assumptions motivated by our work with industrial sponsors. First, we assume only a single product change during the planning horizon. This is largely based on our experience with the auto industry where it was extremely uncommon for the same equipment to be used for more than two generations of products. Typical planning horizons of the useful life of equipment are 10-12 years and manufacturers usually expect only a single product change in that horizon. Therefore, in our models, we assume that there is only one product change over our planning horizon. (Although manufacturers produce a new model year car every year, significant changes requiring tooling occur typically only every 4-5 years. For example, the 1996 Taurus was a new Taurus requiring significant new tooling whereas the Tauruses in previous years only had minor changes.) Second, in our models, we assume that the time when (if ever) reconfiguration becomes necessary is uncertain. For example, styling changes might require new locations for mounting feature on an engine, CAFE (Corporate Average Fuel Economy) requirements might require an engine valve-angle change. It is very hard to have certain information on these requirements 5-10 years in advance. Therefore, from the point of view of individuals responsible for buying tooling, the time when they will need new dedicated tooling or when they will have to reconfigure their equipment is a random variable. (As background, Walton (1997) has several interesting examples of the last minute design changes in the creation of the 1996 Taurus and the effects that these had on tooling and other functions).

The rest of this paper is organized as follows: In Section 2, we present a very basic model that does not take into account machine deterioration. In Section 3, we extend this model to take machine deterioration into account. In Section 4, we undertake a numerical study to explore the sensitivity of decisions to problem parameters. The paper concludes in Section 5.

2 Basic Model

We start with a very basic model to gain insight into the main tradeoffs involved between purchasing dedicated capacity and reconfigurable capacity. We consider the situation of a manufacturer who currently has C units of capacity dedicated to the present product. He can purchase reconfigurable capacity (and dispose of dedicated capacity) which will also produce the next generation product at a net cost of K_1 per unit capacity. We assume that the probability that the next generation product will arrive in any given period is geometric with probability p. When the next generation product arrives, the manufacturer has to immediately convert all his remaining dedicated capacity to be able to produce the next generation product. The conversion cost at that point is K_2 per unit capacity which we assume is larger than K_1. This is due to the fact that 1) the firm may have to lose some capacity and sales if the conversion is done at the last moment rather than planned in advance (as is very typical in the auto industry) and 2) the emergency cost of acquiring the equipment will be larger than ordinary cost. Using a discount factor of δ per unit time, we can then write the dynamic program for computing the optimal decision as follows:

$$V(C) = \min_{0 \le y \le C} \{K_1 y + \delta p K_2 (C - y) + \delta(1-p) V(C-y)\}. \quad (2.1)$$

In the DP formulation, the state variable is the amount of dedicated capacity we have. We can decide to convert some of it at a cost of K_1 per unit now or if we have to produce the next generation product in the next period and we are left with any capacity dedicated to the current product, we can pay K_2 per unit cost then. In this case, we can show that it would never be optimal to convert a part of the dedicated capacity to reconfigurable capacity. One would either convert all or none of it. This is stated in the following

Theorem 1 *1. If $\frac{K_1}{K_2} \ge \frac{\delta p}{1-\delta(1-p)}$, an optimal policy is to keep all current capacity in every state and $V(C) = \frac{C \delta p K_2}{1-\delta(1-p)}$.*

2. If $\frac{K_1}{K_2} < \frac{\delta p}{1-\delta(1-p)}$, an optimal policy is to replace all current capacity in every state and $V(C) = K_1 C$.

Proof: We give the proof for the first part only as the proof for the second part is virtually identical. We prove the result by induction on C. At $C = 1$, $V(1) = \min\{\delta[pK_2 + (1-p)V(1)]; K_1\}$. Suppose an optimal policy is to keep, then

$$\begin{aligned} V(1) &= \delta[pK_2 + (1-p)V(1)] \\ &= \frac{\delta pK_2}{1-\delta(1-p)}. \end{aligned}$$

Therefore, $V(1) = \min\{\frac{\delta pK_2}{1-\delta(1-p)}; K_1\}$. The hypothesis implies that $\frac{\delta pK_2}{1-\delta(1-p)} \leq K_1$. Therefore, $V(1) = \frac{\delta pK_2}{1-\delta(1-p)}$, i.e., the optimal policy at state $C = 1$ is to keep the existing capacity.
Suppose the optimal policy is to keep all dedicated capacity in states $2, \ldots, (k-1)$, then at $C = k$

$$\begin{aligned} V(k) &= \min_{0 \leq y \leq k}\{K_1 y + \delta pK_2(k-y) + \delta(1-p)V((k-y))\} \\ &= \min_{0 \leq y \leq k}\{K_1 y + \delta pK_2(k-y) + \tfrac{\delta(1-p)(k-y)\delta pK_2}{1-\delta(1-p)}\} \\ &= \min_{0 \leq y \leq k}\{K_1 y + \tfrac{\delta pK_2(k-y)}{1-\delta(1-p)}\} \\ &= \min_{0 \leq y \leq k}\{(K_1 - \tfrac{\delta pK_2}{1-\delta(1-p)})y + \tfrac{\delta pK_2 k}{1-\delta(1-p)}\}. \end{aligned}$$

Since $\frac{\delta pK_2}{1-\delta(1-p)} \leq K_1$ then $y = 0$ minimizes $V(k)$. Therefore the optimal policy at state $C = k$ is to keep all current dedicated capacity and $V(k) = \frac{k\delta pK_2}{1-\delta(1-p)}$. Hence, the result is proven.

We note that Theorem 1 implies that under the given cost and probability structure, it is never optimal to carry dedicated and reconfigurable capacity at the same time. One would either have all dedicated or all reconfigurable capacity. It is straightforward to show that this property that it is not optimal to mix dedicated and reconfigurable capacity still holds assuming increasing concave cost functions. That is, if we assume that, the cost for y units of capacity is $K_1(y)$ and $K_2(y)$ (depending on when the capacity is purchased), where K_1 and K_2 are increasing, concave functions of y, the optimal policy still has the property that one would not mix dedicated and reconfigurable capacity. In fact, we formally show this result below where we also assume a more general distribution for the probability of arrival of the next generation product.

We can extend the DP formulation 2.1 by assuming that the probability that the next generation product will arrive in any given period has a general discrete distribution. We assume that at time 0, we are given a probability vector Q which specifies the probability that the next generation product will arrive n periods from now for all n. We also assume that the next generation product must arrive until period $N+1$. We solve the DP assuming we have no new information affecting the arrival of new product. (Should such information arrive changing the probabilities in one of the future periods, one would formulate and solve a new DP.) We define p_n to be the probability that the next generation product will arrive in period n given it has not arrived in periods $1, \ldots, n-1$.

That is $p_n = Q_n/(1 - \sum_{i=1}^{n-1} Q_i)$. We assume that p_n is increasing in n. (Note that this is a discrete version of the Increasing Failure Rate (IFR) requirement.) We also assume increasing and concave costs $K_1(y)$ and $K_2(y)$ with $K_1(0) = K_2(0) = 0$ and $K_1(y) < K_2(y)$ for all $y > 0$.

Define $V(C;n), n = 0,1,\dots,N$ as the minimum expected total cost of having C units of dedicated capacity at time n. We let $V(C;N+1) = K_2(C)$; that is if in period $N+1$, by which time the next generation product has arrived, we still have some dedicated capacity, we immediately need to replace it with capacity for the new product. Then, $V(C;n) = \min_{0 \le y \le C}\{W(C;n;y)\}$ where

$$W(C;n;y) = K_1(y) + \delta p_{n+1} K_2(C - y) + \delta(1 - p_{n+1})V(C - y; n+1)$$

for $n = 0,1,\dots,N$. The optimal policy for this case is described in the following

Theorem 2 *$W(C;n;y)$ is concave in y for fixed C, $V(C;n)$ is concave in C, and $V(0;n) = 0$ for $n = 0,1,\dots,N$. Furthermore, the optimal policy is to either replace all dedicated capacity by reconfigurable capacity or to keep all dedicated capacity in any period.*

Proof: We prove the result by induction on n. First observe that $V(C;N+1) = K_2(C)$ is concave in C by definition.

At $n = N$ and for a fixed C,

$$\begin{aligned} W(C;N;y) &= K_1(y) + \delta p_{N+1} K_2(C - y) + \delta(1 - p_{N+1})V(C - y; N+1) \\ &= K_1(y) + \delta p_{N+1} K_2(C - y) + \delta(1 - p_{N+1})K_2(C - y) \\ &= K_1(y) + \delta K_2(C - y). \end{aligned}$$

The concavity of $W(C;N;y)$ follows from the concavity of K_1 and K_2. This implies that either $y = 0$ or $y = C$ minimizes $V(C;N) = \min_{0 \le y \le C}\{W(C;N;y)\}$. Therefore, $V(C;N) = \min\{K_1(C); \delta K_2(C)\}$ which is also concave in C. Notice that $V(0;N) = 0$ since $K_1(0) = K_2(0) = 0$ by assumption.

Now assume it is also true that $W(C;n;y)$ is concave in y, $V(C;n)$ is concave in C, $V(0;n) = 0$ for $n = k+1, k+2, \dots, N+1$. At $n = k$, and for any fixed C and $0 \le \lambda \le 1$,

$$\begin{aligned} W(C;k;\lambda y_1 + (1-\lambda)y_2) &= K_1(\lambda y_1 + (1-\lambda)y_2) + \delta p_{k+1} K_2(\lambda(C - y_1) + (1-\lambda)(C - y_2)) \\ &\quad + \delta(1 - p_{k+1})V(\lambda(C - y_1) + (1-\lambda)(C - y_2); k+1) \\ &\ge \lambda(K_1(y_1) + \delta p_{k+1} K_2(C - y_1) + \delta(1 - p_{k+1})V(C - y_1)) \\ &\quad + (1-\lambda)(K_1(y_2) + \delta p_{k+1} K_2(C - y_2) + \delta(1 - p_{k+1})V(C - y_2)) \\ &= \lambda W(C;k;y_1) + (1-\lambda)W(C;k;y_2); \end{aligned}$$

where the inequality follows from the fact that $K_1(y)$ and $K_2(y)$ are concave in y by assumption and $V(C;k)$ is a concave funtion by induction hypothesis. Therefore, $W(C;k;y)$ is concave in y. This implies that either $y = 0$ or $y = C$ minimizes $V(C;k)$. Therefore, $V(C;k) = \min\{\delta p_{k+1} K_2(C) + \delta(1 - p_{k+1})V(C;k+1); K_1(C) + \delta(1 - p_{k+1})V(0;k+1)\}$. By the induction hypothesis, $V(0;k+1) = 0$. This implies that

$$V(C;k) = \min\{\delta p_{k+1} K_2(C) + \delta(1 - p_{k+1})V(C;k+1); K_1(C)\}. \tag{2.2}$$

Using the fact that a summation of concave functions is a concave function and a minimum of 2 concave funtions is again a concave function, $V(C;k)$ is, therefore, concave in C. Additionally, since $K_1(0) = K_2(0) = V(0;k+1) = 0$, it implies that $V(0;k) = 0$. This completes the proof .

We have therefore shown that the result that one would not mix reconfigurable and dedicated capacity extends to fairly general assumptions. If we further assume linear costs, i.e., $K_1(y) = K_1 y$ and $K_2(y) = K_2 y$, then Eqn. 2.2 can be rewritten as follows:

$$V(C;k) = \min\{\delta p_{k+1} K_2 C + \delta(1 - p_{k+1})V(C;k+1); K_1 C\} \tag{2.3}$$

for $k = 0, 1, \ldots, N$. We can then determine a critical threshold that provides a result similar to that in Theorem 1. We first need the following

Lemma 1 *Let* $f(\delta;p) = \frac{\delta p}{1-\delta(1-p)}$. *Then* $f(\delta;p) \le \delta$ *for* $0 \le p \le 1; 0 \le \delta < 1$.

Proof: Notice that $f(\delta;0) = 0 \le \delta$. For $p > 0$; $f(\delta;p) = \delta\left(\frac{(1-\delta)}{p} + \delta\right)^{-1}$ which is increasing in p for a fixed δ between 0 and 1. Therefore, the upperbound of $f(\delta;p)$ is $f(\delta;1) = \delta$ for a fixed δ. Therefore, $f(\delta;p) \le \delta$ for all p and any δ.

Theorem 3 1. *Under linear capacity costs K_1 and K_2, if $\frac{K_1}{K_2} \ge \frac{\delta p_{k+1}}{1-\delta(1-p_{k+1})}$, then an optimal policy is to keep all capacity in state $(C;l)$ for all C and $l = 0;1;\ldots;k$.*

2. *Under linear capacity costs K_1 and K_2, if $\frac{K_1}{K_2} < \frac{\delta p_{k+1}}{1-\delta(1-p_{k+1})}$, then an optimal policy is to replace all capacity in state $(C;l)$ for all C and $l = k;k+1;\ldots;N$.*

Proof: (1) From the hypothesis and the fact that $\frac{\delta p_k}{1-\delta(1-p_k)}$ is increasing in k, these imply that for $l = 0;1;\ldots;k$,

$$\delta p_{l+1} K_2 C + \delta(1 - p_{l+1}) K_1 C \le K_1 C.$$

From Eqn. 2.3, $V(C;l+1) = \min\{\delta p_{l+2} K_2 C + \delta(1 - p_{l+2})V(C;l+2); K_1 C\} \le K_1 C$. This implies that

$$V(C;l) \le \delta p_{l+1} K_2 C + \delta(1 - p_{l+1})V(C;l+1)$$

$$
\begin{aligned}
&\leq \delta p_{l+1} K_2 C + \delta(1 - p_{l+1}) K_1 C \\
&\leq K_1 C.
\end{aligned}
$$

This implies that $V(C; l) = \delta p_{l+1} K_2 C + \delta(1 - p_{l+1}) V(C; l+1)$ and the optimal policy in state $(C; l)$ is to keep all capacity.

(2) From the hypothesis and the fact that $\frac{\delta p_k}{1 - \delta(1 - p_k)}$ is increasing in k, these imply that for $l = k, k+1, \ldots, N$,

$$\delta p_{l+1} K_2 C + \delta(1 - p_{l+1}) K_1 C > K_1 C.$$

From Lemma 1 and the hypothesis, it follows that $K_1 < \delta K_2.$ This implies that $V(C; N) = \min\{K_1 C, \delta K_2 C\} = K_1 C$. At $l = N - 1$,

$$
\begin{aligned}
\delta p_N K_2 C + \delta(1 - p_N) V(C; N) &= \delta p_N K_2 C + \delta(1 - p_N) K_1 C \\
&> K_1 C
\end{aligned}
$$

by hypothesis. Therefore, $V(C; N - 1) = K_1 C$.

Assume it is also true that $V(C; m + 1) = K_1 C$. By the same argument as above, it is easy to show that $V(C; m) = K_1 C$. Therefore, it is true that $V(C; l) = K_1 C$ for all $l = k, k+1, \ldots, N$, i.e., it is optimal to replace all capacity in period $l = k, k+1, \ldots, N.$

3 Taking Machine Failures into Account

The basic model in the previous section makes machine purchasing decisions only based on the probability that the next generation product will arrive in a future period. However, a major factor that also drives machine replacement decisions is the age and deterioriation level of the present machine. Therefore, in this section, we extend the model in the previous section to take the age of the machine into account as well. In particular, we assume that both dedicated (DMS) and reconfigurable (RMS) machines have a finite lifetime, which we denote by A_D and A_R, respectively. The machines are subject to catastrophic failures in any given period, and the probability that a dedicated (reconfigurable) machine of age a fails (given it has survived $a - 1$ periods) is given by q_a (r_a). We assume that q_a and r_a is increasing in a, i.e., the lifetime distributions of dedicated and reconfigurable machines satisfy the discrete version of the IFR requirement. We again initially assume that the probability that the next generation product will arrive in any given period is geometric with probability p. (We will then consider more general distributions).

In this section, we focus on the decision of keeping or replacing a single machining system (this could represent a whole dedicated or reconfigurable line). We assume that the capital cost of purchasing a new DMS and RMS to fully satisfy all the demand, if the purchase is planned ahead, is given respectively by K_D and K_R. However, if an emergency purchasing decision is made once a machine has failed or the new product has to be produced, as in the previous section, we assume that additional costs (on top of the capital cost of equipment) are incurred due to lost sales during the time that the new machine arrives, the additional emergency cost, etc. We denote this cost by K_E. We further assume that $K_D \le K_R$ since the RMS has more functionality than DMS. Finally, we let $m_D(a)$ and $m_R(a)$ denote the annual maintenace cost of DMS and RMS of age a, respectively. We assume that $m_D(a)$ and $m_R(a)$ are increasing in a and that these costs are incurred at the end of period. The state variables in this model are the type of current machining system and its age. Using the discount factor of δ per unit time, we can then formulate the dynamic program for computing the optimal decision as follows:

$$
\begin{aligned}
V_D(a) &= \min \begin{cases} \delta\{p[K_R + K_E + V_R^N(0)] + (1-p)[(1-q_{a+1})V_D(a+1) + q_{a+1}V_D(A_D)] + m_D(a)\} \\ K_R + V_R^C(0) \\ K_D + V_D(0), \end{cases} \\
V_R^C(a) &= \min \begin{cases} \delta\{r_{a+1}(p[K_R + K_E + V_R^N(0)] + (1-p)\min\{K_R + K_E + V_R^C(0), K_D + K_E + V_D(0)\}) \\ \quad + (1-r_{a+1})(pV_R^N(a+1) + (1-p)V_R^C(a+1)) + m_R(a)\} \\ K_D + V_D(0) \\ K_R + V_R^C(0) \end{cases} \\
V_R^N(a) &= \min \begin{cases} \delta\{r_{a+1}(K_R + K_E + V_R^N(0)) + (1-r_{a+1})V_R^N(a+1) + m_R(a)\} \\ K_R + V_R^N(0). \end{cases}
\end{aligned} \tag{3.4}
$$

with boundary conditions, $V_D(A_D) = \min\{K_D + K_E + V_D(0), K_R + K_E + V_R^C(0)\}$, $V_R^C(A_R) = \min\{K_R + K_E + V_R^C(0), K_D + K_E + V_D(0)\}$, and $V_R^N(A_R) = K_R + K_E + V_R^N(0)$ where $V_D(a), V_R^C(a), V_R^N(a)$ represent, respectively, the minimum expected discounted cost of operation a DMS, RMS to produce current product generation, and RMS to produce next product generation of age a, respectively. Let $K_{DE} = K_D + K_E$ and $K_{RE} = K_R + K_E$. We can then reduce the above DP formulation to

$$
\begin{aligned}
V_D(a) &= \min \begin{cases} \delta\{m_D(a) + p[K_{RE} + V_R(0)] + (1-p)[(1-q_{a+1})V_D(a+1) + q_{a+1}V_D(A_D)]\} \\ K_R + V_R(0) \\ K_D + V_D(0), \end{cases} \\
V_R(a) &= \min \begin{cases} \delta\{m_R(a) + r_{a+1}(K_{RE} + V_R(0)) + (1-r_{a+1})V_R(a+1)\} \\ K_R + V_R(0). \end{cases}
\end{aligned} \tag{3.5}
$$

with boundary conditions, $V_D(A_D) = \min\{K_{DE} + V_D(0), K_{RE} + V_R(0)\}$ and $V_R(A_R) = K_{RE} + V_R(0)$, (i.e., machines have to be replaced at the end of their lifetimes) where $V_D(a)$ ($V_R(a)$) represents the minimum expected discounted cost of operating a DMS (RMS) of age a.

Note that the reduction to the simpler DP follows because under our formulation, it would never be optimal to buy a DMS once an RMS has been bought. Therefore, under our formulation, when the equipment on hand is dedicated, the choice is between buying a new dedicated, new reconfigurable machine or keeping the existing machine one more period. If we keep the existing machine one more period, then the machine might fail next period or the new generation product might be required, in which case, a new machine would be needed on an emergency basis.

We next derive two lemmas which will be useful in proving that the optimal replacement policy is a threshold policy that is a function of the age of the machine.

Lemma 2 *$V_R(a)$ is increasing in a.*

Proof: We prove by induction on a. At $a = A_R, V_{RE} = K_{RE} + V_R(0)$. It follows that $V_R(A_R - 1) = \min\{\delta m_R(A_R - 1) + \delta[r_{A_R} V_{RE} + (1 - r_{A_R})V_R(A_R)], K_R + V_R(0)\}$.

Since $K_R \le K_{RE}$, it implies that $V_R(A_R - 1) \le V_R(A_R)$. Now, we assume the result is also true for $a = A_R - 2, A_R - 3, \cdots k + 1$. We want to show that $V_R(k) \le V_R(k+1)$.
If $V_R(k+1) = K_R + V_R(0)$, then $V_R(k) \le V_R(k+1)$ since $V_R(k) = \min\{\delta[m_R(k) + r_{k+1}V_{RE} + (1 - r_{k+1})V_R(k+1)], K_R + V_R(0)\}$.
Otherwise, if $V_R(k+1) = \delta[m_R(k+1) + r_{k+2}V_{RE} + (1 - r_{k+2})V_R(k+2)]$, then

$$\begin{aligned}
V_R(k) &\le \delta[m_R(k) + r_{k+1}V_{RE} + (1 - r_{k+1})V_R(k+1)] \\
&\le \delta[m_R(k) + r_{k+1}V_{RE} + (1 - r_{k+1})V_R(k+2)] \\
&\le \delta[m_R(k+1) + r_{k+2}V_{RE} + (1 - r_{k+2})V_R(k+2)] \\
&= V_R(k+1).
\end{aligned}$$

The second inequality follows from the induction hypothesis. The third inequality follows from the increasing breakdown rate and increasing maintenance cost assumption. Therefore, the result holds for all $a = 0, \cdots A_R - 1$.

Lemma 3 *$V_D(a)$ is increasing in a.*

Proof: We prove by induction on a. Let $V_{DE} = V_D(A_D) = \min\{K_{RE} + V_R(0), K_{DE} + V_D(0)\}$. Let $M = \min\{K_R + V_R(0), K_D + V_D(0)\} \le V_D(A_D)$. Then $V_D(A_D - 1) = \min\{\delta(m_D(A_D - 1) +$

$p(K_{RE} + V_R(0)) + (1 - p)[(1 - q_{A_D})V_D(A_D) + q_{A_D}V_{DE}]\}, M\}$. Since $M \le V_D(A_D)$, it implies that $V_D(A_D - 1) \le M \le V_D(A_D)$.

Assume the result is also true for $n = A_D - 2, A_D - 1, \ldots, k + 1$. We want to show that $V_D(k) \le V_D(k+1)$.

If $V_D(k+1) = M$, then $V_D(k) \le V_D(k+1)$ since $V_D(k) = \min\{\delta m_D(k) + \delta[p(K_{RE} + V_R(0)) + (1 - p)[(1 - q_{k+1})V_D(k+1) + q_{k+1}V_{DE}]], M\}$.

Otherwise, if $V_D(k+1) = \delta\{m_D(k+1) + p(K_{RE} + V_R(0)) + (1 - p)[(1 - q_{k+2})V_D(k+2) + q_{k+2}V_{DE}]\}$, then

$$\begin{aligned} V_D(k) &\le \delta\{m_D(k) + p(K_{RE} + V_R(0)) + (1 - p)[(1 - q_{k+1})V_D(k+1) + q_{k+1}V_{DE}]\} \\ &\le \delta\{m_D(k) + p(K_{RE} + V_R(0)) + (1 - p)[(1 - q_{k+1})V_D(k+2) + q_{k+1}V_{DE}]\} \\ &\le \delta\{m_D(k+1) + p(K_{RE} + V_R(0)) + (1 - p)[(1 - q_{k+2})V_D(k+2) + q_{k+2}V_{DE}]\} \\ &= V_D(k+1). \end{aligned}$$

The second inequality follows from the induction hypothesis. The last inequality follows from the assumption that q_k and $m_D(k)$ are increasing in k. Therefore, the result holds for all $a = 0, 1, \ldots, A_D - 1$.

From the above lemmas, we can claim that a threshold policy is an optimal policy. This result is stated in the following

Theorem 4 *If a reconfigurable system is currently used, there exists a threshold T_R such that if $a < T_R$, an optimal policy is to keep, and if $a \ge T_R$, the optimal policy is to replace the current RMS with a new RMS. If the current system is dedicated, there exists a threshold T_D such that if $a < T_D$, an optimal policy is to keep, and if $a \ge T_D$, the optimal policy is to replace with a new system (which might be reconfigurable or dedicated).*

Proof: We prove the existence of T_R only since the proof of the existence of T_D is virtually the same. Lemma 2 proves that $V_R(a)$ is increasing in a which is sufficient to show that $\delta\{m_R(a) + r_{a+1}(K_{RE} + V_R(0)) + (1 - r_{a+1})V_R(a+1)\}$ is increasing in a. On the other hand, $K_R + V_R(0)$ is constant in a. The result follows.

We note that, whereas Theorem 4 indicates that once a reconfigurable machine reaches age $T_R + 1$, it is replaced by another reconfigurable machine, in the case of dedicated machines, when one reaches age $T_D + 1$, it could be replaced by a reconfigurable or dedicated machine. Similarly, Theorem 4 does not specify whether, when a dedicated machine fails, it should be replaced by another dedicated or reconfigurable machine. As an example, with $K_D = 1; K_R = 5; K_E = 3; \delta = 0.8; A_D = A_R = 20$ years, (with both DMS and RMS having discrete uniform lifetime distributions), $m_D(a) = m_R(a) = 0$ for all a, we obtain $T_R = 19$. If $p = 0.3$, $T_D = 17$ years and if one has a 17 yr. old DMS, it is optimal to replace it with a new DMS. On the other hand, if $p = 0.6$, we will replace the current DMS with a new RMS, regardless of its age.

We can extend the DP formulation 3.5 by allowing a more general distribution for the time of arrival of the next generation product. Specifically, as in the previous section, let p_n be the probability that the next generation product will arrive in period n given that it has not arrived in periods $1, \ldots, n-1$ where p_n satisfies the discrete version of the IFR requirement. As in the previous section, we further assume that p_n has finite support such that $p_n = 0$ for $n > N$. The state variables in this model are similar to those in the formulation 3.5 with a time variable n. We define $V_D(a, n)$ as the minimum expected discounted cost of operating a DMS of age a at time n. The dynamic program for this extended model is

$$
\begin{aligned}
V_D(a, n) &= \min \begin{cases} \delta\{m_D(a) + p_{n+1} V_{RE} + p'_{n+1}[q_{a+1} V_D(A_D, n+1) + q'_{a+1} V_D(a+1, n+1)]\} \\ K_R + V_R(0) \\ K_D + V_D(0, n), \end{cases} \\
V_R(a) &= \min \begin{cases} \delta[m_R(a) + r_{a+1} V_R(A_R) + r'_{a+1} V_R(n+1)] \\ K_R + V_R(0), \end{cases}
\end{aligned} \tag{3.6}
$$

with boundary conditions $V_D(a, N) = K_{RE} + V_R(0)$ for all a; $V_D(A_D, n) = \min\{K_{DE} + V_D(0, n), K_{RE} + V_R(0)\}$ for all $n < N$; $V_R(A_R) = K_{RE} + V_R(0)$; where $p'_k = 1 - p_k, q'_k = 1 - q_k, r'_k = 1 - r_k, V_{RE} = K_{RE} + V_R(0)$.

As in the previous case, when the current machine is dedicated, there are three choices. We can replace it with a reconfigurable machine, or a new dedicated machine, or we can use it for one more period. If we replace it with a reconfigurable machine, then the arrival of a new product will not affect anything; however, if we replace it with a dedicated machine, the new product might still arrive in the future necessitating another equipment change. Notice that when we own reconfigurable equipment, we only keep track of equipment age, as the failure of the equipment is the only factor that will cause a costly shutdown. However, with dedicated equipment, we also keep track of actual "calendar time" as this will also affect whether the new product arrives. Therefore, in the above formulation, if we buy a new reconfigurable machine, the discounted cost-to-go from that point onwards is given by $V_R(0)$, whereas if we buy a new dedicated machine at time n, the discounted cost to go is given by $V_D(0, n)$. Finally, if we keep our current dedicated equipment for one more period, with probability p_{n+1}, the new product will arrive in period $n+1$, and even if it doesn't arrive, with probability q_{a+1}, the dedicated equipment will fail. If these events do not happen, then we will have an $a+1$ year old dedicated machine at time $n+1$.

Finally, we note that in our formulation, we assume that when a new equipment is ordered, it arrives in the same period. We can easily extend our formulations in this section to the case where equipment arrives one period later and all the results we present would remain the same.

Since $(i)\, 0 \le \delta < 1$; (ii) all costs are uniformly bounded, and (iii) the problem has finite state space and action space, Eqn. 3.6 can be solved by the value iteration algorithm and the algorithm will provide an optimal policy with any initial value of $V_D(a;n)$ for all a and n. (For more details, see Chapter 6 of Puterman(1994).) In fact, note that the values of $V_R(a)$ can be computed separately (by for example using a value iteration algorithm for $V_R(a)$). Consider a value iteration algorithm to solve Eqn. 3.6 to obtain the $V_D(a;n)$ values once the $V_R(a)$ values have been computed and define $V_D^{k+1}(a;n)$ as the $k-th$ iteration value of the cost to go of owning a dedicated machine of age a at time n:

$$V_D^{k+1}(a;n) = \min\left\{W_K^{k+1}(a;n); W_R^{k+1}(a;n); W_D^{k+1}(a;n)\right\};$$

where

$$\begin{aligned} W_K^{k+1}(a;n) &= \delta\{m_D(a) + p_{n+1}V_{RE} + p'_{n+1}[q_{a+1}V_D^k(A_D;n+1) + q'_{a+1}V_D^k(a+1;n+1)]\}; \\ W_R^{k+1}(a;n) &= K_R + V_R(0); \\ W_D^{k+1}(a;n) &= K_D + V_D^k(0;n); \end{aligned}$$

with the boundary conditions: $V_D^{k+1}(a;N) = V_{RE}$ for all a, $V_D^{k+1}(A_D;n) = \min\{K_{DE} + V_D^k(0;n); K_{RE} + V_R(0)\}$ for all $n < N$. Finally define $V_D^0(a;n) = 0$ for all a and n. Also, observe that $V_D^k(a;n) \le V_{RE}$ for all $k; a$ and n.

By Theorem 6.2.5 of Putterman (1994) $V_D(a;n) = \lim_{k\to\infty} V_D^k(a;n)$ for all a and n. Furthermore, any properties that the recursive value functions $V_D^k(a;n)$ possess will also be carry over to $V_D(a;n)$. We use these properties in the proofs of the following two lemmas.

Lemma 4 *$V_D(a;n) \le V_D(a+1;n)$ for all a and n.*

Proof: We use induction to show that $V_D^k(a;n) \le V_D^k(a+1;n)$ for all a and n, and the result follows from the convergence of the V_D^k functions to V_D. Clearly, $V_D^0(a;n) \le V_D^0(a+1;n)$ for all a and n, since $0 \le 0$. Assume the the result is also true for $k = j$. At $k = j+1$,

$$V_D^{j+1}(a;n) = \min\{W_K^{j+1}(a;n); W_R^{j+1}(a;n); W_D^{j+1}(a;n)\};$$

and

$$V_D^{j+1}(a+1;n) = \min\{W_K^{j+1}(a+1;n); W_R^{j+1}(a+1;n); W_D^{j+1}(a+1;n)\}.$$

To prove that $V_D^{j+1}(a; n) \leq V_D^{j+1}(a+1; n)$, it is sufficient to show that $W_K^{j+1}(a; n) \leq W_K^{j+1}(a+1; n)$ since $W_R^{j+1}(a; n) = W_R^{j+1}(a+1; n)$ and $W_D^{j+1}(a; n) = W_D^{j+1}(a+1; n)$.

$$
\begin{aligned}
W_K^{j+1}(a+1; n) &= \delta\{m_D(a+1) + p_{n+1}V_{RE} + p'_{n+1}[q_{a+2}V_D^j(A_D; n+1) + q'_{a+2}V_D^j(a+2; n+1)]\} \\
&\geq \delta\{m_D(a+1) + p_{n+1}V_{RE} + p'_{n+1}[q_{a+2}V_D^j(A_D; n+1) + q'_{a+2}V_D^j(a+1; n+1)]\} \\
&\geq \delta\{m_D(a) + p_{n+1}V_{RE} + p'_{n+1}[q_{a+1}V_D^j(A_D; n+1) + q'_{a+1}V_D^j(a+1; n+1)]\} \\
&= W_K^{j+1}(a; n).
\end{aligned}
$$

The first inequality follows by the induction hypothesis and the second inequality follows since q_a and $m_D(a)$ are increasing in a and $V_D^j(A_D; n+1) \geq V_D^j(a+2; n+1)$ by induction hypothesis. The result is then true at $k = j+1$. Therefore, for every k, $V_D^k(a; n) \leq V_D^k(a+1; n)$ for all a and n. This completes the proof.

The following corollary is a consequence of the proof of Lemma 4.

Corollary 1 *$W_K^k(a; n)$ is increasing in a for all k and n.*

Lemma 5 *$V_D(a; n) \leq V_D(a; n+1)$ for all a and n.*

Proof: Once again, it is sufficient to prove that $V_D^k(a; n) \leq V_D^k(a; n+1)$ for all a and n, and all k. We prove this result by induction on k. This holds trivially at $k = 0$. Assume that the result is also true for $k = j$. At $k = j+1$,

$$V_D^{j+1}(a; n) = \min\{W_K^{j+1}(a; n); W_R^{j+1}(a; n); W_D^{j+1}(a; n)\};$$

and

$$V_D^{j+1}(a; n+1) = \min\{W_K^{j+1}(a; n); W_R^{j+1}(a; n); W_D^{j+1}(a; n)\}.$$

We want to show that $(i) W_K^{j+1}(a; n) \leq W_K^{j+1}(a; n+1)$; $(ii) W_R^{j+1}(a; n) \leq W_R^{j+1}(a; n+1)$; and $(iii) W_D^{j+1}(a; n) \leq W_D^{j+1}(a; n+1)$. However, $W_R^{j+1}(a; n) = K_R + V_R(0) = W_R^{j+1}(a; n+1)$. Therefore, we only need to show that (i) and (iii) are true.

$$
\begin{aligned}
W_K^{j+1}(a; n+1) &= \delta\{m_D(a) + p_{n+2}V_{RE} + p'_{n+2}[q_{a+1}V_D^j(A_D; n+2) + q'_{a+1}V_D^j(a+1; n+2)]\} \\
&\geq \delta\{m_D(a) + p_{n+2}V_{RE} + p'_{n+2}[q_{a+1}V_D^j(A_D; n+1) + q'_{a+1}V_D^j(a+1; n+1)]\} \\
&\geq \delta\{m_D(a) + p_{n+1}V_{RE} + p'_{n+1}[q_{a+1}V_D^j(A_D; n+1) + q'_{a+1}V_D^j(a+1; n+1)]\} \\
&= W_K^{j+1}(a; n).
\end{aligned}
$$

The first inequality follows by the induction hypothesis. The second inequality follows since p_n is increasing in n by assumption and the fact that $V_D^j(a; n) \leq V_{RE}$ for all a and n.

Similarly,

$$\begin{aligned} W_D^{j+1}(a, n+1) &= K_D + V_D^j(0, n+2) \\ &\geq K_D + V_D^j(0, n+1) \\ &= W_D^{j+1}(a, n). \end{aligned}$$

The inequality follows by the induction hypothesis. The result is then true for $k = j + 1$. Therefore, for every k, $V_D^k(a, n) \leq V_D^k(a, n+1)$ for all a and n.

The following corollary is a consequence of the proof of Lemma 5.

Corollary 2 *1.* $W_K^k(a, n)$ *is increasing in* n *for all* a.

2. $W_D^k(a, n)$ *is increasing in* n *for any given* a *and is constant in* a *for any given* n.

Using the above lemmas and corollaries, we can derive the following result which partially characterizes the structure of the optimal policy, when we have a DMS to begin with. The decision to replace or keep an RMS is exactly the same as in Theorem 4.

Theorem 5 *There exists a switching function* $g(a)$ *such that for* $n \geq g(a)$, *purchasing a RMS is preferable to keeping an existing DMS for one more period, and for* $n < g(a)$, *the reverse holds. Furthermore,* $g(a)$ *is decreasing in* a. *There exists a simple threshold* n_R *such that for* $n \geq n_R$, *buying a RMS is preferable to buying a DMS, and for* $n < n_R$, *the reverse holds. Finally, there exists a switching function* $f(n)$ *such that for* $a \geq f(n)$, *buying a new DMS is preferable to keeping an old DMS for one more period, and the reverse holds true for* $a < f(n)$.

Proof: The existence and monotonicity of $g(a)$ follows from the fact that $W_K^k(a, n)$ is increasing in a and n for any k (Corollary 1 and Corollary 2 part 1) whereas $W_R^k(a, n)$ is a constant for all a, n and any k. Similary, the existence of the threshold n_R follows from the fact that $W_D^k(a, n)$ is increasing in n but constant in a for any k (Corollary 2 part 2) while $W_R^k(a, n)$ is a constant in a, n for any k. Finally, the existence of $f(n)$ from Corollary 1 and Corollary 2 part 2. .

Figure 1 shows a typical optimal policy for Eqn. 3.6 when the firm currently has a DMS. In this example, $K_D = 1, K_R = 5, K_E = 3, \delta = 0.8, A_D = A_R = 20$ years, $N = 9$ years, $m_D(a) = m_R(a) = 0$ for all a. Additionally, we assume that the arrival time of next generation product and lifetime of each machine are distributed according to a discrete uniform distribution. As seen in Figure 1, in the early periods, the optimal decision when the current DMS is new is to keep operating the current DMS for one more period. The optimal decision, however, changes to buying a new DMS when the

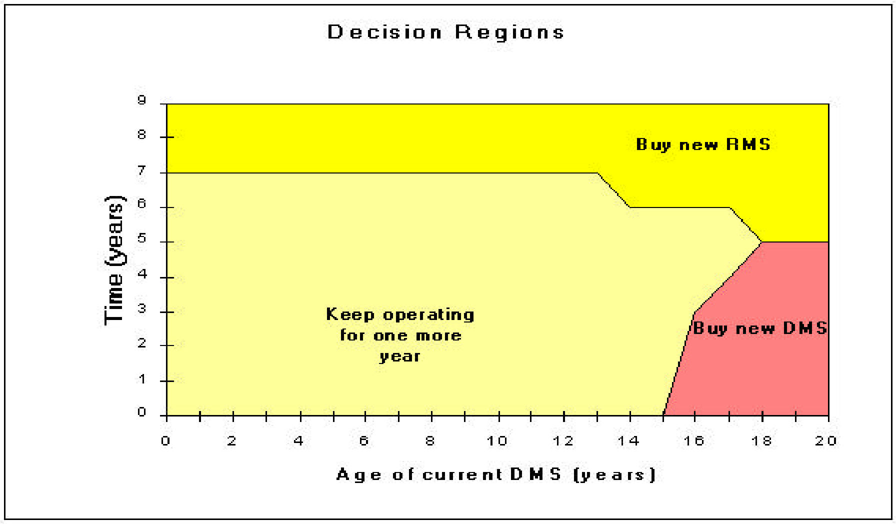

Figure 1: Optimal decision regions when $K_D = 1; K_R = 5; K_E = 3; \pm = 0{:}8$.

age of current DMS is close to A_D. On the other hand, the optimal decision is to replace the current DMS with a new RMS in later periods. Notice that the threshold between keeping the current DMS for one more period and buying a new RMS is monotonic as stated in the above theorem. The threshold between buying a new DMS and buying a new RMS is constant as stated in the above theorem. In this example, the threshold between between keeping the current DMS and buying new DMS is also monotonic but this is not always the case.

Figure 2 provides an example with a nonmonotonic threshold between keeping the current DMS and buying a new RMS. The example includes $K_D = 1; K_R = 3{:}47007; K_E = 6{:}557; \pm = 0{:}81742; A_D = A_R = 20$ periods, (with the distributions discrete uniform again),$N = 9$ periods, and $p_i = 0; i = 0; \ldots; 6; p_7 = p_8 = 0{:}7; p_9 = 1$. As can be seen in Figure re±g:COUNTERX, the threshold between keeping the current DMS and buying a new DMS is no longer monotonic.

4 Numerical Results and Sensitivity Analysis

In this section we will provide some numerical examples to demonstrate how our decision regions are a®ected by problem parameters. We ¯rst provide numerical examples for the case with a geometric probability for new product introduction formulated in Eqn 3.5. Initially, consider the base case with parameters $K_D = 1; K_E = 3; \pm = 0{:}8$, and $A_D = A_R = 20$ years. In all our examples maintenance costs for recon¯gurable and dedicated equipment were zero. The breakdown time of each system is discrete uniformly distributed over its physical lifetime. We are interested in how optimal decisions change as a function of changes in K_R, the cost of new recon¯gurable capacity, and p, the probability

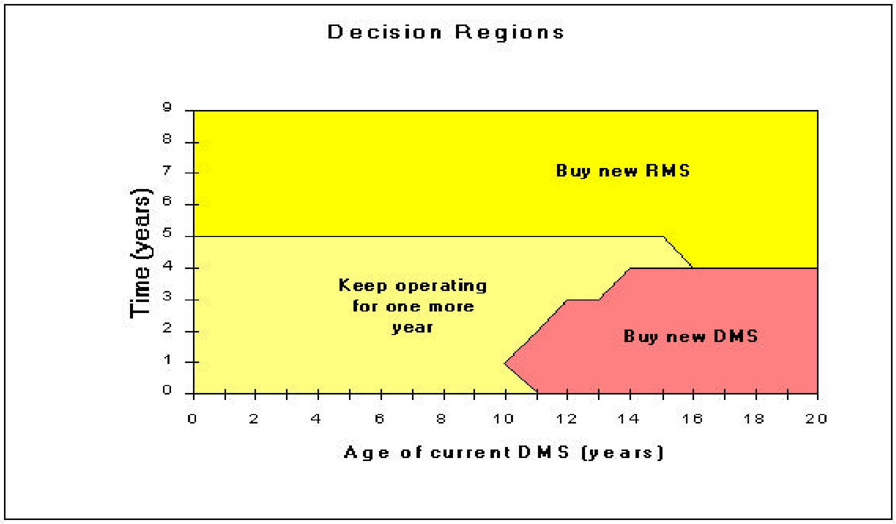

Figure 2: Counterexample to monotonicity of threshold between Keep DMS and Buy DMS.

a new product will be introduced in the next period. We remind the reader that when owning a DMS, there exists a threshold such that if the DMS is older than the threshold age, it is optimal to replace it with either a new DMS or a RMS. In Figure 3, we have plotted how optimal decisions change as a function of changes in K_R for the case with $p = 0.3$. For example, when $K_R = 1$ (i.e., the cost of a new RMS is the same as a new DMS), it is in fact optimal to buy a new RMS immediately regardless of the age of the current DMS. When $K_R = 3$, it is optimal to replace the existing DMS, with a RMS only if the existing DMS is 11 years old or older. Finally, if $K_R = 5$, it is in fact optimal to keep the existing DMS until it is 17 years old (assuming the new product has not been introduced by then) and then to replace it with a new DMS.

In the example above, when p is increased (next generation product arrives sooner on expectation), we intuitively expect that we are more likely to switch from the current DMS to a new RMS. Figure 4 confirms this intuition. This figure plots the optimal decisions as a function of K_R when as $p = 0.6$. It is clear that the region where it is optimal to buy a new RMS has greatly increased. In fact, unless $K_R \geq 8$, we never replace an existing DMS with a new DMS. Furthermore, as long as $K_R \leq 5$, we replace an existing DMS with a RMS regardless of the age of the DMS.

We now consider examples with more complicated distributions for the arrival time of the next generation product. Consider the base case with the parameters given above, and $K_R = 5$. Also, assume that the arrival time of the next generation product is uniformly distributed over $1, \ldots, N = 9$ years. Figure 1 plots the optimal decisions as a function of the age of the existing DMS and time. When the existing dedicated machine is still new and time is close to 0 (i.e., the arrival of the next generation product is still far into the future), it is optimal to keep operating the current dedicated

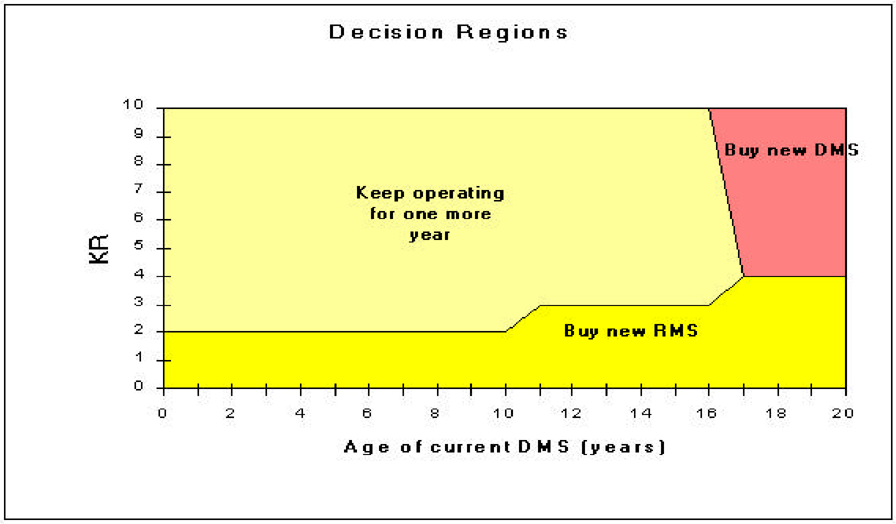

Figure 3: Optimal decision regions with di®erent value of K_R when $p = 0.3$.

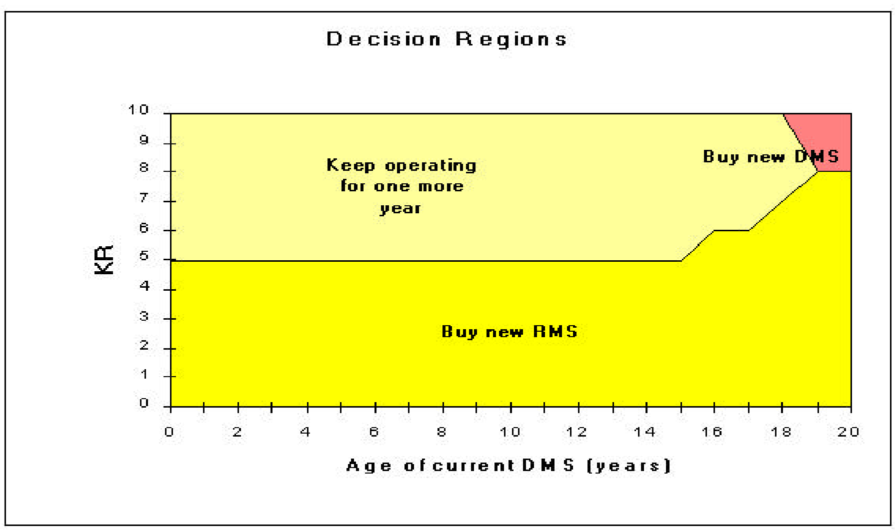

Figure 4: Optimal decision regions with di®erent value of K_R when $p = 0.6$.

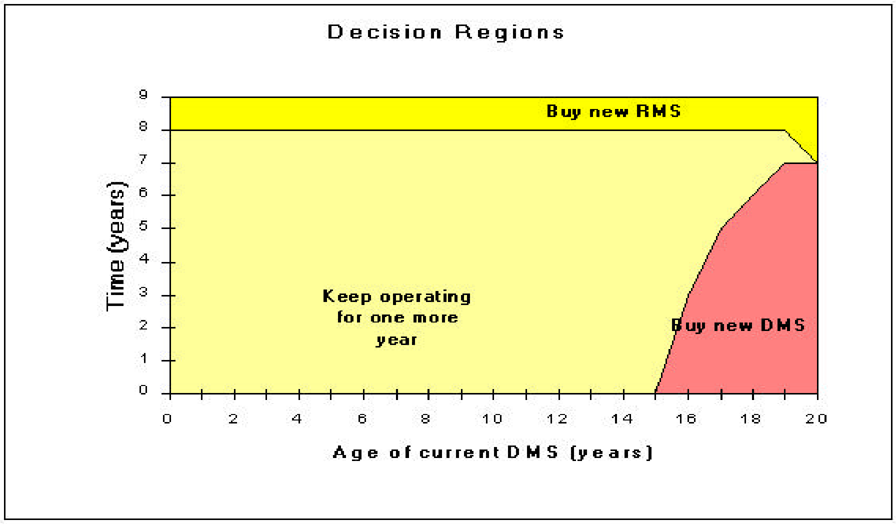

Figure 5: Optimal decision regions when $K_D = 1, K_R = 10, K_E = 3$, and $\delta = 0.8$.

machine for one more period. However, if time is close to N (e.g., 7 years after the start of the problem), the optimal decision is to buy a new reconfigurable machine as the likelihood that the next generation product will arrive soon is very high.) Similarly, when the age of current dedicated machine is close to its physical lifetime, it is optimal to replace it by a dedicated machine in the initial periods (i.e., time close to 0,) and by a new reconfigurable machine in later periods (time close to N).

We are first interested in seeing the effect of a more expensive RMS on the optimal decisions. Figure 5 shows the optimal decisions when K_R increases from 5 to 10. As we would expect, we are more willing to continue operating the current DMS in this case since a new RMS has become very expensive and K_E is relatively small when compared with K_R. In contrast, if K_R is decreased from 5 to 3, we are more willing to replace the current DMS with a new machine sooner as we show in the largely expanded region for buying a new RMS in Figure 6.

Now, once again consider the base case with $K_R = 5$ and assume that the next generation product will arrive at either time $7, 8,$ or 9 with probability 1/3 for each period. The optimal decision regions is shown in Figure 7. What is interesting in this example compared to the base case (shown in Figure 1) where a discrete uniform distribution between 1 and 9 was used for the arrival time of the next generation product is that the optimal policy regions have hardly changed. Whereas in the base case the probability that the next generation product arrives in period k is $1/(10 - k)$, it is 0 in this case for $k = 1, \ldots, 6$. However, the p_k values are the same for the two cases for $k = 7, 8, 9$. This example demonstrates that the model is not overly sensitive to small changes in p, a desirable characteristic since p is typically the hardest parameter to estimate in practice. The optimal policy structure in

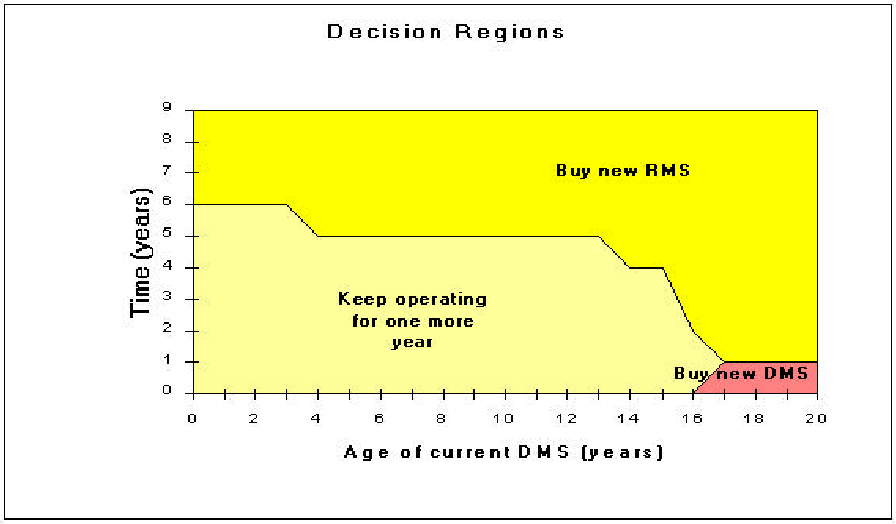

Figure 6: Optimal decsion regions when $K_D = 1; K_R = 3; K_E = 3$, and $\delta = 0.8$.

this case is very similar to the case shown in Figure 1.

We finally consider the effects of changing machine reliability. Consider the same example as in Figure 7 but further assume that the lifetime of DMS is discrete uniform between 10 and 20 (i.e., in this case we have made the machine more reliable than before as we assume that it can not fail in the first 9 periods, and the probability that it will fail in any period after 9 given it has not failed until then is the same as in the previous example shown in Figure 7. Figure 8 shows the optimal policy structure for this case. Comparing the two figures, we note that we tend to keep an existing DMS for a shorter time for the case with improved reliability!! Notice that though this might seem unintuitive at first, it can be explained by the fact that, when we replace, for example, a 12 yr. old DMS in Figure 8 , we are getting a new DMS that is trouble free for at least 9 years. However, in the example in Figure 7, even a new DMS has a fair likelihood of failing and therefore there is not as much point in replacing it. This example shows that changes in parameters might change decision regions in what might at first sight appear to be unintuitive ways.

5 Conclusions and Further Research

In this paper, we considered the question of when it is economically advantageous for a firm to invest in reconfigurable rather than dedicated capacity. Influenced by our work with auto manufacturers, and machine tool builders, we considered models where at most one reconfiguration is required over the lifetime of the equipment. Our models considered both the stochastic nature of the time when the next reconfiguration will arise as well as uncertainty in machine reliability. Our models have

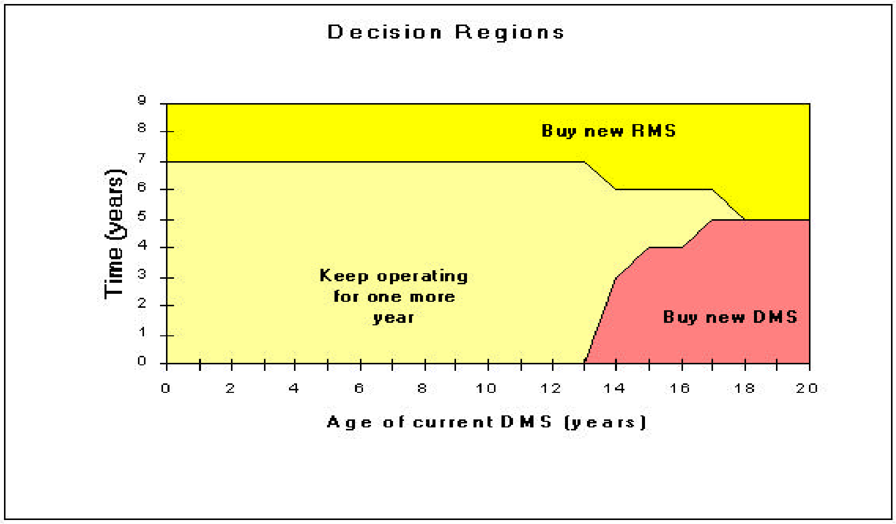

Figure 7: Optimal decision regions when $K_D = 1$, $K_R = 5$, $K_E = 3$, ± = 0:8, and arrival of next generation product discrete uniform between 7 and 9.

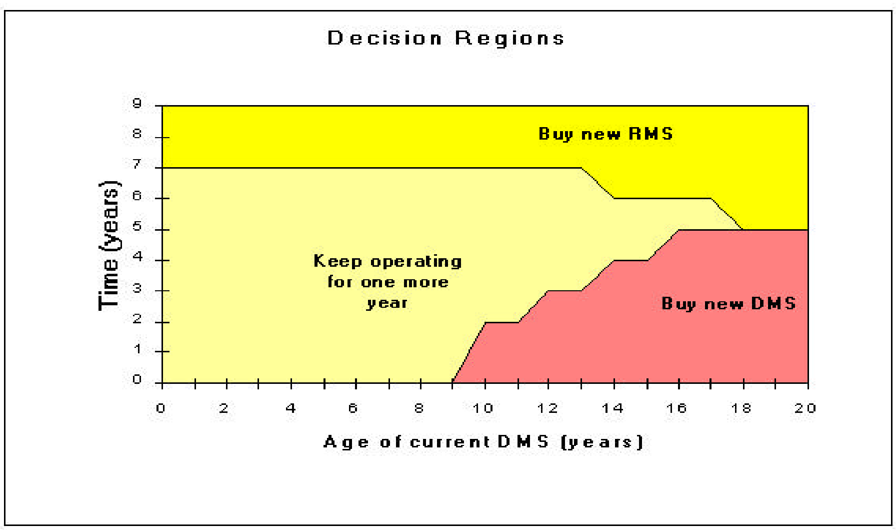

Figure 8: Optimal decision regions for same parameters as in Figure 6 except for improved reliability for DMS

resulted in a spreadsheet model tested thoroughly by our industry partners in their own organizations for decisions involving recon¯gurable machine purchasing decisions. (Interested readers can obtain a copy of the spreadsheet by contacting the authors.) We believe that our models provide an important analytical tool in decisions that auto companies and machine tool makers make in purchasing or building recon¯gurable equipment.

We have also been able to characterize the structure of the optimal policy analytically and shown the e®ect of changes in problem parameters through numerical examples. For our most complicated model allowing general distributions for machine failures as well as time of next product arrival, the optimal policy (when current equipment is a DMS) is characterized by two switching curves (one of them monotonic) and a threshold.

Our models indicate the key parameters that ¯rms need to take into account when deciding to invest in recon¯gurable machinery. As discussed previously, investment in recon¯gurable machinery can be viewed as a hedging strategy where the ¯rm is hedging against the probability that the ¯rm will have to introduce a new model that can not be produced by the current machinery. Our models explore when such hedging makes sense. As our discussion in Section 4 indicates, hedging by investing in recon¯gurable machinery becomes more appealing as

1. the probability that a model change will occur increases
2. the cost of hedging (i.e., the cost of recon¯gurable equipment versus new dedicated equipment) decreases
3. the cost of being caught without the capability to produce the new model increases
4. the age of the current dedicated equipment increases thus increasing the likelihood of a catasphoric failure as well as maintenance costs.

Although our models serve as a useful ¯rst tool for understanding the basic tradeo®s between purchasing a new DMS, RMS or keeping old equipment, (and have been useful in the automobile industry), further research would extend their applicability to greater domains. For example, an interesting extension would consider the possibility of several di®erent product introductions over the planning horizon. Also, models that allow for equipment prices to change over time would also be interesting. (We note that our model formulations can be easily changed to allow equipment prices that are a function of time, however our structural results will not survive this change.) Also, we assume in this paper, that the demand for the two generation products are the same. Models that consider stochastic nonstationary demand would be of interest. This will lead to a much more complicated formulation than the one we presented as the decision maker now has to decide on a whole portfolio of machines.

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