The Role of Wholesale-Salespersons and Incentive Plans in Promoting Supply Chain Performance

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Abstract

Almost all research into the operations and behavior of salespeople has focused on sales at the retail level. But in many supply chains, manufacturers use a sales force to promote sales and improve coordination with retailers. In this note, we add an important piece to the science of sales operations by investigating the impact of two primary tasks performed by wholesale-salespersons: (1) enhancing retail demand, and (2) convincing the retailer to order more stock (i.e., enhancing wholesale demand). We show how the sales force can play an important role in supply chain coordination, even when it does not directly promote retail level demand. We also examine the relative effectiveness of the following compensation schemes in salesperson performance: (1) a salary plan in which the salesperson's effort is observable (first-best salary), and (2) a commission plan based on retailer order quantity. We find that although the commission plan is less efficient than the first-best salary plan in motivating the salesperson, it is more robust to uncertainty in performance and motivation parameters. Finally, we compare a wholesale-salesperson with a traditional retail-salesperson and conclude that although a retail-salesperson is more efficient at increasing supply chain profit than is a wholesale-salesperson, the wholesale-salesperson raises manufacturer profit more than does a retail-salesperson (even if both promote the same retail demand). Furthermore, we show that wholesale-salespersons promote higher wholesale and retail prices than do retail-salespersons.

Keywords: Salesforce, Principle-Agent Problem, Supply Chain Coordination, Compensation

1 Introduction

A salesforce represents a major investment for most firms, typically costing from 5 to 40 percent of sales (Zoltners et al. 2001). The majority of research on salesforce management has focused on *retail salespersons* who market directly to customers. In this paper, we study a different kind of salesperson – a *wholesale-salesperson* who markets products or services in a business-to-business setting. The sole responsibility of a retail-salesperson is to promote retail demand (sales). In contrast, a wholesale-salesperson promotes both wholesale demand by influencing ordering decisions of the downstream firms (e.g., retailers) and final retail demand by helping to improve sales and marketing practices.

For example, in the automobile industry, manufacturers sell vehicles to dealers who sell them to customers. Wholesale-salespersons serve as the primary contact between the manufacturer and the dealers, while retail-salespersons sell cars directly to customers. We performed a six-month field study of the vehicle salesforce at a major auto manufacturer and observed the activities of wholesale-salespersons. In addition to a host of specific job activities, we noted that:

- Wholesale-salespersons working for automobile manufacturers play two key roles: (i) promoting wholesale demand (i.e., convincing dealers to take more cars than they would on their own), and (ii) promoting demand at the customer level (i.e., helping dealers to enhance their sales),
- Wholesale-salespersons working for the automobile manufacturers are paid on a strict salary basis, but are evaluated with respect to monthly sales targets. Previously, our client firm offered commissions to these salesperson but apparently that these were ineffective and discontinued them.
- Retail-salespersons working for the dealer are almost universally paid through compensation schemes that make heavy use of commissions.

These observations suggested to us that there is something fundamentally different about wholesale-salespersons and retail-salespersons, which might impact the relative effectiveness of salary and commissions as compensation schemes. In this note, we examine this difference by creating a model that represents the manufacturer, dealer and salesperson as three separate, utility-maximizing parties, and use it to investigate the following questions:

1. What is the role of a wholesale-salesperson in coordinating a supply chain and how can the manufacturer motivate the wholesale salesperson to improve supply chain performance?
2. How efficient and robust is a commission plan in cases where the wholesale-salesperson's task is to promote retail demand compared with the case where the wholesale salesperson's task is to push more inventory downstream (i.e., promoting wholesale demand)?
3. Which type of salesperson has more impact on supply chain performance, a wholesale-salesperson or a retail-salesperson?
4. Is a commission plan more effective in motivating a retail salesperson or a wholesale-salesperson? For which of these is the commission plan more robust with respect to uncertainty in the salesperson's effectiveness/disutility?

We show that the presence of a wholesale-salesperson can make both the manufacturer and the dealer better off by either promoting wholesale demand (even if this does not result in stronger retail demand), or by promoting retail demand, or both. The influence of a wholesale-salesperson on wholesale demand (i.e., the retailer's order quantities) raises the possibility that such a salesperson can play a role in supply chain coordination. It is well known that decentralized supply chains are inefficient due to double-marginalization. Such inefficiency can be reduced through better coordination of the decisions by various parties. Considerable research has examined ways to achieve better objective alignment through risk sharing and demand/supply information sharing (see de Kok and Graves 2003 for a literature review). Our paper is the first of which we are aware that describes the role of a wholesale-salesperson in supply chain coordination.

The other contribution of our paper is a comparison of the efficiency and robustness of commission plans for wholesale- and retail-salespersons. Our results provide evidence that a commission plan is more efficient in maximizing total supply chain profits when applied to retail-salespersons than when applied to wholesale-salespersons. While this may seem logical, given that wholesale-salespersons are one level removed from the customers, it is not obvious. Indeed, our client had to discover this result the hard way (i.e., by offering commissions and being unsatisfied with the impact they had on wholesale-salesperson performance).

2 Literature Review

The literature relevant to the role of salespeople in supply chains can be classified into four streams: (1) supply chain coordination, and (2) agency problem research (in economics, marketing and operations management), (3) the interface between sales and operations, and (4) vertical control problems.

Extensive research has been devoted to improving supply chain efficiency through coordination via contracts (see Cachon 2003). For example, in a single manufacturer/single retailer newsvendor setting, researchers have shown that a number of contract mechanisms, including buybacks, quantity-flexibility, sales-rebate contracts, and revenue sharing, can coordinate retailer order quantity (e.g., Tsay et al. 1998). However, for systems in which firms (manufacturer and retailer) have price-setting power, Cachon (2003) showed that most of these mechanisms distort retailer incentives and fail to coordinate the supply chain. In the setting of this paper, where both the manufacturer and the retailer have price-setting power, we show that hiring a wholesale-salesperson can result in supply chain coordination.

Several disciplines, including economics, marketing and management science/operations research, have studied the agency problem of how to motivate agents with misaligned objectives. Specific research topics have included the impact of moral hazard (e.g., Holmstrom and Milgrom 1987), allocation of a salesperson's effort across multiple tasks (e.g., Meyer and Vickers 1997), relative performance evaluation and team impact (e.g., Hansen 1997), and reputation in multi-period environment (Lazear and Moore 1984). In economics, the majority of work on the principal-agent problem has assumed pricing and production decisions to be exogenous and independent of incentives, which limits applicability in operations management settings. In marketing, the pioneering work is that of Farley (1964), which studied the alignment of the objectives of the salesperson and the firm, with the goal of eliminating the need for costly surveillance and monitoring (see Coughlan 1993, and Misra et al. 2003 for reviews of the resulting stream of research). Work in the marketing area is (i) primarily aimed at the retail sales setting, and (ii) focuses on the relationship between the salesforce and the firm, without consideration of the effect of salespeople in supply chain relationships and market demand.

A limited amount of work has been done on the interface between sales and operations management. We found that almost all of this work has been directed at facilitating inventory and production decisions. Porteus and Whang (1991) explored the tension between manufacturing (focus on efficiency) and marketing (focus on customer satisfaction), and proposed an incentive structure that optimally delegates the stocking decision to the marketing manager. They also provided a comprehensive literature review of early work on the manufacturing/marketing interface. More recently, Chen, considered a system with a manufacturer and a salesperson (who directly sells products to customers) and proposed compensation packages that motivate the salesperson's effort to smooth demand (Chen 2000) and to disclose market information (Chen 2005) with the goal of assisting production decisions. Our paper differs from Chen (2000) and Chen (2005) in two regards: (i) our paper addresses a non-retail (supply chain) setting, and (ii) prices are not exogenously given but are instead decision variables of the firms.

Although in the OM literature supply chain problems are seldom presented in the context of vertical control theory, the economics and industrial organization literature often use this concept to depict supply chains (see Tirole 1998). Examples of studies that use vertical control to analyze supply chains are Bresnahan et al. (1985) on dealers and manufacturer margin, Gallini et al.(1983) on monopolistic competition, Greenhut and Ohta (1976,1979) and Schmalensee (1973) on mergers and vertical integration.

Our model, similar to most supply chain models in the OM literature, can be considered as a version of a vertical control problem with a third party, i.e., the salesperson. However, what distinguishes our contribution from the existing literature on vertical control is that we focus on the behavioral dynamics of the third party. Specifically, we study how different compensation plans motivate the third party and affect its impacts in coordinating the supply chain. We also study how these plans differ in motivating the two different types of salespersons, i.e., the wholesale- and the retail-salespersons. To the best of our knowledge, these issues have not been addressed in the vertical control literature.

3 Model Basics

We consider a single-product supply chain with an upstream manufacturer (e.g., auto maker) and a downstream retailer (e.g., dealership), in which a salesperson is hired by the manufacturer to increase the demand either at the customer level or at the retail level. We assume that the manufacturer's production capacity and quantity are exogenous long-term decisions. This is reasonable in the auto industry where these decisions are usually determined in advance of pricing and ordering decisions because they require considerable capital investment and scheduling commitment.

We model this system as a single-period ordering and pricing problem. Facing a fixed production level x (with production cost c per unit), the manufacturer chooses wholesale price p_w while the retailer selects order quantity Q and retail price p_r. Both the manufacturer and the retailer seek to maximize their respective profits. In addition to pricing, the manufacturer can influence the retailer's decisions via the wholesale-salesperson's selling effort $e(\geq 0)$. It induces this effort by paying the wholesale-salesperson a commission rate of $\alpha(\geq 0)$ and a fixed amount S. At the end of the period, left over inventory incurs a unit holding cost h_w for the manufacturer and h_r for the retailer. To be complete, we assume unmet dealer orders cost the manufacturer s_w per unit and unmet market demand costs the retailer s_r per unit.

We assume the following sequence of events: (1) the manufacturer determines its production level x, which is known by the salesperson, the wholesale price p_w, and the commission plan parameters (a commission rate α and a fixed amount S), (2) the salesperson decides whether to accept the compensation package offered by the manufacturer, and if so, chooses her effort level e, (3) the retailer decides whether to accept the supply chain contract, and if so, orders quantity Q and then sets retail price p_r, and finally (4) market demand is realized and customers make purchases based on retail price.

We model demand $D(p_r)$ as a linear function of the retail price:

$$D(p_r) = a - bp_r \tag{1}$$

where a is the base demand and b is the demand elasticity $(a, b \geq 0)$. Following the work of Corbett and Karmarkar (2001) and Weng (1995), we assume deterministic linear demand. Although this demand function is simple, it captures the essential dynamics of pricing and market demand. We also assume a symmetric information environment in which the cost parameters, profit and utility functions are known to all parties in the supply chain. However, since exact values of the salesperson's effectiveness and disutility parameters would be difficult to obtain in practice, we also conduct a series of numerical analysis that examine the robustness of different compensation plans with respect to errors in these estimates.

3.1 Objectives of Individual Parties

3.1.1 Manufacturer's Problem

Given a fixed production level x, the manufacturer sets the wholesale price p_w and compensation payment W based on the commission plan parameters (the commission rate α and the fixed amount S) in order to maximize its profit. We define $\Pi_w(p_w, \alpha, S)$ to be the manufacturer's profit, which can be expressed as:

$$\Pi_w(p_w, \alpha, S) \quad = \quad p_w \min\{Q(p_w, e), x\} - cx - h_w[x - Q(p_w, e)]^+ - s_w[Q(p_w, e) - x]^+ - W(\alpha, S, e), \qquad (2)$$

where $Q(p_w, e)$ is the retailer order quantity when the wholesale price is p_w, the salesperson spends effort e on her task and $W(\alpha, S, e)$ is the compensation paid to the salesperson based on a fixed amount S and at a commission rate α for an effort level e devoted to the task of demand promotion or inventory allocation.

We assume that the production level x is greater than the retailer's optimal order quantity without a salesperson Q_{ns}. In other words, the manufacturer will hire a salesperson to perform demand promotion activities only when its production level is sufficient to more than meet the retailer's order in the absence of a salesperson.

3.1.2 Salesperson's Problem

The salesperson, who is aware of the manufacturer's production level x, wholesale (p_w) and compensation decisions, exerts effort e on her selling task. This results in a retailer order of size $Q(p_w, e) \leq x$.[1] We model the salesperson as a risk neutral agent who seeks to maximize utility, given by

$$U(e) = C_1 W(\alpha, S, e) - C_2 V(e),$$

where $W(\alpha, S, e)$ is the compensation that results from commission rate α, fixed amount S, and effort level e; $V(e)$ is the disutility of exerting effort e; C_1 is the utility per commission dollar earned, and C_2 is the coefficient of disutility ($C_1, C_2 \geq 0$). For simplicity, we normalize utility units so that $C_1 = 1$ and $C_2 = C$. To represent $V(e)$ as convex and increasing in effort (see Baker 1992), we assume throughout that $V(e) = e^2$ (see Kalra et al. 2003). Finally, we define the salesperson's individual rationality (IR)

[1] Note that it does not make sense for the manufacturer to set wholesale price, or for the salesperson to exert effort, such that the resulting order quantity is greater than x, the manufacturer's production level.

constraint as $U^*(e) \geq U_{min}$, where U_{min} is the minimum utility needed to retain the salesperson. In practice, this retention utility, U_{min} can be interpreted as the utility of the salesperson's best outside option.

3.1.3 Retailer's Problem

In response to the manufacturer's pricing decision and the salesperson's selling effort, the retailer determines his order quantity Q and retail price p_r to maximize profit. The retailer's problem is a price-setting problem (Petruzzi and Dada, 1999).

We define $\Pi_r(p_r, Q)$ to be the retailer's profit, which can be written as:

$$\Pi_r(p_r, Q) \quad = \quad p_r \min\{D(p_r), Q(p_w, e)\} - p_w Q(p_w, e) - h_r[Q(p_w, e) - D(p_r)]^+ - s_r[D(p_r) - Q(p_w, e)]^+. \tag{3}$$

Equation (3) is general; however, it can be easily shown that the optimal order quantity for the retailer is an inventory clearing quantity, i.e., $Q^* = D(p_r)$. Therefore, Equation (3) reduces to:

$$\Pi_r(p_r) = \Pi_r(p_r, Q^*) \quad = \quad (p_r - p_w)Q^*.$$

Moreover, since $Q \leq x$ and $Q^* = D(p_r)$, the manufacturer's profit (Equation (2)) reduces to

$$\Pi_w(p_w, \alpha, S) = p_w Q(p_w, e) - cx - h_w\big(x - Q(p_w, e)\big) - W(\alpha, S, e)$$

The retailer's individual rationality (IR) constraint to participate in the contract is $\Pi_r^*(p_r, Q) \geq R_{min}$, where $R_{min} \geq 0$ is the minimum acceptable profit for the retailer. We assume that this minimum acceptable profit is less than or equal to the total supply chain profit in a centralized supply chain ($R_{min} \leq \Pi^*_{total,ct}$).

Total supply chain profit is the sum of the monetary income of the manufacturer, the retailer and the salesperson:

$$\Pi_{total}(p_w, p_r, \alpha, S, Q, e) = \Pi_w(p_w, \alpha, S) + \Pi_r(p_r, Q) + W(\alpha, S, e)$$

3.2 Compensation Plans

Compensation is one of the most important means for motivating salespeople. Hence, the structure of the compensation plan is an important decision for the manufacturer. In this section, we examine the effectiveness of a linear commission plan for a supply chain with a wholesale-salesperson.

To be viable, a compensation must be feasible according to the following definition.

Definition 1: *In a supply chain with a salesperson, a compensation plan is feasible if there exists a solution such that, under the production level and parameter settings, (i) the salesperson minimum utility is met (i.e., $U^*(e) \geq U_{min}$), and (ii) the retailer minimum profit is met (i.e., $\Pi_r^*(p_r, e) \geq R_{min}$).*

Note that the existence of a feasible plan does not mean that the manufacturer can always increase his/her profit if it hires a salesperson. In On-Line Appendix B, we show that there exists a threshold on the salesperson's minimum utility above which it is not beneficial for the manufacturer to hire a salesperson.

3.2.1 First-Best Salary Plan

We first establish a benchmark case in which effort is assumed to be observable and contractible. Under this plan: (i) the manufacturer informs the salesperson about his production level x, (ii) the manufacturer specifies the salesperson's effort level e and offers a fixed salary $W(\alpha, S, e) = W(e)$ as compensation, (iii) the salesperson compares the compensation with her minimum utility, and accepts the contract (plan) if $U(e) \geq U_{min}$. Because it assumes effort is completely observable and minimum salesperson utility is known, this compensation plan gives the manufacturer complete control over the salesperson's effort. It is, by definition, the most effective feasible plan from the manufacturer's perspective. Hence, this benchmark case is also the first-best solution to the principal-agent (manufacturer-salesperson) problem, which we label the *first-best salary plan*.

Note that the first-best salary plan is in fact an effort-contingent plan that can be implemented only when the salesperson's effort is observable/contractable. Since the manufacturer can contract and observe the salesperson's effort, it is easy to show that regardless of whether the salesperson spends her effort on retail demand promotion or on wholesale demand promotion, the salesperson's utility is always at her minimum retaining level under the first-best salary plan.

In practice, the performance of a first-best salary plan can be achieved when (i) the manufacturer and the salesperson are the same person, (ii) the manufacturer can price the business relationship as a project and sell it to the salesperson, or (iii) salary and sales (and hence effort) expectation are well-defined with clear consequences (e.g., ex-post penalties are charged). Our client's policy of paying wholesale-salespersons a fixed salary and holding them to sales volume targets resembles the first-best salary plan, but only approximately, since neither effort nor minimum salesperson utility are observable.

3.2.2 Quantity Based Commission Plan

Because effort is not usually observable or contractible, firms often use commission plans based on observable sales quantities. To allow us to evaluate such plans, we assume: (i) the manufacturer informs the salesperson about his production level x, (ii) the manufacturer tells the salesperson that her salary is a commission plan in which she receives a fixed amount S and α for each unit ordered by the retailer, i.e., $W(\alpha, S, e) = S + \alpha Q(p_w, e)$, (iii) the salesperson either accepts the contract or rejects it, depending on the amount of effort she plans to spend. With these, we can state the salesperson optimization problem as :

$$\max_{0 \le e \le 1} \quad U(e) = \quad \big(S + \alpha\ Q(p_w, e)\big) - C\ e^2$$

3.3 Effect of Salesperson's Effort

We model the effect of *retail demand promotion* by the salesperson via an increase in the base demand parameter a, which is a linear function of the salesperson's effort.

$$a = a_0\big(1 + \lambda_m e\big) \tag{4}$$

Hence, a_0 is the base demand without salesperson's promotional effort, e is the amount of salesperson's demand promotion effort, and $\lambda_m(\ge 0)$ is the salesperson's effectiveness at demand promotion.

We represent the additional units ordered due to the salesperson's effort on *wholesale demand promotion* (i.e., increasing the retailer's order size) as $\Delta(e)$. Hence, if the retailer's optimal order quantity without a salesperson is Q_{ns}, the actual order quantity he will place is $Q = Q_{ns} + \Delta(e)$.

Similar to the sales-response formulation in Lucas et al. (1975), and Kalra et al. (2003), we model the additional order quantity as a linear function of salesperson effort:

$$\Delta(e) = \lambda_i e$$

where $\lambda_i(\ge\ 0)$ parameterizes the salesperson's effectiveness in convincing the retailer to increase his order size. In practice, λ_i is positively correlated with how convincing and skillful the salesperson is, and also depends on the level of confidence and trust the retailer has towards the salesperson.

Beyond increasing the retailer order quantity, the effort of the salesperson will have two other impacts: (i) the retailer will adjust the retail price, and (ii) the manufacturer will adjust the wholesale price.

For the remainder of the paper, we label the supply chain with a salesperson who devotes her effort to retail demand promotion as the "demand promotion model" and the case where she focuses on promoting wholesale demand (which results in moving more inventory downstream) as the "inventory allocation model."

4 Role of the Salesperson in Supply Chain Coordination

We can now examine the impact of the wholesale-salesperson on the performance of the supply chain in both the demand promotion and the inventory allocation models, and under both the first-best salary plan and the linear commission plan.

4.1 Inventory Allocation Model

In this section we focus on supply chains in which the role of the salesperson is to promote wholesale demand. To promote wholesale demand (i.e., convince retailers to order more vehicles than they would on their own), we observed that salespersons made use of two basic types of policies: information and incentives. Information policies consisted of providing dealers with information that might convince them to order more vehicles. This included demand forecasts carried out by the manufacturer, sales figures for dealers of other brands in the region, and price elasticity data to help dealers project profits from various price points. Incentive policies consisted of rewards for meeting sales targets. These included plaques, trips, prizes and other considerations from the manufacturer at the end of the year.

While it is obvious why a manufacturer who has excess inventory would want the retailer to order more, it is not so clear that elevating retailer order quantities will benefit the retailer or the supply chain as a whole. In this section, we evaluate the impact of a salesperson whose sole role is to encourage the retailer to order more inventory.

Theorem 1 *In the inventory allocation model under either a feasible first-best salary plan or a feasible commission plan, if it is beneficial for the manufacturer to hire a salesperson, then*

(*i*) *the resulting optimal retailer profit can be greater than that in a supply chain without a salesperson.*

(*ii*) *the resulting total supply chain profit is at least as large as that in a supply chain without a salesperson,*

The above result offers an interesting link between the behavioral dynamics of organizational relationships and the analytic dynamics of supply chains. By building a relationship of trust with a retailer, a manufacturer's sales representative can persuade the retailer to give up his myopic optimum in favor of an ordering policy that is more attractive to the manufacturer. But, because of double marginalization in the decentralized supply chain, the new solution (i.e., higher volume, lower retail price) turns out to be more profitable for both the retailer and the manufacturer. The additional profit also covers the compensation of the salesperson.

From the perspective of supply chain coordination, the wholesale-salesperson acts like a quantity discount contract to coordinate the supply chain. But, since a salesperson can devote different levels of effort to different retailers, she is analogous to a customized system of quantity discounts, which offers different discounts to different retailers. Such differentiation can clearly improve performance. However, since the Robinson-Patman Act prohibits preferential pricing[2], such differentiation between retailers is not possible via contracts. Hence, wholesale-salespersons may be preferable to contracts as supply chain coordination mechanisms in some settings.

4.2 Demand Promotion Model

In this section we focus on supply chains in which the role of the salesperson is to promote retail demand (i.e., demand at the customer level). In out field study we observed that, wholesale-salespersons practiced a variety of techniques (which varied among individuals) to help dealers promote retail demand. Some of them emphasized improving the sales techniques (e.g., learning the best selling points of each vehicle, comparing vehicles with competitors from other brands, matching option packages to specific customers, methods of price negotiation, etc.) of retail-salespersons at the dealership, while others stressed marketing methods (e.g., local print and electronic advertising, planning of sales events, layout of marketing materials inside the dealership, etc.) by the dealership itself.

Similar to the case of inventory allocation, we can demonstrate the following theorem.

Theorem 2 *In the demand promotion model under either a feasible first-best salary plan or a feasible commission plan, if it is beneficial for the manufacturer to hire a salesperson, then*

(*i*) *the resulting optimal retailer profit is always greater than that in a supply chain without a salesperson.*

(*ii*) *the resulting total supply chain profit is always greater than that in a supply chain without a salesperson,*

5 Effectiveness of the Commission Plan

To deepen our understanding of the salesperson's role in a supply chain, we explored the relative effectiveness of the commission plan through a numerical study. We considered 2160 cases for the demand promotion setting and 2160 cases for the inventory allocation setting generated by considering a wide range for system parameter values (see On-Line Appendix A for the details of the experimental design).

5.1 Commission Plan versus First-Best Salary Plan

To gain insight into the efficiency of the commission plan, we compared the performance of the first-best salary plan with that of the commission plan. We define the *inefficiency* of the commission (COM) plan

[2]Note that the Robinson-Patman act only applies to pricing, not salesperson's effort. Nothing in the act would prevent the manufacturer from having the salesperson spend a lot more time (under demand promotion model or inventory allocation model) at one retailer than at another.

in motivating the salesperson relative to the first-best salary (FBS) plan as follows:

$$\Gamma^{total} = \frac{\Pi^{total}_{COM} - \Pi^{total}_{FBS}}{\Pi^{total}_{FBS}} \tag{5}$$

Our numerical study shows that, under the demand promotion model, the average and maximum inefficiency of the commission plan based on the total supply chain profit ($\Gamma^{total,demand}$) are -9% and -14%, respectively. Under the inventory allocation model, the average and maximum inefficiency ($\Gamma^{total,inventory}$) are -4%, and -11%, respectively. This leads us to conclude:

Observation 1: *The commission plan is less efficient in motivating the wholesale-salesperson in the demand promotion model than in the inventory allocation model.*

It is interesting to see that the inefficiency of the commission plan in the inventory allocation model is almost half of that in the demand promotion model. The reason is that in the demand promotion model, the impact of increasing total demand on total supply chain profit is much greater than the impact of pushing stock downstream in the inventory allocation model. Thus, replacing a commission plan with the first-best plan will have a more significant impact on the supply chain profit if the wholesale-salesperson performs demand promotion than if she promotes inventory allocation.

5.2 Sensitivity Analysis

To further examine the practical differences between the first-best salary plan and the commission plan, we examined the robustness of the compensation plans with respect to (unavoidable) errors in salesperson parameter estimates. Specifically, we consider cases in which the manufacturer does not know the actual values of the salesperson's parameters. Instead, the manufacturer must use estimates of these parameters. We compared supply chain performance as optimized by the manufacturer for cases with and without knowledge of the actual values of salesperson parameters in order to evaluate the sensitivity to manufacturer's errors in estimating these parameters.

In each case of our numerical study, we computed the optimal decisions for the retailer and the manufacturer assuming they optimize based on the *estimated* salesperson parameters. However, the salesperson optimizes based on her *actual* parameters. Then, we fed these pricing, ordering and effort decisions into the supply chain and compared the system performance with the situation where the retailer and the manufacturer know the actual salesperson parameters. We observed the errors in the estimates of the salesperson's responsiveness to a compensation plan (which is represented by the ratio of the salesperson's effectiveness and her disutility attitude, i.e., $\frac{\lambda^2}{C}$) ranging from -50% to $+50\%$ in increments of 10%. We define the *robustness* of a compensation plan based on the manufacturer profit

(ρ_w) and the total supply chain profit (ρ_{total}) as follows:

$$\rho_w = \frac{\Pi_{w,estimate} - \Pi_{w,actual}}{\Pi_{w,actual}} \qquad \text{and} \qquad \rho_{total} = \frac{\Pi_{total,estimate} - \Pi_{total,actual}}{\Pi_{total,actual}}$$

where $\Pi_{w,estimate}$ ($\Pi_{total,estimate}$) is the optimal manufacturer profit (total supply chain profit) achieved when the manufacturer and the retailer only have an estimate of the salesperson parameters, and $\Pi_{w,actual}$ ($\Pi_{total,actual}$) is the optimal manufacturer profit (total supply chain profit) achieved when the salesperson parameters are known to all parties.

We found that, in both the demand promotion and the inventory allocation models, the commission plan is more robust than the first-best salary plan with respect to errors in salesperson parameters. Specifically, we found that, the average robustness in terms of manufacturer profit and the average robustness based on total supply chain profit for the first-best salary plan are -112% and -83% under the inventory allocation model, and -138% and -107% under the demand promotion model. These numbers are -34% and -29% for the commission plan under the inventory allocation model, and -39% and -33% for the commission plan under the demand promotion model.

This suggests that the commission plan is more robust than the first-best salary plan to errors in the estimate of salesperson responsiveness. The reason is that payment to the salesperson is proportional to the *results* of her effort as opposed to the effort level itself. Even though the commission rate offered by the manufacturer may not be optimal, the salesperson still has control over her compensation level under the commission plan, and hence is motivated to adjust her effort level to maximize her income according to her actual parameter value. However, the first-best salary plan uses a fixed payment scheme, which makes it quite rigid. For a specific effort level, if the manufacturer offers too high a salary, the salesperson will enjoy a large surplus. If the manufacturer offers too low a salary, salesperson will not be willing to participate in the plan. This makes the first-best salary plan less robust than the commission plan. Hence, we conclude the following:

Observation 2*: In both the demand promotion and the inventory allocation models, the commission plan is more robust than the first-best salary plan with respect to uncertainty in salesperson effectiveness/disutility attitude.*

Despite the fact that the first-best salary plan has the potential to perform better than the commission plan, our results suggest that if the manufacturer does not have very good knowledge of salesperson characteristics, the relative robustness of the commission plan may offset its potential inefficiency.

6 Wholesale versus Retail Salesperson

A wholesale-salesperson differs from a traditional retail-salesperson by playing an intermediary role in allocating inventory within a supply chain. Although the wholesale-salesperson is paid by the manufacturer, her performance is influenced by the decisions of both the manufacturer and the retailer. In contrast, a retail-salesperson is paid by the retailer and her performance is only directly influenced by the retailer. In a vehicle supply chain, a retail-salesperson works for a dealership and sells directly to customers, while a wholesale-salesperson works for the manufacturer and sells to the dealerships, but may also help promote retail demand. In this section, we contrast the performance and compensation effectiveness of a wholesale-salesperson with those of a retail-salesperson.

To examine the performance of a retail-salesperson, we consider a single-product supply chain with one manufacturer and one retailer, in which the salesperson is hired by the retailer to sell to the customers. Since inventory allocation is not a role of a retail-salesperson, we focus exclusively on the demand promotion role of the salesperson in our comparison. We model the retail system as a single-period ordering and pricing problem with the following sequence of events: (1) the manufacturer determines the wholesale price, (2) the retailer decides whether to accept the supply chain contract, and if so, places an order and then sets his retail price, (3) the salesperson decides whether to accept the compensation package offered by the retailer, and if so, chooses her effort level, and (4) market demand is realized and customers make purchases based on retail price.

Similar to our models with a wholesale-salesperson, we assume information symmetry for the retail-salesperson model (i.e., cost parameters, salesperson's parameters, profit and utility functions are known to all). We first compare the differences in performance of a retail-salesperson and a wholesale-salesperson under the first-best salary plan. Then, we extend our comparison to the efficiency and robustness of the commission plan. We label the environment with a manufacturer, a retailer, and a retail-salesperson as the *retail model* (Figure 1.*Top*); and the environment with a manufacturer, a wholesale-salesperson and a retailer as the *wholesale model* (Figure 1.*Bottom*).

The analysis and characterization of the structure of the optimal pricing and compensation policy for the retail model under the first-best salary plan and the commission plan are given in On-Line Appendix C.

For each case in our numerical study, we compared the supply chain performance of a retail model with that of a wholesale model. We observed that the retail-salesperson generates on average 18% (maximum of 31%) higher profit for the supply chain than does the wholesale-salesperson under the first-best salary plan, and these numbers are 21% and 29% under the commission plan. A wholesale-

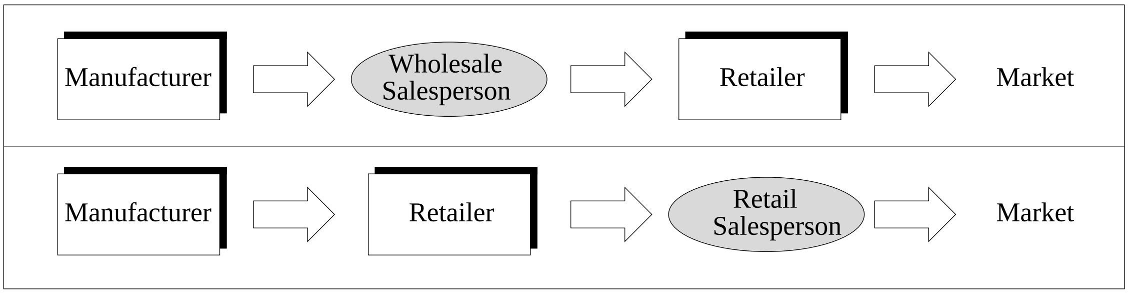

Figure 1: *Top:* Wholesale Model, *Bottom:* Retail Model.

salesperson is less effective than a retail-salesperson because the impact of the selling effort is hampered by double marginalization in the decentralized supply chain. The closer the salesperson is to the market (i.e., the lower she is in the supply chain), the more direct her effect on market demand. The underlying reason is that a wholesale-salesperson has a stronger influence on wholesale pricing than does a retail-salesperson. In the wholesale model, the manufacturer tends to set a higher wholesale price in order to limit the amount of compensation paid to the salesperson. However, in the retail model, the salesperson is hired and paid by the retailer. Hence, the manufacturer tends to set a lower wholesale price without concern for sharing the profits with the retail-salesperson. A lower wholesale price stimulates a larger retailer order quantity and hence increases the overall supply chain profit. Hence, we conclude the following:

Observation 3*: In the demand promotion model, a retail-salesperson is more effective at increasing supply chain profit than is a wholesale-salesperson.*

Comparing systems with a retail-salesperson and a wholesale-salesperson, we also found that, the retail-salesperson generates on average 26% higher profit for the retailer than does the wholesale-salesperson under the first-best salary plan. This number is 32% under the commission plan. Hence, if we were to shift the salesperson in a supply chain from being a wholesale-salesperson to being a retail-salesperson, the retailer would benefit from the profit increase.

Observation 4*: In the demand promotion model, the retailer benefits the most from the retail-salesperson, while the manufacturer benefits most from a wholesale-salesperson. Furthermore, the retail-salesperson puts more effort in promoting demand than the wholesale-salesperson.*

Note that, in the demand promotion case, both the wholesale-salesperson and retail-salesperson perform the same task of promoting demand at the retail level, which in turn increases the demand for the manufacturer's product. So it is interesting that the manufacturer does not gain much from the retail-salesperson. The reason is that the optimal amount of demand promotion is larger when the salesperson works for the retailer than when she works for the manufacturers.

Furthermore, from Observations 3 and 4, we can see that if the salesperson costs the same, then from a total supply chain profit perspective, the retail salesperson is the better hire. If the same firm owned both the manufacturing function and the retail function, they would clearly only use retail salespersons. But, since they are separate, the manufacturer has incentive to hire a wholesale salesperson.

But suppose that the retailer knew that the manufacturer were going to hire a wholesale salesperson and proposed that he can be payed by the manufacture with the same salary to hire a retail salesperson. Since total supply chain profits will be larger (by Observation 3), the retailer can pay the manufacturer back, so that the manufacturer will make the same profit as he would with a wholesale salesperson, and the retailer will be better off. That is, there is some reallocation of profits such that both parties prefer the retail salesperson to the wholesale salesperson. So, in an ideal world we should not see wholesale salespersons. But in the real world, where we don't have first best salary plans, where effort is not contractible, where there is not enough information and enforcement to ensure an equitable split of the profits, a wholesale salesperson would make sense. Another reason for wholesale salespersons is the inventory allocation function, and indeed, we have shown that this role can help coordinate the supply chain (in Section 4.1).

We also observed the following in our numerical study:

Observation 5: *In the demand promotion model, both the wholesale-salesperson's utility and the retail-salesperson's utility are always at the retention level.*

In the case of a wholesale-salesperson, the manufacturer can always design a commission plan (with two parameters: commission rate α and fixed amount S, i.e., $W = \alpha Q + S$) to drive the salesperson's utility to its minimum by setting the commission rate α to be the optimal α under the pure commission plan (i.e., $W = \alpha Q$) and setting the fixed amount S (which is negative) to be the difference between U_{min} and the optimal wholesale-salesperson's utility under the pure commission plan. In the case of a retail-salesperson, since the retailer's action (i.e., setting the commission plan) depends only on the salesperson's utility function, under symmetric information, the retailer can easily design the commission plan such that it drives the retail-salesperson's utility to its minimum. Note that it is easy (and optimal) for the manufacturer to design a first-best salary plan that drives wholesale-salesperson's utility to its minimum, because effort is contractible under the first-best salary plan.

We also compared the performance of the retail and wholesale models under the first-best plan and the best commission plan to check the inefficiency of the commission plan in motivating the salesperson relative to the first-best plan (results omitted for brevity). We observed that the gap between the first-best profit and the profit under the best commission plan is similar in the retail model and the wholesale model. That is, using a commission plan is less effective than the first-best plan in motivating the retail-salesperson.

In order to compare the robustness of the first-best salary plan and commission plan in wholesale- and retail-salesperson models, we also conducted a sensitivity analysis for the retail model as we did for the wholesale model in Section 5.2. Under the commission plan, we found that the average robustness of manufacturer profit and total supply chain profit over the range of salesperson parameter errors are -31% and -26%, respectively. In contrast, under the first-best salary plan, the average robustness of manufacturer profit and total supply chain profit over the range of salesperson parameter errors are -123% and -105%, respectively.

As in the wholesale model, the commission plan is more robust to changes in salesperson parameters than is the first-best salary plan in the retail model. In fact, we observed that, the commission plan is even more robust in the retail model ($\bar{\rho}_{w,retail} = -31\%$ and $\bar{\rho}_{total,retail} = -26\%$) than in the wholesale model ($\bar{\rho}_{w,wholesale} = -39\%$ and $\bar{\rho}_{total,wholesale} = -33\%$). Hence, we conclude the following:

Observation 6: *When salesperson effort is devoted to demand promotion, the commission plan is more robust to uncertainty in the salesperson effectiveness/disutility attitude in a retail model than in a wholesale model.*

The intuition behind this finding is that a wholesale-salesperson has a greater degree of influence over the wholesale price than does the retail-salesperson. Hence, an inaccurate estimate of the parameters of a wholesale-salesperson hurts both the effectiveness of compensation plans and the wholesale pricing decision. Observation 6 suggests that commission plans are more attractive as compensations schemes for retail-salespersons than for wholesale-salespersons. This general conclusion aligns with experimental and field research on salesperson compensation showing that salary plans tend to play a more important role than incentive plans in salesperson compensation when (1) there is a stronger interest in instilling a long-term orientation in the salesperson, (2) the need for personal selling versus general advertising is greater, and (3) selling tasks are complex (Smyth 1968, John and Weitz 1989, and Menguc and Barker 2003). These characteristics are more typical of the environment of a wholesale-salesperson than that of a retail-salesperson.

Finally, we make the following observation about the relative impact of retail- and wholesale-salespersons on prices:

Observation 7: *In general, under both commission and first-best salary plans, the wholesale (/retail)* *price in a supply chain with a wholesale-salesperson is higher than that in a supply chain with a retail-salesperson.*

In our numerical study we observed that, the *retail price* in the supply chain with wholesale-salesperson is, on average 31% (maximum of 59%) higher than that in the supply chain with retail-salesperson under the commission plan. These two numbers are 23% and 83% under the first-best salary plan. The difference in retail price increases as the demand becomes more sensitive to price (i.e., price-elasticity decreases), and salesperson becomes more effective (i.e., C-ratio increases).

In contrast, the *wholesale price* in the supply chain with wholesale-salesperson is, on average 67% (maximum of 83%) higher than that in the supply chain with a retail-salesperson under the commission. These two numbers are 33% and 65% under the first-best salary plan. The difference in wholesale price increases as price elasticity decreases and the production to base demand ratio (x/a_0) is high (i.e., manufacturer has more inventory to clear).

The intuition behind Observation 7 is as follows. The manufacturer in the wholesale model tends to set a higher wholesale price in order to compensate for the amount that the manufacturer pays to the salesperson. However, in the retail model, the salesperson is hired and paid by the retailer; hence, the manufacturer tends to set a lower wholesale price without concern for sharing the profit with the retail-salesperson. The higher wholesale price in the wholesale model also drives the retail price to be higher than in the retail model. Higher prices are one more reason manufacturers may want to make use of wholesale-salespersons.

7 Summary

This paper represents a first step in the analysis of the role and effect of a wholesale-salesperson in increasing the efficiency of supply chains. It compares the impact of a salesperson on supply chain performance when she is hired by the manufacturer (i.e., a wholesale-salesperson) with that when she is hired by the retailer (i.e, a retail-salesperson). We show that although a retail-salesperson is more efficient at increasing supply chain profit than is a wholesale-salesperson, the wholesale-salesperson raises manufacturer profit more than does a retail-salesperson (even if both promote the same retail demand). Furthermore, we show that wholesale-salespersons promote higher wholesale and retail prices than do retail-salespersons. Comparing two compensation plans, we show that although the commission plan is less efficient than the first-best plan in motivating the salesperson, it is more robust to uncertainty about salesperson characteristics in both the wholesale model and the retailer model.

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ON-LINE APPENDIX A

A.1. Experimental Design

We considered 2160 cases for the demand promotion setting and 2160 cases for the inventory allocation setting generated by considering all combinations of the following parameter values:

- *Production level ratio* x/a_0: This ratio compares the production level to the base demand. We considered values of 0.5, 0.75, 1, 2.5 and 5.0. Note that 0.5 represents cases where the manufacturer is under capacitated where 5.0 represents the cases where the manufacturer is over capacitated.
- *Demand elasticity* b: This is a relative measure of the sensitivity of demand to retail price. We considered values of $b = 0.577$, 1.000 and 1.732, which correspond to slopes of 30°, 45° and 60°, respectively.
- *Manufacturer's holding cost ratio* h_w/c: This ratio characterizes the manufacturer's holding cost per item per period relative to the unit production cost. We considered values of 0.15, 0.30 and 0.45 (pointed out as acceptable range in most Operations Management books).
- *Compensation-Price efficiency* (λ^2/bC) *ratio (i.e., CP ratio):* This ratio, which is shown to play an important role in our model, corresponds to the impact of a salesperson on the supply chain performance. For example, in the inventory allocation model, the manufacturer would only hire a salesperson if λ_i^2/bC is greater than 1. The numerator of this ratio corresponds to the effectiveness of the salesperson in her tasks (i.e., demand promotion or inventory allocation), and as it increases, the salesperson can have a higher impact on system performance. The denominator of the ratio corresponds to salesperson disutility and demand elasticity which, as they increase, the impact of the salesperson on system performance decreases. Thus, we change this ratio in our numerical study to generate different scenarios for the impact of the salesperson on system performance. We considered values for λ^2/bC ranging from 0.25 to 2 in increments of about 0.25.
- *Minimum retailer profit ratio* $R_{min}/\Pi_{r,ns}$: This ratio compares the retailer's minimum acceptable profit to his profit in a supply chain without a salesperson. We considered values of 0.5 and 1.0. Note that 1.0 represents the case where the retailer would not participate in a supply chain with a salesperson unless his profit is at least as high as that in a supply chain without a salesperson. The ratio of 0.5 allows us to generate more cases in which there exist feasible commission plans.
- *Minimum salesperson utility ratio* $U_{min}/\Pi_{r,ns}$: This is the ratio of the salesperson's retention utility (normalized to dollar units) to the retailer's profit in a supply chain without a salesperson. We considered values of 0.05, 0.1, 0.2. A low utility ratio represents cases where the best alternative to working in the supply chain offers a fairly low utility value to the salesperson (only 5% of $\Pi_{r,ns}$). A high utility ratio represents cases where the best alternative to working in the supply chain offers a fairly high utility value to the salesperson (20% of $\Pi_{r,ns}$).

A.2. Wholesale Versus Retail Salesperson

To compare the relative inefficiency and the robustness of a commission plan for a retail-salesperson and a wholesale-salesperson, we consider a commission scheme for the retail-salesperson in which the salesperson's compensation is a linear function of the market demand:

$$W(\alpha, S, e) = S + \alpha D(e).$$

Note that, in the retail model, the commission plan based on market demand is the same as the commission plan we have been using for the wholesale-salesperson. This is because, in a demand promotion model, the retailer always prices its products such that there is no leftover inventory (see the On-Line Appendix B). Therefore, a commission plan based on order quantity in a retail model is the same as a commission plan based on market demand.

We evaluated the performance of the commission plan under the retail model by first determining the structure of the optimal policies (see the On-Line Appendix C). Then, we conducted a numerical analysis and compared the supply chain performance with that of a wholesale model.

ON-LINE APPENDIX B

Proofs of Analytical Results

Before, we present the proofs of our analytical results, we first introduce the following definitions:

- **SC1:** A supply chain with a single manufacturer, a single retailer and no salesperson. In this appendix, we use subscript "ns" to refer to this supply chain.
- **SC2:** A supply chain with a manufacturer, a salesperson and a retailer.

We also define *feasibility* of compensation plans as follows:

Feasibility of a Compensation Plan: *A compensation plan is feasible if there exists a solution such that, under the production level and parameter settings, the salesperson minimum utility and the retailer minimum profit are both met, i.e., the IR constraints of the salesperson and the retailer are satisfied ($U^*(e) \geq U_{min}$ and $\Pi_r^*(p_r, e) \geq \Pi_{r,min}$).*

Note that the existence of a feasible plan does not mean that the manufacturer can always increase his/her profit if it hires a salesperson. In other words, it does not mean that the profit in **SC2** is higher than that in **SC1**.

In this On-Line Appendix, Theorem 1, which includes both the first-best-salary plan and the commission plan is divided into Theorem 1a (which corresponds to the results for the first-best-salary plan) and Theorem 1b (which corresponds to the results for the commission plan). Theorems 1a and 1b are proven separately in On-Line Appendix B1. Similarly, Theorem 2 is proven in On-Line Appendix B2.

B.1. Supply Chain without a Salesperson (SC1)

We first consider the case where the objective of the manufacturer and the retailer is to maximize their individual profits (i.e., decentralized supply chain). We use subscript "ns" to refer to the corresponding single-product *decentralized* supply chain without a salesperson.

When the manufacturer has sufficient production capacity (i.e., can support production level $x \geq \frac{a+h_w b}{4}$), the optimal strategies for the manufacturer and the retailer in a single period problem without a salesperson are (Petruzzi and Dada 1999):

$$
\begin{array}{rcrcl}
 & & p^*_{w,ns} & = & \frac{a-h_w b}{2b} \\
 & & p^*_{r,ns} & = & \frac{3a-h_w b}{4b} \\
D^*_{ns} & = & Q^*_{ns} & = & \frac{(a-p^*_w b)}{2} = \frac{a+h_w b}{4} \\
 & & \Pi^*_{w,ns} & = & \frac{(a+h_w b)^2}{8b} - x(c+h_w) \\
 & & \Pi^*_{r,ns} & = & \frac{(a+h_w b)^2}{16b} \\
 & & \Pi^*_{total,ns} & = & \frac{3(a+h_w b)^2}{16b} - x(c+h_w)
\end{array}
$$

We then consider the case where the objective of the manufacturer and the retailer is to maximize the total supply chain profit (i.e., centralized supply chain). We use subscript "ct" to refer to the corresponding single-product *centralized* supply chain without a salesperson. The optimal strategies for the centralized supply chain without a salesperson are (Petruzzi and Dada 1999):

$$
\begin{array}{rcrcl}
 & & p^*_{ct} & = & \frac{a-h_w b}{2b} \\
Q^*_{ct} & = & D^*_{ct} & = & \frac{a+h_w b}{2} \\
\Pi^*_{total,ct} & = & \Pi^*_{ct} & = & \frac{(a+h_w b)^2}{4b} - x(c+h_w)
\end{array}
$$

B.2. General Results for the Supply Chain with a Salesperson (SC2)

In this section, we examine the general characteristics of a supply chain with a salesperson, regardless of the type of salesperson (i.e., demand promotion or inventory allocation) or the structure of the compensation plan (i.e., first-best salary plan or commission plan).

PROPOSITION B1 *Regardless of the structure of salesperson's compensation plan, if in* **SC2** *the corresponding order quantity is greater than or equal to* $Q^*_{ns} = \frac{a+h_w b}{4}$, *but is less than or equal to* $\bar{Q}^*_{ns} = \frac{3(a+h_w b)}{4}$, *then such a supply chain with a salesperson generates at least as much total profit as one without a salesperson.*

Proof: We first consider a supply chain without a salesperson (i.e., **SC1**). The total supply chain profit as a function of order quantity is:

$$\begin{aligned}\Pi_{total,sc1}(Q) &= \Pi_{w,sc1}(Q) + \Pi_{r,sc1}(Q) \\ &= p_w Q - cx - h_w(x-Q) + p_r Q - p_w Q \\ &= p_r Q - cx - h_w(x-Q).\end{aligned}$$

Since the order quantity is the same as the demand level (i.e., $Q = D$), we can write the retail price as a function of order quantity: $p_r = \frac{a-Q}{b}$. The total supply chain profit then becomes:

$$\Pi_{total,sc1}(Q) = \frac{a-Q}{b}Q - cx - h_w(x-Q),$$

$\Pi_{total,sc1}(Q)$ is concave in Q and is maximized at $Q^*_{sc1} = \frac{a+h_w b}{2}$, which is also the optimal order quantity in the centralized supply chain without a salesperson (i.e., $Q^*_{sc1} = Q^*_{ct}$).

In a decentralized supply chain without a salesperson, since the manufacturer and the retailer maximize their respective profits, the total supply chain profit is less than that in a centralized system because of double marginalization. The corresponding optimal order quantity is $Q^*_{ns} = \frac{a+h_w b}{4} < Q^*_{ct}$.

Since $\Pi_{total,sc1}(Q)$ is concave in Q and $Q^*_{ns} < Q^*_{ct}$, there is always a $\bar{Q}^*_{ns}$ such that $\Pi_{total,sc1}(\bar{Q}^*_{ns}) = \Pi_{total,sc1}(Q^*_{ns})$ and $\bar{Q}^*_{ns} > Q^*_{ct}$. Moreover, $\bar{Q}^*_{ns} = \frac{3(a+h_w b)}{4}$. Therefore, we can conclude that,

$$\Pi_{total,sc1}(Q)|_{Q^*_{ns} \le Q \le \bar{Q}^*_{ns}} \ge \Pi_{total,sc1}(Q^*_{ns}) = \Pi^*_{total,ns} \tag{6}$$

Now we consider the case where the supply chain has a salesperson (i.e., **SC2**). The total supply chain profit function for this system is:

$$\begin{aligned}\Pi_{total,sc2}(Q) &= \Pi_{w,sc2}(Q) + \Pi_{r,sc2}(Q) + W(Q) \\ &= p_w Q - cx - h_w(x-Q) - W(Q) + p_r Q - p_w Q + W(Q) \\ &= p_r Q - cx - h_w(x-Q) \\ &= \frac{a-Q}{b}Q - cx - h_w(x-Q),\end{aligned}$$

which follows the same structure as that in the system without a salesperson. In other words, the total supply chain profit as a function of the order quantity in **SC2** is the same as that in **SC1** (i.e., $\Pi_{total,sc2}(Q) = \Pi_{total,sc1}(Q)$). Hence, we can conclude from Equation (6) that in a supply chain with a salesperson, for any order quantity Q, the total supply chain profit has the following characteristics:

$$\Pi_{total,sc2}(Q)|_{Q^*_{ns} \le Q \le \bar{Q}^*_{ns}} \ge \Pi_{total,sc1}(Q^*_{ns}) = \Pi^*_{total,ns} \qquad \blacksquare$$

To simplify notation, unless needed, we omit (e) from $p_w(e)$, $p_r(e)$, $U(e)$ and $W(e)$; we also omit (p_w, e) from $Q(p_w, e)$ and $p_r(p_w, e)$; and we omit (p_r, Q) from $\Pi_r(p_r, Q)$.

ON-LINE APPENDIX B-1

Proofs for Inventory Allocation Model

In this On-Line Appendix, we present the proofs of our results for the inventory allocation model. Theorem 1, which includes both the first-best-salary plan and the commission plan is divided into Theorem 1a (which corresponds to the results for the first-best-salary plan) and Theorem 1b (which corresponds to the results for the commission plan). Theorems 1a and 1b are proven separately. Before we present the proofs of Theorems 1a and 1b, we need to present the following results.

B.3. Inventory Allocation Salesperson Model under the First-Best Salary Plan

In this section we present our analytical results for the inventory allocation salesperson under the first-best salary plan.

LEMMA B1 *In the inventory allocation model, if the first-best salary plan is feasible, the optimal retail price is:*

$$p_r^*(p_w, e) \quad = \quad \frac{a + p_w b - 2\lambda_i e}{2b}. \tag{7}$$

Proof: Given a wholesale price (p_w) and a salesperson effort level (e), the retailer's optimal order quantity is the sum of what he would have ordered without a salesperson (Q_{ns}^*) and the additional items that the salesperson is able to convince him to order:

$$Q^* = Q_{ns}^* + \Delta(e) = \frac{1}{2}(a - bp_w) + \lambda_i e.$$

The retailer profit for *any* price-setting newsvendor problem is maximized when $Q = D(= a - bp_r)$. So we have

$$\frac{1}{2}(a - bp_w) + \lambda_i e = a - bp_r.$$

Hence, the optimal retail price is

$$p_r^*(p_w, e) = \frac{a + p_w b - 2\lambda_i e}{2b}. \qquad \blacksquare$$

LEMMA B2 *In the inventory allocation model, the first-best salary plan is feasible when $x \geq \sqrt{R_{min} b}$.*

Proof: Recall that a compensation plan is feasible if both the retailer and the salesperson are willing to participate in the game. Hence, a compensation plan is feasible when, under production level x, there exists an effort level $e \geq 0$ such that the IR constraints of the salesperson and the retailer are met (i.e., $U \geq U_{min}$, $\Pi_r \geq R_{min}$).

<u>*Retailer's IR Constraint:*</u> From Lemma B1, we know that given a wholesale price (p_w) and salesperson effort level (e), the retailer sets the retail price at $p_r^*(p_w, e) = \frac{a+p_w b-2\lambda_i e}{2b}$. Thus, the retailer profit function becomes:

$$\Pi_r = \frac{(a - p_w b + 2\lambda_i e)(a - p_w b - 2\lambda_i e)}{4b}.$$

Hence, $\Pi_r \geq R_{min}$ if

$$p_w \leq \frac{a - 2\sqrt{\lambda_i^2 e^2 + R_{min} b}}{b} \equiv p_w^u,$$

where p_w^u is defined as an upper bound for the wholesale price corresponding to the highest wholesale price the retailer will accept to stay in the game.

<u>*Salesperson's IR Constraint:*</u> In **SC2**, the manufacturer satisfies the salesperson's IR constraint.[3] Furthermore, we know that the salesperson's IR constraint is met at its minimum under the first-best salary plan, i.e., $U = U_{min}$.

<u>*Production Constraint:*</u> Note that it does not make sense for the manufacturer to set wholesale price, or for the salesperson to exert effort, such that the resulting order quantity is greater than x, the manufacturer's production level. Hence, this imposes a lower bound p_w^l on the wholesale price such that $Q^* \leq x$, where

$$p_w^l = arg_{p_w}\Big\{Q^* = x\Big\} = \frac{a + 2\lambda_i e - 2x}{b}.$$

Therefore, the first-best salary plan is only feasible if there exists an effort $e \geq 0$ such that $p_w^l \leq p_w \leq p_w^u$. Comparing equations p_w^u and p_w^l, we find that

$$p_w^u - p_w^l = \frac{2(x - \sqrt{\lambda_i^2 e^2 + R_{min} b} - \lambda_i e)}{b},$$

[3] Note that in **SC2**, we assume that the manufacturer is willing to pay the salesperson as much as necessary to keep the salesperson in the supply chain. Later we will show whether it is beneficial for the manufacturer to do this.

and is non-negative when $e \leq \frac{x^2 - R_{min}b}{2\lambda_i x}$. We define the maximum feasible effort as

$$e_f = \frac{x^2 - R_{min}b}{2\lambda_i x},$$

which is non-negative if $x \geq \sqrt{R_{min}b}$ (note that $R_{min}b \geq 0$ because $R_{min} \geq 0$ and $b \geq 0$ by assumption). Hence, $x \geq \sqrt{R_{min}b}$ is a necessary condition for the first-best salary plan to be feasible. ■

Condition $x \geq x_{min} = \sqrt{R_{min}b}$ implies that there exists a threshold x_{min} on production level x, below which the retailer's individual rationality constraint is not satisfied. Furthermore, the threshold x_{min} increases with retailer's minimum acceptable profit (R_{min}) and demand elasticity (b). This is intuitive, since as the retailer's minimum acceptable profit increases, to have a feasible compensation plan, the minimum production level x_{min} should also increase to provide enough items that can result in higher profit (than R_{min}) for the retailer. On the other hand, as demand elasticity b increases, the decrease in price results in a large increase in demand. To take advantage of the increase in demand, there should be enough items available to the retailer (i.e., x_{min} should be larger), so he can increase his profit beyond the minimum acceptable. This is the main reason that systems with higher demand elasticity require higher minimum production threshold (x_{min}) to have a feasible commission plan.

LEMMA B3 *In the inventory allocation model, if the first-best salary plan is feasible, the structure of the optimal wholesale price under the first-best salary plan is:*

When $R_{min} \leq \frac{(a+h_w b-3x)x}{b}$, *then*

$$p_w^* = \begin{cases} p_w^o & : \text{ if } 0 \leq e \leq e_c^l \\ p_w^l & : \text{ if } e_c^l \leq e \leq e_f \end{cases}$$

When $R_{min} > \frac{(a+h_w b-3x)x}{b}$, *then*

$$p_w^* = \begin{cases} p_w^o & : \text{ if } 0 \leq e \leq e_c^u \\ p_w^u & : \text{ if } e_c^u \leq e \leq e_f \end{cases}$$

where

$$p_w^o = \frac{a + 2\lambda_i e - h_w b}{2b}, \qquad p_w^l = \frac{a + 2\lambda_i e - 2x}{b}, \qquad p_w^u = \frac{a - 2\sqrt{\lambda_i^2 e^2 + R_{min}b}}{b},$$

$$e_f = \frac{x^2 - R_{min}b}{2\lambda_i x}, \qquad e_c^l = \frac{4x - (a + h_w b)}{2\lambda_i}, \qquad e_c^u = \frac{2\sqrt{(a + h_w b)^2 - 12R_{min}b} - (a + h_w b)}{6\lambda_i}.$$

Proof: The outline of the proof is as follows. (1) We determine the exact value of maximum feasible effort for $e \geq 0$. (2) We study the manufacturer's problem and define the optimal wholesale price without taking into account any constraints. (3) We examine the production constraint and the retailer's IR constraint, and determine the overall optimal wholesale price structure.

Maximum feasible effort (e_f): Recall that, in the proof of Lemma B2, we defined the maximum feasible effort as

$$e_f = \frac{x^2 - R_{min}b}{2\lambda_i x},$$

which is non-negative when $x \geq \sqrt{R_{min}b}$.

Manufacturer's Problem: Note that it does not make sense for the manufacturer to set wholesale price, or for the salesperson to exert effort, such that the resulting order quantity is greater than x, the manufacturer's production level. So we focus on the case where $Q^* \leq x$ and the manufacturer's profit is

$$\Pi_w(p_w, e) = p_w Q^* - cx - h_w(x - Q^*) - U_{min} - Ce^2.$$

Given any effort level e, the manufacturer profit function Π_w is maximized at

$$p_w^o = \frac{a + 2\lambda_i e - h_w b}{2b}.$$

Note that p_w^o is the optimal wholesale price without taking into account the production constraint or the retailer's IR constraint. To incorporate these, recall from the proof of Lemma B2 that the production constraint imposes a lower bound on the wholesale price, p_w^l, and the retailer's IR constraint imposes an upper bound on the wholesale price, p_w^u. Hence, p_w^l is the optimal wholesale price in cases where the production constraint is binding, while p_w^u is the optimal wholesale price in cases where the retailer's IR constraint is binding. Let e_c^l be the effort level that results in $p_w^o = p_w^l$ and e_c^u be the effort level that results in $p_w^o = p_w^u$.

Comparing p_w^o and p_w^l, we get:

$$p_w^o - p_w^l = \frac{-a - 2\lambda_i e - h_w b + 4x}{2b},$$

and hence

$$e_c^l = arg_e\Big\{p_w^o = p_w^l\Big\} = \frac{4x - (a + h_w b)}{2\lambda_i}.$$

Since $p_w^o - p_w^l$ is decreasing in e, we know that when the effort level $e \geq e_c^l$, then $p_w^o < p_w^l$, meaning that the production constraint is binding. Moreover, when the effort level $e \leq e_c^l$, then $p_w^o \geq p_w^l$, meaning that the production constraint is not binding. Hence, if we neglect the effect of the retailer's IR constraint, the optimal wholesale price has the following structure:

$$p_w^* = \begin{cases} p_w^o & : \text{ if } 0 \leq e \leq e_c^l \\ p_w^l & : \text{ if } e_c^l \leq e \leq e_f \end{cases} \tag{8}$$

On the other hand, since e_c^u is the effort level that results in $p_w^o = p_w^u$, then by comparing p_w^u with p_w^o, we get

$$p_w^u - p_w^o = \frac{a + h_w b - 2\lambda_i e - 4\sqrt{\lambda_i^2 e^2 + R_{min} b}}{2b},$$

and hence

$$e_c^u = arg_e\Big\{p_w^u = p_w^o\Big\} = \frac{2\sqrt{(a + h_w b)^2 - 12 R_{min} b} - (a + h_w b)}{6\lambda_i}.$$

Since $p_w^u - p_w^o$ is decreasing in e, we know that when the effort level $e > e_c^u$, then $p_w^u < p_w^o$, meaning that the retailer's IR constraint is binding. Moreover, when the effort level $e \leq e_c^u$, then $p_w^u \geq p_w^o$, meaning that the retailer's IR constraint is not binding. Hence, if we neglect the effect of the production constraint, the optimal wholesale price has the following structure:

$$p_w^* = \begin{cases} p_w^o & : \text{ if } 0 \leq e \leq e_c^u \\ p_w^u & : \text{ if } e_c^u \leq e \leq e_f \end{cases}$$

Comparing the two critical efforts e_c^l and e_c^u, we find that

$$e_c^l - e_c^u = \frac{6x - \sqrt{(a + h_w b)^2 - 12 R_{min} b} - (a + h_w b)}{3\lambda_i},$$

which is greater than zero if $R_{min} > \frac{(a + h_w b - 3x)x}{b}$ and smaller than zero if $R_{min} < \frac{(a + h_w b - 3x)x}{b}$. Hence, we finally get:

When $R_{min} \leq \frac{(a + h_w b - 3x)x}{b}$, *then*

$$p_w^* = \begin{cases} p_w^o & : \text{ if } 0 \leq e \leq e_c^l \\ p_w^l & : \text{ if } e_c^l \leq e \leq e_f \end{cases}$$

When $R_{min} > \frac{(a + h_w b - 3x)x}{b}$, *then*

$$p_w^* = \begin{cases} p_w^o & : \text{ if } 0 \leq e \leq e_c^u \\ p_w^u & : \text{ if } e_c^u \leq e \leq e_f \end{cases} \qquad \blacksquare$$

PROPOSITION B2 *In the inventory allocation model, if the first-best salary plan is feasible, the optimal strategy for the manufacturer under the first-best salary plan is:*

(i) If $\lambda_i^2 > 2bC$ and $R_{min} \leq \frac{(a+h_w b-3x)x}{b}$, then

$$p_w^* = p_w^l = \frac{a + 2\lambda_i e - 2x}{b}, \qquad e^* = e_f = \frac{x^2 - R_{min} b}{2\lambda_i x}$$

(ii) If $\lambda_i^2 \leq 2bC$ and $R_{min} \leq \frac{(a+h_w b-3x)x}{b}$, or $\lambda_i^2 \leq 2bC$, and $\frac{(a+h_w b-3x)x}{b} < R_{min} \leq \frac{C(a+h_w b)^2(bC-2\lambda_i^2)}{4(\lambda_i^2-2bC)^2}$ then

$$p_w^* = p_w^o = \frac{a + 2\lambda_i e - h_w b}{2b}, \qquad e^* = e^{o*} = \frac{\lambda_i(a + h_w b)}{2(2bC - \lambda_i^2)}$$

(iii) If $\lambda_i^2 \leq 2bC, R_{min} > \frac{(a+h_w b-3x)x}{b}$ and $R_{min} > \frac{C(a+h_w b)^2(bC-2\lambda_i^2)}{4(\lambda_i^2-2bC)^2}$, or $\lambda_i^2 > 2bC$ and $R_{min} > \frac{(a+h_w b-3x)x}{b}$ then

$$p_w^* = p_w^u = \frac{a - 2\sqrt{\lambda_i^2 e^2 + R_{min} b}}{b}, \qquad e^* = \min\Big\{\max\{e^{u*}, e_c^u\}, e_f\Big\}$$

where

$$e_c^u = \frac{2\sqrt{(a + h_w b)^2 - 12R_{min} b} - (a_0 + h_w b)}{6\lambda_i}$$

and e^{u*} is the root of the following:

$$\Big((4\lambda_i^2 C^2 b + 16\lambda_i^4 C)\ e^4 + (-4\lambda_i^3 aC - 4\lambda_i^3 h_w bC)\ e^3 + (4R_{min} b^2 C^2 + 16R_{min} b\lambda_i^2 C)\ e^2 + (-4R_{min} b\lambda_i^3 h_w - 4R_{min}\lambda_i^3 a - 4R_{min} b\lambda_i aC - 4R_{min} b^2 h_w \lambda_i C)\ e - 4\lambda_i^2 R_{min}^2 b + R_{min} b^2 h_w^2 \lambda_i^2 + R_{min}\lambda_i^2 a^2 + 2R_{min} b\lambda_i^2 a h_w\Big) = 0$$

Proof: The outline of the proof is as follows. For every case, we find the optimal effort level that maximizes the manufacturer profit. Then using Lemma B3, we find the corresponding optimal wholesale price.

To save space, we only present the proof for the first part of case (i). The proof for cases (ii) and (iii) are similar and are therefore omitted.

When $\lambda_i^2 > 2bC$ and $R_{min} \leq \frac{(a+h_w b-3x)x}{b}$, from Lemma 9 we know that the optimal wholesale price has the following structure:

$$p_w^* = \begin{cases} p_w^o & : \text{ if } 0 \leq e \leq e_c^l \\ p_w^l & : \text{ if } e_c^l \leq e \leq e_f \end{cases} \tag{9}$$

We will show that when $0 \leq e \leq e_c^l$, the manufacturer's optimal profit function is increasing in e. While, when $e_c^l \leq e \leq e_f$, the manufacturer's optimal profit function is increasing in e with its maximum at e_f. This implies that when $\lambda_i^2 > 2bC$ and $R_{min} \leq \frac{(a+h_w b-3x)x}{b}$, the optimal effort level e^* that results in the maximum value for the manufacturer profit should be within the range of $e_c^l \leq e \leq e_f$, which according to Lemma B3 results in $p_w^* = p_w^l$. Also the corresponding optimal effort level $e^* = e_f$.

<u>Case (ia):</u> In this case, we have $0 \leq e \leq e_c^l$. From Lemma B3, we know that $p_w^* = p_w^0$. The manufacturer profit function is

$$\Pi_w(p_w^0) = \frac{1}{8b}\Big(4(\lambda_i^2 - 2bC)e^2 + 4\lambda_i(a + h_w b)e + (a + h_w b)^2\Big) - x(c + h_w) - U_{min},$$

which is convex in e, since

$$\frac{d^2}{d^2 e}\Pi_w(p_w^u) = \lambda_i^2 - 2bC > 0$$

and is minimized at

$$e^{o*} = \frac{\lambda_i(a + h_w b)}{2(-\lambda_i^2 + 2bC)} < 0,$$

which is outside of the region $0 \leq e \leq e_c^l$. So, the manufacturer's profit $\Pi_w(p_w^0)$ is increasing in e in the range of $0 \leq e \leq e_c^l$.

<u>Case (ib):</u> In this case, we have $e_c^l \leq e \leq e_f$. From Lemma B3, we know that $p_w^* = p_w^l$. The manufacturer profit function is

$$\Pi_w(p_w^l) = \frac{1}{b}\Big((a + \lambda_i e - 2x) - bCe^2\Big) - x(c + h_w) - U_{min},$$

which is concave in e, since

$$\frac{d^2}{d^2 e}\Pi_w(p_w^u) = -bC < 0$$

and is maximized at

$$e^{l*} = \frac{\lambda_i x}{bC}.$$

Comparing e^{l*} and e^l_c, we find that

$$e^{l*} - e^l_c = \frac{bC(a + h_w b) + 2x(\lambda_i^2 - 2bC)}{2bC\lambda_i},$$

which is greater than zero when $x > \frac{(a+hwb)bC}{2(-\lambda_i^2+2bC)}$. Since $\lambda_i^2 > 2bC$, we know that $\frac{(a+hwb)bC}{2(-\lambda_i^2+2bC)} < 0$, and hence $e^{l*} > e^l_c$ for all $x > 0$.

Comparing e^{l*} and e_f, we find that

$$e^{l*} - e_f = \frac{(2\lambda_i^2 - bC)x^2 + R_{min}b^2C}{2xbC\lambda_i},$$

which is greater than zero when $R_{min} > \frac{(-2\lambda_i^2+bC)x^2}{b^2C)}$. Since $\lambda_i^2 > 2bC > bC$, we know that $\frac{(-2\lambda_i^2+bC)x^2}{b^2C)} < 0$, and we also know that $R_{min} \geq 0$ by assumption. Therefore, $e^{l*} > e_f$ (i.e., e^{l*} is outside of the region $e^l_c \leq e \leq e_f$). Thus, we conclude that, the manufacturer's optimal profit function $\Pi_w(p^0_w)$ is increasing in e and is therefore maximized at e_f in the range of $e^l_c \leq e \leq e_f$.

It is easy to show that the manufacturer profit function is continuous in e within these two regions. Furthermore, at the critical effort level e^l_c, we have $\Pi_w(p^l_w, e^l_c) = \Pi_w(p^o_w, e^l_c)$. Thus, the manufacturer profit function is continuous for $0 \leq e \leq e_f$.

We can combine the results in Cases (ia) and (ib), and conclude that the manufacturer's optimal profit function is increasing in e within the region $0 \leq e \leq e_f$ and is maximized at e_f. Since e_f is in the region $e^l_c \leq e \leq e_f$, from Lemma 9, we have $p^*_w = p^l_w$. Thus, if there exists a feasible first-best salary plan, then the optimal solution for the manufacturer is

$$p^*_w = p^l_w, \qquad e^* = e_f.$$

This completes the proof for case (i). ■

B.4. Inventory Allocation Model: Comparing SC2 with SC1 Under the First-Best Salary Plan

In this section, we examine the conditions under which it is more *beneficial* for the manufacturer to hire a salesperson than not to hire one, i.e., the manufacturer has greater profit in **SC2** than in **SC1**. The proof of Proposition 8 is similar to those for the demand promotion model and is therefore omitted.

PROPOSITION B3 *In the inventory allocation model under the first-best salary plan, it is beneficial for the manufacturer to hire the salesperson if $U_{min} \leq U_{i1}$ where*

$$U_{i1} = \begin{cases} u^l_c & : \text{ if } \lambda_i^2 > 2bC \text{ and } R_{min} \leq \frac{(a+h_w b-3x)x}{b} \\ u^o_c & : \text{ if } \lambda_i^2 \leq 2bC \text{ and } R_{min} \leq \frac{(a+h_w b-3x)x}{b}, \\ & \quad \text{or } \lambda_i^2 \leq 2bC \text{ and } \frac{(a+h_w b-3x)x}{b} < R_{min} \leq \frac{C(a+h_w b)^2(bC-2\lambda_i^2)}{4(\lambda_i^2-2bC)^2} \end{cases}$$

where

$$\begin{aligned} u^l_c &= \frac{1}{-8bx^2\lambda_i^2}\Big(-8a\lambda_i^2x^3 + 8\lambda_i^2x^2R_{min}b + 8\lambda_i^2x^4 + 2Cb^3R_{min}^2 \\ &\quad -4Cb^2R_{min}x^2 + 2Cbx^4 + \lambda_i^2x^2a^2 + 2\lambda_i^2x^2ah_wb + \lambda_i^2x^2h_w^2b^2 - 8x^3b\lambda_i^2h_w\Big), \\ u^o_c &= \frac{\lambda_i^2(a + h_w b)^2}{8b(2bC - \lambda_i^2)}. \end{aligned}$$

Proof: Assuming that the first-best salary plan is feasible, we compare the manufacturer profit in a supply chain with a salesperson (**SC2**) with that in a supply chain without a salesperson (**SC1**). We examine the two possible optimal solutions ($p_w^* = p_w^l,\ e^* = e_f$) and ($p_w^* = p_w^o,\ e^* = e_o^*$) for the manufacturer in **SC2**.

Case 1: In this case, we consider $p_w^* = p_w^l$ and $e^* = e_f$. By Proposition B2, we know $p_w^* = p_w^l$ and $e^* = e_f$ when $\lambda_i^2 > 2bC$ and $R_{min} \le \frac{(a+h_w b-3x)x}{b}$. Comparing the manufacturer's optimal profit in **SC2** with that in **SC1**, we find that $\Pi_w^*(p_w^l, e_f) = \Pi_{w,ns}^*$ when $U_{min} = u_c^l$ where

$$\begin{aligned} u_c^l &= \frac{1}{-8bx^2\lambda_i^2}\Big(-8a\lambda_i^2 x^3 + 8\lambda_i^2 x^2 R_{min} b + 8\lambda_i^2 x^4 + 2Cb^3 R_{min}^2 \\ &\quad -4Cb^2 R_{min} x^2 + 2Cbx^4 + \lambda_i^2 x^2 a^2 + 2\lambda_i^2 x^2 a h_w b + \lambda_i^2 x^2 h_w^2 b^2 - 8x^3 b \lambda_i^2 h_w \Big). \end{aligned}$$

Moreover, $\Pi_w^*(p_w^l, e_f) - \Pi_{w,ns}^*$ is decreasing in U_{min}, and hence, $\Pi_w^*(p_w^l, e_f) \ge \Pi_{w,ns}^*$ when $U_{min} \le u_c^l$.

Case 2: In this case, we consider $p_w^* = p_w^o$ and $e^* = e^{o*}$. By Proposition 7, we know one of the necessary conditions for $p_w^* = p_w^o$ and $e^* = e^{o*}$ is $\lambda_i^2 \le 2bC$. Comparing the manufacturer's optimal profit in **SC2** with that in **SC1**, we find that $\Pi_w^*(p_w^o, e^{o*}) = \Pi_{w,ns}^*$ when $U_{min} = u_c^o$ where

$$u_c^o = \frac{\lambda_i^2 (a + h_w b)^2}{8b(2bC - \lambda_i^2)}.$$

Moreover, $\Pi_w^*(p_w^o, e^{o*}) - \Pi_{w,ns}^*$ is non-increasing in U_{min} (given $\lambda_i^2 \le 2bC$), and hence, $\Pi_w^*(p_w^o, e^{o*}) \ge \Pi_{w,ns}^*$ when $U_{min} \le u_c^o$.

Remark: We know from Proposition B2 that $p_w^* = p_w^u$ and $e^* = \min\Big\{\max\{e^{u*}, e_c^u\}, e_f\Big\}$ when $\lambda_i^2 \le 2bC, R_{min} > \frac{(a+h_w b-3x)x}{b}$ and $R_{min} > \frac{C(a+h_w b)^2(bC-2\lambda_i^2)}{4(\lambda_i^2-2bC)^2}$, or $\lambda_i^2 > 2bC$ and $R_{min} > \frac{(a+h_w b-3x)x}{b}$. It can be shown numerically that $\Pi_w^*(p_w^u, e^*) \ge \Pi_{w,ns}^*$ for some values of U_{min}. However, since e^{u*} cannot by expressed in closed form, we cannot specify the maximum allowable U_{min} in order for $\Pi_w^*(p_w^u, e^*) \ge \Pi_{w,ns}^*$. ■

THEOREM 1a *In the inventory allocation model under a feasible first-best salary plan, if it is beneficial for the manufacturer to hire a salesperson, then*

(*i*) *the resulting optimal retailer profit can be greater than that in a supply chain without a salesperson, and*

(*ii*) *the resulting total supply chain profit is at least as large as than that in a supply chain without a salesperson.*

Proof for part (i): We prove this part by showing there is at least one set of feasible input parameters such that the manufacturer earns a greater profit with an inventory allocation salesperson in the supply chain than without one (i.e., $\Pi_w^* \ge \Pi_{w,ns}^*$), and the retailer also earns at least as much profit with such a salesperson as without one (i.e., $\Pi_r^* > \Pi_{r,ns}^*$).
Consider the case where (1) $R_{min} = \Pi_{r,ns}^* = \frac{(a+h_w b)^2}{16b}$, (2) $bC < \lambda_i^2 < 2bC$, and (3) $x = Q_{ns}^* = \frac{a+h_w b}{4}$. By substitution, we find that $\frac{(a+h_w b-3x)x}{b} = \frac{(a+h_w b)^2}{16b}$ and hence $R_{min} \le \frac{(a+h_w b-3x)x}{b}$ holds. Therefore, this is Case (ii) in Proposition B2, and hence, the manufacturer's optimal strategy is $e^* = e^{o*} = \frac{\lambda_i(a+h_w b)}{2(-\lambda_i^2+2bC)}$ and $p_w^* = p_w^o = \frac{a+2\lambda_i e - h_w b}{2b}$. The manufacturer profit is at least as large as that without a salesperson ($\Pi_w^o \ge \Pi_{w,ns}^*$) if

$$e^{o*} \ge \underline{e}^h \equiv \frac{-\lambda_i(a + h_w b) + \sqrt{\lambda_i^2(a + h_w b)^2 + 8bU_{min}(\lambda_i^2 - 2bC)}}{2(\lambda_i^2 - 2bC)},$$

and $e^{o*} \ge \underline{e}^h$ when $\lambda_i^2 < 2bC$ and $U_{min} \ge 0$. Also, since $R_{min} = \Pi_{r,ns}^*$, we know that $\Pi_r^* \ge \Pi_{r,ns}^*$ must hold. So, under the above three conditions, both the retailer and the manufacturer can be at least as well off in a supply chain with a salesperson as that one without.

Proof for part (ii): We examine the three possible optimal wholesale price cases from Proposition B2, i.e.,

- Case 1: $p_w^* = p_w^l$
- Case 2: $p_w^* = p_w^o$
- Case 3: $p_w^* = p_w^u$

The proofs of the second and third cases are similar to that of the first case and are therefore omitted.

Case 1: From Proposition B2, we know that the optimal wholesale price is $p_w^* = p_w^l$ when $\lambda_i^2 > 2bC$ and $R_{min} \leq \frac{(a+h_w b-3x)x}{b}$. We prove the result for this case by showing that the model meets the conditions in Proposition B1 including (1) $Q^* \geq Q_{ns}^* = \frac{a+h_w b}{4}$, and (2) $Q^* \leq \bar{Q}_{ns}^* = \frac{3(a+h_w b)}{4}$.

Condition 1: We know, by the definition of p_w^l, the resulting ordering quantity equals to the production level, i.e., $Q^*(p_w^l) = x$. Since we have assumed that $x \geq Q_{ns}^*$, $Q^*(p_w^l) \geq Q_{ns}^*$ holds.

Condition 2: When $p_w^* = p_w^l$, we know that $R_{min} \leq \frac{(a+h_w b-3x)x}{b}$. Also, recall that we assumed $R_{min} \geq 0$. $\frac{(a+h_w b-3x)x}{b} \geq 0$ implies that $0 \leq x \leq \frac{a+h_w b}{3}$. Moreover, the optimal ordering quantity $Q^*(p_w^l) = x$. Therefore, $Q^*(p_w^l)(= x) \leq \frac{a+h_w b}{3} < \bar{Q}_{ns}^*(= \frac{3(a+h_w b)}{4})$.

In summary, $Q_{ns}^* \leq Q^*(p_w^l) < \bar{Q}_{ns}^*$ when the first-best salary plan is feasible. By Proposition B1, we conclude that $\Pi_{total}^*(p_w^l) \geq \Pi_{total,ns}^*$. ■

B.5. Inventory Allocation Model under the Commission Plan

In this section we present our analytical results for the inventory allocation salesperson under the commission plan.

LEMMA B4 *In the inventory allocation model under the commission plan, if there exists a feasible commission plan, given a wholesale price (p_w) and commission plan parameters (α and S), the optimal salesperson effort (e^*), retailer order quantity (Q^*) and retail price (p_r^*) are:*

$$\begin{aligned} e^*(p_w, \alpha) &= \frac{\lambda_i \alpha}{2C} \\ Q^*(p_w, \alpha) &= \frac{1}{2}\Big(a - p_w b + \frac{\lambda_i^2 \alpha}{C}\Big) \\ p_r^*(p_w, \alpha) &= \frac{1}{2b}(a + p_w b - \frac{\lambda_i^2 \alpha}{C}). \end{aligned}$$

Proof: We study the optimal responses of the salesperson and the retailer given a wholesale price p_w and a commission rate α as follows.

Salesperson's Problem: Given the wholesale price, p_w, and commission plan parameters, α and S, salesperson's utility is:

$$U(e) = (S + \alpha Q) - Ce^2 = \alpha \frac{(a - p_w b)}{2} + \alpha \lambda_i e - Ce^2 + S,$$

which is maximized at

$$e^* = \frac{\alpha \lambda_i}{2C}.$$

Note that $Q^* = Q_{ns}^* + \lambda_i e$, where $Q_{ns}^* = \frac{1}{2}(a - p_w b)$. Hence, the corresponding order quantity is

$$Q^*(p_w, \alpha) = \frac{1}{2}(a - p_w b) + \lambda_i e^* = \frac{1}{2}\Big(a - p_w b + \frac{\lambda_i^2 \alpha}{C}\Big).$$

Retailer's Problem: Given the wholesale price p_w and a salesperson effort level e, the retailer orders quantity Q^*. The retailer's profit is

$$\Pi_r = p_r(a - bp_r) - p_w Q^*,$$

and the optimal retail price is

$$p_r^*(p_w, \alpha) = arg_{p_r}\Big\{D = Q^*\Big\} = \frac{aC + p_w bC - \lambda_i^2 \alpha}{2bC}. \quad ■$$

PROPOSITION B4 *In the inventory allocation model, if there exists a feasible commission plan, the manufacturer's optimal wholesale price under the commission plan is as follows:*

(i) If $\lambda_i^2 > bC$, $R_{min} > \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, *and* $x \geq Q_{ns}^*$, *then*

$$p_w^* = p_w^u \qquad : \qquad \text{if } \ 0 \leq \alpha \leq \alpha_f$$

(ii) If $\lambda_i^2 > bC$, $R_{min} \leq \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, *and* $x \geq Q_{ns}^*$, *then*

- *If* $x \geq x_c$, *then*

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^u \\ p_w^u & : \quad \text{if } \ \alpha_c^u \leq \alpha \leq \alpha_f \end{cases}$$

- *If* $x < x_c$, *then*

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^l \\ p_w^l & : \quad \text{if } \ \alpha_c^l \leq \alpha \leq \alpha_f \end{cases}$$

(iii) If $\lambda_i^2 \leq bC$ *and* $x \geq Q_{ns}^*$, *then*

$$p_w^* = \begin{cases} p_w^l & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^l \\ p_w^o & : \quad \text{if } \ \alpha_c^l \leq \alpha \leq \alpha_c^u \\ p_w^u & : \quad \text{if } \ \alpha_c^u \leq \alpha \leq \alpha_f \end{cases}$$

where

$$p_w^o = \frac{1}{2b}\left(a - h_w b + \alpha b + \frac{\lambda_i^2\alpha}{C}\right), \qquad p_w^l = \frac{a-2x}{b} + \frac{\lambda_i^2\alpha}{bC}, \qquad p_w^u = \frac{aC - \sqrt{\lambda_i^4\alpha^2 + 4R_{min}C^2 b}}{bC},$$

$$x_c = \frac{\lambda_i^2(a+h_w b) + \sqrt{\lambda^4(a+h_w b)^2 - 4R_{min}b(Cb+3\lambda_i^2)(\lambda_i^2 - bC)}}{2(3\lambda_i^2+bC)},$$

$$\alpha_f = \frac{(x^2 - R_{min}b)C}{\lambda_i^2 x}, \qquad \alpha_c^l = \frac{(4x - a - h_w b)C}{\lambda_i^2 - bC},$$

$$\alpha_c^u = \frac{\left(-(a+h_w b)(bC+\lambda_i^2) + 2\sqrt{\lambda_i^4(a+h_w b)^2 - 4bR_{min}(3\lambda_i^2+bC)(\lambda_i^2-bC)}\right)C}{(3\lambda_i^2+bC)(\lambda_i^2-bC)}.$$

Proof: The outline of the proof is as follows. We first study the effects of the retailer's IR constraint and production constraint on the wholesale price individually. Then, we combine the two effects and determine the maximum feasible commission rate. Lastly, we determine the optimal wholesale price structure for different system settings.

<u>Retailer's IR Constraint</u>: We know from Lemma B4 that $p_r^*(p_w, \alpha) = \frac{1}{2b}(a + p_w b - \frac{\lambda_i^2\alpha}{C})$, so the retailer's profit is

$$\begin{aligned} \Pi_r &= p_r^*(a - bp_r^*) - p_w Q^* \\ &= \frac{(aC - p_w bC + \lambda_i^2\alpha)(aC - p_w bC - \lambda_i^2\alpha)}{4bC^2} \end{aligned}$$

In order to satisfy the retailer's IR constraint ($\Pi_r \geq R_{min}$), the wholesale price should be

$$p_w \geq \frac{aC + \sqrt{\lambda_i^4\alpha^2 + 4R_{min}bC^2}}{bC} \qquad \text{or} \qquad p_w \leq \frac{aC - \sqrt{\lambda_i^4\alpha^2 + 4R_{min}bC^2}}{bC}.$$

We find that when $p_w = \frac{aC+\sqrt{\lambda_i^4\alpha^2+4R_{min}bC^2}}{bC}$, the corresponding optimal order quantity $Q^* = -\frac{-\sqrt{\lambda_i^4\alpha^2+4R_{min}bC^2}+\lambda_i^2\alpha}{2C}$, which is less than zero for $\alpha \geq 0$. Since Q^* is decreasing in p_w, when $p_w \geq \frac{aC+\sqrt{\lambda_i^4\alpha^2+4R_{min}bC^2}}{bC}$, the optimal order quantity $Q^* \leq -\frac{-\sqrt{\lambda_i^4\alpha^2+4R_{min}bC^2}+\lambda_i^2\alpha}{2C} \leq 0$. Hence, we focus on the second condition

$$p_w \leq \frac{aC - \sqrt{\lambda_i^4\alpha^2 + 4R_{min}bC^2}}{bC},$$

and define the upper bound on p_w imposed by the retailer's IR constraint as follows:

$$p_w^u = \frac{aC - \sqrt{\lambda_i^4\alpha^2 + 4R_{min}bC^2}}{bC}.$$

Note that p_w^u is an upper bound on the wholesale price corresponding to the highest wholesale price the retailer will accept to stay in the game. Hence, p_w^u is the optimal wholesale price when the retailer's IR constraint is binding.

<u>Production Constraint</u>: Note that it does not make sense for the manufacturer to set wholesale price, or for the salesperson to exert effort, such that the resulting order quantity is greater than x, the manufacturer's production level. This imposes lower bound p_w^l on the wholesale price such that $Q^* \leq x$, where

$$p_w^l = arg_{p_w}\Big\{Q^* = x\Big\} = \frac{-2xC + aC + \lambda_i^2\alpha}{bC}.$$

Note that p_w^l is the lower bound on the wholesale price corresponding to the lowest wholesale price the manufacturer can use to ensure no shortage.

<u>Manufacturer's Problem</u>: Given the optimal responses of the retailer (Q^*) and the salesperson (e^*), the manufacturer profit is

$$\Pi_w = \frac{(aC - p_w bC + \lambda_i^2\alpha)(p_w + h_w - \alpha)}{2C} - x(c + h_w) - S,$$

which is concave in p_w and maximized at

$$p_w^o = \frac{1}{2b}\Big(a - h_w b + b\alpha + \frac{\lambda_i^2\alpha}{C}\Big),$$

for all $\alpha \geq 0$. Note that p_w^o is the optimal wholesale price without taking into account the production constraint or the retailer's IR constraint.

Next we consider the effect of production and retailer's IR constraints on the wholesale price. Let α_c^u be the commission rate that results in $p_w^o = p_w^u$. Let α_c^l be the commission rate that results in $p_w^o = p_w^l$.

Comparing p_w^o and p_w^u, we find that $p_w^o = p_w^u$ when

$$\alpha = \alpha_c^u \equiv arg_\alpha\Big\{p_w^o = p_w^u\Big\} = \frac{\Big(-(a + h_w b)(bC + \lambda_i^2) + 2\sqrt{\lambda_i^4(a + h_w b)^2 - 4bR_{min}(3\lambda_i^2 + bC)(\lambda_i^2 - bC)}\Big)C}{(3\lambda_i^2 + bC)(\lambda_i^2 - bC)}.$$

Since $p_w^u - p_w^o$ is decreasing in α. So, when $\alpha > \alpha_c^u$, $p_w^u < p_w^o$, which means that the retailer's IR constraint is binding; when $\alpha \leq \alpha_c^u$, $p_w^u \geq p_w^o$, which means that the retailer's IR constraint is not binding.

Note that, when $\lambda_i^2 > bC$, if $R_{min} > \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, α_c^u does not exist because $p_w^u < p_w^o$ for all real values of α.

Hence, if we neglect the effect of the production constraint, the optimal wholesale price has the following structure:
(i) If $\lambda_i^2 > bC$ and $R_{min} > \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, then

$$p_w^* = p_w^u \qquad : \qquad \alpha \geq 0$$

(ii) If $\lambda_i^2 > bC$ and $R_{min} \leq \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, or $\lambda_i^2 \leq bC$, then

$$p_w^* = \begin{cases} p_w^o & : \text{ if } 0 \leq \alpha \leq \alpha_c^u \\ p_w^u & : \text{ if } \alpha \geq \alpha_c^u \end{cases}$$

Comparing p_w^o and p_w^l, we find that $p_w^o = p_w^l$ when

$$\alpha = \alpha_c^l \equiv arg_\alpha\Big\{p_w^o = p_w^l\Big\} = \frac{(4x - a - h_w b)C}{\lambda_i^2 - bC}.$$

- When $\lambda_i^2 > bC$, then $p_w^o - p_w^l$ is decreasing in α. So, when $\alpha > \alpha_c^l$, $p_w^l > p_w^o$, which means that the production constraint is binding; when $\alpha \leq \alpha_c^l$, $p_w^l \leq p_w^o$, which implies that the production constraint is not binding. If we neglect the effect of the retailer's IR constraint, the optimal wholesale price has the following structure:

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } 0 \leq \alpha \leq \alpha_c^l \\ p_w^l & : \quad \text{if } \alpha \geq \alpha_c^l \end{cases}$$

- When $\lambda_i^2 \leq bC$, then $p_w^o - p_w^l$ is increasing in α. So, when $\alpha < \alpha_c^l$, $p_w^l > p_w^o$, which means that the production constraint is binding; when $\alpha \geq \alpha_c^l$, $p_w^l \leq p_w^o$, which implies that the production constraint is not binding. If we neglect the effect of the retailer's IR constraint, the optimal wholesale price has the following structure:

$$p_w^* = \begin{cases} p_w^l & : \quad \text{if } 0 \leq \alpha \leq \alpha_c^l \\ p_w^o & : \quad \text{if } \alpha \geq \alpha_c^l \end{cases}$$

Comparing α_c^u and α_c^l,

$$\alpha_c^u - \alpha_c^l = \frac{2C\Big(\lambda_i^2(a+h_w b) - 2x(bC+3\lambda_i^2) + \sqrt{\lambda^4(a+h_w b)^2 - 4R_{min}b(\lambda_i^2 - bC)(bC+3\lambda_i^2)}\Big)}{(\lambda_i^2 - bC)(3\lambda_i^2 + bC)}.$$

- If $\lambda_i^2 > bC$, we find that $\alpha_c^u = \alpha_c^l$ when

$$x = x_c \equiv \frac{\lambda_i^2(a+h_w b) + \sqrt{\lambda^4(a+h_w b)^2 - 4R_{min}b(\lambda_i^2 - bC)(3\lambda_i^2 + bC)}}{2(3\lambda_i^2 + bC)},$$

  and $\alpha_c^u - \alpha_c^l$ is decreasing in x. When $x \geq x_c$, then $\alpha_c^u \leq \alpha_c^l$, which implies that the retailer's IR constraint dominates the production constraint; when $x < x_c$, then $\alpha_c^u > \alpha_c^l$, which implies that the production constraint dominates the retailer's IR constraint. Moreover, x_c is decreasing in R_{min}, and is minimized at $R_{min} = \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$ and maximized at $R_{min} = 0$. Moreover, when $\lambda_i^2 > bC$, for $R_{min} \leq \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, then $x_c > Q_{ns}^*$.
- If $\lambda_i^2 \leq bC$, then $\alpha_c^u \geq \alpha_c^l$ when $x > Q_{ns}^*$.

Based on the above findings, if we neglect the feasibility requirement, we can summarize the optimal wholesale price structure as the following:

(i) If $\lambda_i^2 > bC$, $R_{min} > \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, and $x > Q_{ns}^*$, then

$$p_w^* = p_w^u \qquad : \qquad \alpha \geq 0$$

(ii) If $\lambda_i^2 > bC$, $R_{min} \leq \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, and $x > Q_{ns}^*$, then

- If $x \geq x_c$ (which results in $\alpha_c^u \leq \alpha_c^l$), then

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } 0 \leq \alpha \leq \alpha_c^u \\ p_w^u & : \quad \text{if } \alpha \geq \alpha_c^u \end{cases}$$

- If $x < x_c$ (which results in $\alpha_c^u > \alpha_c^l$), then

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } 0 \leq \alpha \leq \alpha_c^l \\ p_w^l & : \quad \text{if } \alpha \geq \alpha_c^l \end{cases}$$

(iii) If $\lambda_i^2 \leq bC$ and $x > Q_{ns}^*$, then

$$p_w^* = \begin{cases} p_w^l & : \quad \text{if } 0 \leq \alpha \leq \alpha_c^l \\ p_w^o & : \quad \text{if } \alpha_c^l \leq \alpha \leq \alpha_c^u \\ p_w^u & : \quad \text{if } \alpha \geq \alpha_c^u \end{cases}$$

Maximum Feasible Commission Rate: We define α_f to be the maximum feasible commission rate such that both the retailer's IR constraint and the production constraint are met. In the following, we determine the value of α_f and incorporate its effect on the optimal wholesale price structure.

Comparing p_w^u and p_w^l, we find that, $p_w^u = p_w^l$ when

$$\alpha = \alpha_f \equiv \frac{(x^2 - R_{min}b)C}{\lambda_i^2 x}.$$

Now, we incorporate the maximum feasible commission rate to the optimal wholesale price structure and conclude the following:

(i) If $\lambda_i^2 > bC$, $R_{min} > \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, and $x \geq Q_{ns}^*$, then

$$p_w^* = p_w^u \qquad : \quad \text{if } 0 \leq \alpha \leq \alpha_f$$

(ii) If $\lambda_i^2 > bC$, $R_{min} \leq \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, and $x \geq Q_{ns}^*$, then

- If $x \geq x_c$, then

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } 0 \leq \alpha \leq \alpha_c^u \\ p_w^u & : \quad \text{if } \alpha_c^u \leq \alpha \leq \alpha_f \end{cases}$$

- If $x < x_c$, then

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } 0 \leq \alpha \leq \alpha_c^l \\ p_w^l & : \quad \text{if } \alpha_c^l \leq \alpha \leq \alpha_f \end{cases}$$

(iii) If $\lambda_i^2 \leq bC$ and $x \geq Q_{ns}^*$, then

$$p_w^* = \begin{cases} p_w^l & : \quad \text{if } 0 \leq \alpha \leq \alpha_c^l \\ p_w^o & : \quad \text{if } \alpha_c^l \leq \alpha \leq \alpha_c^u \\ p_w^u & : \quad \text{if } \alpha_c^u \leq \alpha \leq \alpha_f \end{cases} \qquad \blacksquare$$

LEMMA B5 *In the inventory allocation model, the commission plan is feasible if* $x \geq x_{min} = \sqrt{R_{min}b}$.

Proof: The outline of this is proof is as follows. We first determine the feasibility condition on the production level imposed by the retailer's IR constraint and the production constraint, and then we determine the feasibility condition imposed by the salesperson's IR constraint.

Retailer's IR Constraint and Production Constraint: Recall from Proposition B4 that, in order to meet both the production and the retailer's IR constraints, $\alpha_f = \frac{(x^2-R_{min}b)C}{\lambda_i^2 x} \geq 0$ must hold. So, $x \geq \sqrt{R_{min}b}$ is a necessary condition for a commission plan to be feasible in an inventory allocation model.

Salesperson's IR Constraint: We claim that, in **SC2**, the manufacturer can always satisfy the salesperson's IR constraint by adjusting the fixed amount S in the commission plan. We examine this claim for the following three cases:

1. $\lambda_i^2 > bC$, $R_{min} > \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, and $x \geq Q_{ns}^*$
2. $\lambda_i^2 > bC$, $R_{min} \leq \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, and $x \geq Q_{ns}^*$
3. $\lambda_i^2 \leq bC$ and $x \geq Q_{ns}^*$

We only present the proof for case 1 in which $\lambda_i^2 > bC$, $R_{min} > \frac{\lambda_i^4(a+h_w b)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$, and $x \geq Q_{ns}^*$. The proofs for cases 2 and 3 are similar and are therefore omitted.

__Case 1:__ From Proposition B4, we know that, in this case, $p_w^* = p_w^u$. The corresponding salesperson utility is

$$U(p_w^u, \alpha, S) = \frac{\alpha(2\sqrt{\lambda_i^4\alpha^2 + 4R_{min}bC^2} + \lambda_i^2\alpha)}{4C} + S,$$

and $U(p_w^u, \alpha, S) \geq U_{min}$ if $\alpha \geq \underline{\alpha}^u$ where $\underline{\alpha}^u$ is the lower bounds on the commission rate imposed by the salesperson's IR constraint. Define $\tilde{U}_{min} = U_{min} - S$, we have

$$\underline{\alpha}^u = \frac{2\sqrt{3\Big(- 2R_{min}bC^2 - \lambda_i^2\tilde{U}_{min}C + 2\sqrt{R_{min}^2 b^2 C^4 + R_{min}bC^3\lambda_i^2\tilde{U}_{min} + \lambda_i^4\tilde{U}_{min}^2 C^2}\Big)}}{3\lambda_i^2}.$$

To meet the salesperson's IR constraint (i.e., $U(p_w^u, \alpha, S) \geq U_{min}$) when $0 \leq \alpha \leq \alpha_f$, there should exist a commission rate α in region $0 \leq \alpha \leq \alpha_f$ such that $\alpha \geq \underline{\alpha}^u$. So, it is necessary for $\underline{\alpha}^u \leq \alpha_f$.

Comparing $\underline{\alpha}^u$ and α_f, we find that $\underline{\alpha}^u \leq \alpha_f$ if $u_{c2}^u \leq \tilde{U}_{min} \leq u_{c1}^u$, where

$$u_{c1}^u = \frac{(3x^2 + R_{min}b)(x^2 - R_{min}b)C}{4\lambda_i^2 x^2}(\geq 0),$$

and

$$u_{c2}^u = -\frac{(x^2 + 3R_{min}b)(x^2 - R_{min}b)C}{4\lambda_i^2 x^2}(\leq 0).$$

Therefore, when $0 \leq \alpha \leq \alpha_f$, we have $U(p_w^u, \alpha, S) \geq U_{min}$ if $u_{c2}^u \leq U_{min} - S \leq u_{c1}^u$, which can always be satisfied by adjusting the value of S in the commission plan. So we conclude that the manufacturer can always satisfy the salesperson's IR constraint by adjusting the fixed amount S in a commission plan. ■

B.6. Inventory Allocation Model: Comparing SC2 with SC1 Under the Commission Plan

In this section, we examine the conditions under which it is more beneficial for the manufacturer to hire an inventory allocation salesperson under a commission plan than not to hire one, i.e., the manufacturer has greater profit in **SC2** than in **SC1**.

THEOREM 1b *In the inventory allocation model under a feasible commission plan, if it is beneficial for the manufacturer to hire a salesperson, then*

(*i*) *the resulting optimal retailer profit can be greater than that in a supply chain without a salesperson, and*

(*ii*) *the resulting total supply chain profit is at least as large as than that in a supply chain without a salesperson.*

Proof for part (i): We prove this part by showing there is at least one set of feasible input parameters such that (1) the manufacturer earns a greater profit with an inventory allocation salesperson in the supply chain than without one (i.e., $\Pi_w^* > \Pi_{w,ns}^*$), and (2) the retailer also earns at least as much profit with such a salesperson as without one (i.e., $\Pi_r^* \geq \Pi_{r,ns}^*$).

Consider the case where

- $\lambda_i^2 > bC$,
- $x \geq x_c$,
- $U_{min} = \frac{C(3\lambda_i^2+bC)(\lambda_i^2-bC)(13\lambda_i^4+2bC\lambda_i^2+b^2C^2)(a+h_wb)^2}{256\lambda_i^6(\lambda_i^2+bC)^2}$, and
- $R_{min} = \Pi_{r,ns}^*$, which guarantees that $\Pi_r^* \geq \Pi_{r,ns}^*$ must hold.

So, in the following we will show that under the above conditions, the manufacturer earns a greater profit with an inventory allocation salesperson in the supply chain than without one.

Since $R_{min} = \Pi_{r,ns}^*$, $R_{min} \leq \frac{\lambda_i^4(a+h_wb)^2}{4b(3\lambda_i^2+bC)(\lambda_i^2-bC)}$ holds. Also, $\lambda_i^2 > bC$ and $x \geq x_c$, so this is Case (ii) in Proposition B4. By substitution, we find that when $R_{min} = \Pi_{r,ns}^*$, $\alpha_c^u = 0$. So, by Proposition B4, the optimal wholesale price under a feasible commission plan is $p_w^* = p_w^u$ for all $0 \leq \alpha \leq \alpha_f$.

When $p_w^* = p_w^u$, the manufacturer profit in **SC2** is

$$\Pi_w(p_w^u) = \frac{(aC + h_w bC - \lambda_i^2\alpha - bC\alpha)(\lambda_i^2\alpha + \sqrt{\lambda_i^4\alpha^2 + 4bC^2R_{min}})}{2bC^2} - x(h_w + c) - 2R_{min} - S,$$

which is concave in α and is maximized at α^{u*} where

$$\alpha^{u*} = \frac{C\Big(\lambda_i^4(a + h_w b)^2 - 4R_{min}b(\lambda_i^2 + bC)^2\Big)}{2\lambda_i^4(a + h_w b)(\lambda_i^2 + bC)},$$

and it is easy to see that $\alpha^{u*} \leq \alpha_f$. Given $R_{min} = \Pi^*_{r,ns}$, we have

$$\tilde{\alpha} \equiv \alpha^{u*}(R_{min} = \Pi^*_{r,ns}) = \frac{C(3\lambda_i^2 + bC)(\lambda_i^2 - bC)(a + h_w b)}{4\lambda_i^4(\lambda_i^2 + bC)}.$$

Also,

$$S^* = \frac{C(3\lambda_i^2 + bC)(\lambda_i^2 - bC)(13\lambda_i^4 + 2bC\lambda_i^2 + b^2C^2)(a + h_w b)^2}{256\lambda_i^6(\lambda_i^2 + bC)^2} - U_{min}.$$

Now, we need to show that $\Pi_w(p_w^u, \tilde{\alpha}, S^*) > \Pi^*_{w,ns}$ when

- $R_{min} = \Pi^*_{r,ns}$, and
- $U_{min} = \frac{C(3\lambda_i^2+bC)(\lambda_i^2-bC)(13\lambda_i^4+2bC\lambda_i^2+b^2C^2)(a+h_wb)^2}{256\lambda_i^6(\lambda_i^2+bC)^2}$, which implies that $S^* = 0$.

$$\Pi_w(p_w^* = p_w^u, \alpha, S^* = 0) = \frac{(aC + h_w bC - \lambda_i^2\alpha - bC\alpha)(\lambda_i^2\alpha + \sqrt{\lambda_i^4\alpha^2 + 4bC^2R_{min}})}{2bC^2} - x(h_w + c) - 2\frac{(a + h_w b)^2}{16b} \geq \Pi^*_{w,ns},$$

when

$$\alpha \geq \hat{\alpha} \equiv \frac{2C(\lambda_i^2 - bC)(a + h_w b)}{(\lambda_i^2 + bC)(3\lambda_i^2 - BC)},$$

and it is easy to see that $\hat{\alpha} \leq \alpha_f$.

Comparing $\hat{\alpha}$ with $\tilde{\alpha}$, we have

$$\hat{\alpha} - \tilde{\alpha} = \frac{(\lambda_i^2 - bC)^2(a + h_w b)C}{4\lambda_i^4(3\lambda_i^2 - BC)} > 0.$$

Recall that, the manufacturer profit function is concave in α, so we have $\Pi_w(p_w^* = p_w^u, \alpha^* = \tilde{\alpha}, S^* = 0) > \Pi^*_{w,ns}$.

Therefore, under the above conditions, both the retailer and the manufacturer can be at least as well off in a supply chain with a salesperson as that without one.

Proof for part (ii): We prove this part of the theorem by showing that $\Pi^*_{total} \geq \Pi^*_{total,ns}$ holds for all of the three possible optimal wholesale price cases from Proposition B4, i.e.,

- Case 1-a: $p_w^* = p_w^l$, when $\lambda_i^2 > bC$;
- Case 1-b: $p_w^* = p_w^l$, when $\lambda_i^2 \leq bC$;
- Case 2-a: $p_w^* = p_w^o$, when $\lambda_i^2 > bC$;
- Case 2-b: $p_w^* = p_w^o$, when $\lambda_i^2 \leq bC$;
- Case 3-a: $p_w^* = p_w^u$, when $\lambda_i^2 > bC$;
- Case 3-b: $p_w^* = p_w^u$, when $\lambda_i^2 \leq bC$.

The proofs of all other cases are similar to that of the **Case 1-a** and are therefore omitted.

Case 1-a: We will show that the model meets the conditions in Proposition B1 including (1) $Q^* \geq Q^*_{ns} = \frac{a+h_wb}{4}$, and (2) $Q^* \leq \bar{Q}^*_{ns} = \frac{3(a+h_wb)}{4}$.

Condition 1: We know, by the definition of p_w^l, that the resulting ordering quantity equals to the production level, i.e., $Q^*(p_w^l) = x$. Since we have assumed that $x \geq Q^*_{ns}$, then $Q^*(p_w^l) \geq Q^*_{ns}$.

Condition 2: By Proposition B4, when $\lambda_i^2 > bC$ and $p_w^* = p_w^l$, $Q_{ns}^* \le x < x_c$. Recall that

$$x_c = \frac{\lambda_i^2(a+h_w b) + \sqrt{\lambda^4(a+h_w b)^2 - 4R_{min}b(3\lambda_i^2 + Cb)(\lambda_i^2 - bC)}}{2(3\lambda_i^2 + bC)} < \bar{Q}_{ns}^* = \frac{3(a+h_w b)}{4}$$

when $\lambda_i^2 > bC$. Moreover, the optimal ordering quantity $Q^*(p_w^l) = x$. Therefore, $Q^*(p_w^l) = x < x_c < \bar{Q}_{ns}^*$.

In summary, $Q_{ns}^* \le Q^*(p_w^l) < \bar{Q}_{ns}^*$ when an inventory allocation salesperson is hired under the commission plan. By Proposition B1, we conclude that $\Pi_{total}^*(p_w^l) \ge \Pi_{total,ns}^*$. ■

ON-LINE APPENDIX B-2

Proofs for Demand Promotion Model

In this On-Line Appendix, we present the proofs of our results for the demand promotion model. Theorem 2, which includes both the first-best-salary plan and the commission plan is divided into Theorem 2a (which corresponds to the results for the first-best-salary plan) and Theorem 2b (which corresponds to the results for the commission plan). Theorems 2a and 2b are proven separately. Before we present the proofs of Theorems 2a and 2b, we need to present the following results.

B.7. Demand Promotion Salesperson Model Under The First-Best Salary Plan

To simplify notation, unless needed, we omit (e) from $p_w(e)$, $p_r(e)$, $U(e)$ and $W(e)$; we omit (p_w, e) from $Q(p_w, e)$ and $p_r(p_w, e)$; and we omit (p_r, Q) from $\Pi_r(p_r, Q)$.

LEMMA B6 *In the demand promotion model under the first-best-salary plan, given a certain wholesale price p_w and a specified selling effort level e, the optimal retailer order quantity and retail price are:*

$$Q^*(p_w, e) = \frac{a_0(1+\lambda_m e) - p_w b}{2}, \tag{10}$$

$$p_r^*(p_w, e) = \frac{a_0(1+\lambda_m e) + p_w b}{2b}. \tag{11}$$

Proof: Given a wholesale price (p_w), the retailer profit for *any* price-setting problem is maximized when $Q = D$ (Petruzzi and Dada 1999). The retailer profit is:

$$\Pi_r(p_r, Q) = p_r(a - bp_r) - p_w Q = (p_r - p_w)(a - bp_r).$$

Given a fixed Q, the profit function $\Pi_r(p_r, Q)$ is concave in p_r and is maximized at:

$$p_r^*(p_w, e) = \frac{a + p_w b}{2b} = \frac{a_0(1+\lambda_m e) + p_w b}{2b}.$$

Substituting p_r^* back to Π_r, it is clear that Π_r is also concave in Q and is maximized at:

$$Q^*(p_w, e) = \frac{a - p_w b}{2} = \frac{a_0(1+\lambda_m e) - p_w b}{2}. \qquad ■$$

LEMMA B7 *In the demand promotion model, the first-best salary plan is feasible when $x \ge \sqrt{R_{min} b}$.*

Proof: Recall that a compensation plan in **SC2** is feasible if both the retailer and the salesperson are willing to participate in the game. Hence, a compensation plan is feasible when, under production level x, there exists an effort level $e > 0$ such that the IR constraints of the salesperson and the retailer are met (i.e., $U \ge U_{min}$,

$\Pi_r \geq R_{min}$). To find the conditions under which these constraints are met, we first examine the strategies of the manufacturer and the retailer using backward induction.

Retailer's Problem: From Lemma B6, we know that given a wholesale price (p_w) and salesperson effort level (e), the retailer orders a quantity of $Q^*(p_w, e)$ and sets the retail price at $p_r^*(p_w, e)$ as shown in (10) and (11). Thus, the retailer profit function becomes:

$$\Pi_r = \frac{(a_0 + a_0\lambda_m e - p_w b)^2}{4b}.$$

Hence, $\Pi_r \geq R_{min}$ if

$$p_w \leq \frac{a_0\lambda_m e + a_0 - 2\sqrt{R_{min}b}}{b} = p_w^u, \tag{12}$$

where p_w^u is an upper bound for the wholesale price corresponding to the highest wholesale price the retailer will accept to stay in the game.

Salesperson's Problem: In **SC2**, the manufacturer satisfies the salesperson's IR constraint.[4] Furthermore, we know that the salesperson's IR constraint is met at its minimum under the first-best salary plan, i.e., $U = U_{min}$.

Manufacturer's Problem: Note that it does not make sense for the manufacturer to set wholesale price, or for the salesperson to exert effort, such that the resulting order quantity is greater than x, the manufacturer's production level. Hence, this imposes a lower bound p_w^l on the wholesale price such that $Q^* \leq x$, where

$$p_w^l = arg_{p_w}\Big\{Q^* = x\Big\} = \frac{a_0 + a_0\lambda_m e - 2x}{b}. \tag{13}$$

Therefore, the first-best salary plan is only feasible if there exists an effort $e > 0$ such that $p_w^l \leq p_w \leq p_w^u$. Comparing Equations (12) and (13), we find that

$$p_w^u - p_w^l = \frac{2(x - \sqrt{R_{min}b})}{b},$$

and is non-negative if $x \geq \sqrt{R_{min}b}$ (note that $R_{min}b \geq 0$ because $R_{min} \geq 0$ and $b \geq 0$ by assumption). Hence, $x \geq \sqrt{R_{min}b}$ is a necessary condition for the first-best salary plan to be feasible. ■

LEMMA B8 *In the demand promotion model, if there exists a feasible first-best salary plan, there are two thresholds, e_c^l and e_c^u, on the salesperson's effort level that determine the structure of the manufacturer's optimal wholesale price under the first-best salary plan as follows:*

$$p_w^* = \begin{cases} p_w^u = \frac{a_0(1+\lambda_m e) - 2\sqrt{R_{min}b}}{b} & : \ if \ \ 0 \leq e \leq e_c^u \\ p_w^o = \frac{a_0(1+\lambda_m e) - h_w b}{2b} & : \ if \ \ e_c^u \leq e \leq e_c^l \\ p_w^l = \frac{a_0(1+\lambda_m e) - 2x}{b} & : \ if \ \ e_c^l \leq e \end{cases}$$

where

$$e_c^l = \frac{4x - (a_0 + h_w b)}{a_0\lambda_m}, \qquad e_c^u = \frac{4\sqrt{R_{min}b} - (a_0 + h_w b)}{a_0\lambda_m}.$$

Proof: Given a fixed effort level, the manufacturer profit with retailer's optimal order quantity Q^* (Equation (10)) is:

$$\begin{aligned} \Pi_w(p_w, e) &= p_w Q^* - cx - h_w(x - Q^*) - U_{min} - Ce^2 \\ &= p_w \frac{a_0(1+\lambda_m e) - p_w b}{2} - cx - h_w\Big(x - \frac{a_0(1+\lambda_m e) - p_w b}{2}\Big) - U_{min} - Ce^2. \end{aligned}$$

Given any effort level e, the manufacturer profit function Π_w is maximized at

$$p_w^o = \frac{a_0(1+\lambda_m e) - h_w b}{2b}. \tag{14}$$

[4]Note that in **SC2**, we assume that the manufacturer is willing to pay the salesperson as much as necessary to keep the salesperson in the supply chain. Later in Proposition B7, we show whether it is beneficial for the manufacturer to do this.

Note that p_w^o is the optimal wholesale price without taking into account of the production constraint or the retailer's IR constraint. To incorporate these constraints, recall from Lemma B7 that the production constraint imposes a lower bound on the wholesale price, p_w^l, and the retailer's IR constraint imposes an upper bound on the wholesale price, p_w^u. Hence, p_w^l is the optimal wholesale price in cases where the production constraint is binding, while p_w^u is the optimal wholesale price in cases where the retailer's IR constraint is binding. Let e_c^l be the effort level that results in $p_w^o = p_w^l$.

Using Equations (13) and (14), we get:

$$p_w^o - p_w^l = \frac{-a_0 - a_0\lambda_m e - h_w b + 4x}{2b},$$

and hence

$$e_c^l = arg_e\Big\{p_w^o = p_w^l\Big\} = \frac{4x - (a_0 + h_w b)}{a_0\lambda_m}. \tag{15}$$

Since $p_w^o - p_w^l$ is decreasing in e, we know that when the effort level $e \geq e_c^l$, then $p_w^o < p_w^l$, meaning that the production constraint is binding. Moreover, when the effort level $e \leq e_c^l$, then $p_w^o \geq p_w^l$, meaning that the production constraint is not binding. Hence, if we neglect the effect of the retailer's IR constraint, the optimal wholesale price has the following structure:

$$p_w^* = \begin{cases} p_w^o & = \frac{a_0(1+\lambda_m e) - h_w b}{2b} & : \quad \text{if } \; 0 \leq e \leq e_c^l \\ p_w^l & = \frac{a_0(1+\lambda_m e) - 2x}{b} & : \quad \text{if } \; e_c^l \leq e. \end{cases} \tag{16}$$

On the other hand, if e_c^u is the effort level that results in $p_w^o = p_w^u$, then by comparing p_w^u (Equation (12)) with p_w^o (Equation (14)), we get

$$p_w^u - p_w^o = \frac{a_0\lambda_m e + a_0 - 4\sqrt{R_{min} b} + h_w b}{2b},$$

and hence

$$e_c^u = arg_e\Big\{p_w^u = p_w^o\Big\} = \frac{4\sqrt{R_{min} b} - (a_0 + h_w b)}{a_0\lambda_m}. \tag{17}$$

Since $p_w^u - p_w^o$ is increasing in e, we know that when the effort level $e < e_c^u$, then $p_w^u < p_w^o$, meaning that the retailer's IR constraint is binding. Moreover, when the effort level $e \geq e_c^u$, then $p_w^u \geq p_w^o$, meaning that the retailer's IR constraint is not binding. Hence, if we neglect the effect of the production constraint, the optimal wholesale price has the following structure:

$$p_w^* = \begin{cases} p_w^u & = \frac{a_0(1+\lambda_m e) - 2\sqrt{R_{min} b}}{b} & : \quad \text{if } \; 0 \leq e \leq e_c^u \\ p_w^o & = \frac{a_0(1+\lambda_m e) - h_w b}{2b} & : \quad \text{if } \; e_c^u \leq e. \end{cases} \tag{18}$$

Comparing the two critical efforts e_c^l (Equation (15)) and e_c^u (Equation (17)), we find that

$$e_c^l - e_c^u = \frac{4(x - \sqrt{R_{min} b})}{a_0\lambda_m},$$

which is greater than zero for any feasible first-best salary plan (as shown in Lemma 2 that $x \geq \sqrt{R_{min} b}$ guarantees feasibility). Hence, $e_c^u \leq e_c^l$ for all feasible first-best salary plans, and we can combine Equations (16) and (18) to get:

$$p_w^* = \begin{cases} p_w^u & = \frac{a_0(1+\lambda_m e) - 2\sqrt{R_{min} b}}{b} & : \quad \text{if } \; 0 \leq e \leq e_c^u \\ p_w^o & = \frac{a_0(1+\lambda_m e) - h_w b}{2b} & : \quad \text{if } \; e_c^u \leq e \leq e_c^l \\ p_w^l & = \frac{a_0(1+\lambda_m e) - 2x}{b} & : \quad \text{if } \; e_c^l \leq e \end{cases} \qquad \blacksquare$$

PROPOSITION B5 *In the demand promotion model, if there exists a feasible first-best salary plan, the optimal strategy for the manufacturer under the first-best salary plan is as follows:*

(*i*) *If* $a_0^2\lambda_m^2 \leq 8bC$, *and* $x < x_c$, *or if* $a_0^2\lambda_m^2 > 8bC$, *then*

$$p_w^* = p_w^l = \frac{a_0(1+\lambda_m e^*) - 2x}{b}, \qquad e^* = e^{l*} = \frac{a_0\lambda_m x}{2bC}.$$

(ii) If $a_0^2\lambda_m^2 \leq 8bC$, $x \geq x_c$ and $R_{min} \leq R_c$, then

$$p_w^* = p_w^o = \frac{a_0(1+\lambda_m e^*) - h_w b}{2b}, \qquad e^* = e^{o*} = \frac{a_0\lambda_m(a_0 + h_w b)}{8bC - a_0^2\lambda_m^2}.$$

(iii) If $a_0^2\lambda_m^2 \leq 8bC$, $x \geq x_c$ and $R_{min} > R_c$, then

$$p_w^* = p_w^u = \frac{a_0(1+\lambda_m e^*) - 2\sqrt{R_{min}b}}{b}, \qquad e^* = e^{u*} = \frac{a_0\lambda_m\sqrt{R_{min}b}}{2bC}.$$

where

$$x_c = \frac{2bC(a_0 + h_w b)}{8bC - a_0^2\lambda_m^2} \quad \text{and} \quad R_c = \frac{4(a_0 + h_w b)^2 C^2 b}{(8bC - a_0^2\lambda_m^2)^2}. \tag{19}$$

Proof: The outline of the proof is as follows. For every case, we find the optimal effort level that maximizes the manufacturer profit. Then using Lemma B8, we find the corresponding optimal wholesales price.

To save space, we only present the proofs for the first part of case (i) when $a_0^2\lambda_m^2 \leq 8bC$ and $x < x_c$. The proof for the case when $a_0^2\lambda_m^2 > 8bC$, and cases (ii) and (iii) are similar and are therefore omitted.

When $a_0^2\lambda_m^2 \leq 8bC$ and $x < x_c$, from Lemma B7 we know that the first-best salary plan is feasible only if $x \geq \sqrt{R_{min}b}$. Hence, for this case where $x < x_c$, there exists at least one feasible plan if

$$x_c = \frac{2bC(a_0 + h_w b)}{8bC - a_0^2\lambda_m^2} > \sqrt{R_{min}b} \qquad \text{or} \qquad R_{min} < \frac{4(a_0 + h_w b)^2 bC^2}{(8bC - a_0^2\lambda_m^2)^2} = R_c$$

We now study the behavior of the manufacturer profit function based on effort level e in the following three cases:

(a) $0 \leq e \leq e_c^u$
(b) $e_c^u \leq e \leq e_c^l$
(c) $e_c^l \leq e$

We will show that when $0 \leq e \leq e_c^l$, the manufacturer's optimal profit function is increasing in e. On the other hand, when $e \geq e_c^l$ the manufacturer's optimal profit function is concave in e with its maximum within region $e \geq e_c^l$. This implies that when $a_0^2\lambda_m^2 \leq 8bC$ and $x < x_c$, the optimal effort level e^* that results in the maximum value for the manufacturer profit should be within region $e \geq e_c^l$, which according to Lemma B8 results in $p_w^* = p_w^l$. We also drive the corresponding optimal effort level e^*.

Case (ia): In this case, we have $0 \leq e \leq e_c^u$. From Lemma B8, we conclude that $p_w^* = p_w^u$. The manufacturer profit function is therefore:

$$\Pi_w(p_w^u) = \frac{1}{b}\left\{\Big(a_0(1+\lambda_m e) + h_w b\Big)\sqrt{R_{min}b}\right\} - (U_{min} + 2R_{min} + Ce^2) - x(c + h_w),$$

which is concave in e, since

$$\frac{d^2}{d^2e}\Pi_w(p_w^u) = -2C < 0$$

and is maximized at

$$e^{u*} = \frac{a_0\lambda_m\sqrt{R_{min}b}}{2bC}. \tag{20}$$

Comparing e^{u*} and e_c^u from Equation (17), we find that:

$$e^{u*} - e_c^u = \frac{2bC(a_0 + h_w b) + (a_0^2\lambda_m^2 - 8bC)\sqrt{R_{min}b}}{2a_0 bC\lambda_m}.$$

It is easy to show that when $R_{min} = R_c$, then we have $e^{u*} - e_c^u = 0$, or $e^{u*} = e_c^u$. Moreover, when $a_0^2\lambda_m^2 \leq 8bC$, then $e^{u*} - e_c^u$ is decreasing in R_{min}. Therefore,

- If $R_{min} < R_c$, we have $e^{u*} > e_c^u$. In this case, e^{u*} corresponding to the maximum of the manufacturer profit function is outside region $0 \leq e \leq e_c^u$. Since the manufacturer profit function is concave, and since $e^{u*} > e_c^u$, then $\Pi_w(p_w^u)$ is increasing in e when $0 \leq e \leq e_c^u$.

- If $R_{min} > R_c$, then as discussed above, there exists no feasible plan.

Thus, we conclude that, if there exists a feasible plan, then the manufacturer's optimal profit function is increasing in e when $0 \leq e \leq e_c^u$.

Case (ib): In this case, we have $e_c^u \leq e \leq e_c^l$. From Lemma B8, we conclude that $p_w^* = p_w^o$. The manufacturer profit function is therefore:

$$\Pi_w(p_w^o) = \frac{1}{8b}\Big((a_0 + h_w b)^2 + 2a_0\lambda_m e(a_0 + h_w b) + e^2(a_0^2\lambda_m^2 - 8bC) - 8b(cx + h_w x + U_{min})\Big),$$

where

$$\frac{d^2}{d^2e}\Pi_w(p_w^o) = \frac{a_0^2\lambda_m^2 - 8bC}{4b}. \tag{21}$$

When $a_0^2\lambda_m^2 \leq 8bC$, then (21) is non-positive and therefore the manufacturer profit function $\Pi_w(p_w^o)$ is concave in e and is maximized at

$$e^{o*} = \frac{a_0\lambda_m(a_0 + h_w b)}{8bC - a_0^2\lambda_m^2}. \tag{22}$$

Comparing e^{o*} with e_c^l (see Equation (15)), it is easy to show that if $x < x_c$ (where $x_c = \frac{2bC(a_0+h_w b)}{8bC - a_0^2\lambda_m^2} \geq \sqrt{R_{min} b}$), then $e^{o*} > e_c^l$. Since the manufacturer's objective function $\Pi_w(p_w^o)$ is concave in e when $e_c^u \leq e \leq e_c^l$, and since $e^{o*} > e_c^l$, then $\Pi_w(p_w^o)$ is increasing in e when $e_c^u \leq e \leq e_c^l$, and its maximum occurs at e_c^l.

Case (ic): In this case we have $e \geq e_c^l$. From Lemma B8 we conclude that $p_w^* = p_w^l$. The manufacturer profit function is therefore:

$$\Pi_w(p_w^l) = \frac{1}{b}\Big(xa_0(1 + \lambda_m e) - bCe^2 - 2x^2 - xbc - U_{min} b\Big),$$

which is concave in e, since

$$\frac{d^2}{d^2e}\Pi_w(p_w^l) = -2C < 0$$

and is maximized at e^{l*}, where

$$e^{l*} = \frac{a_0\lambda_m x}{2bC}. \tag{23}$$

Comparing e^{l*} with e_c^l (see Equation (15)), we find that

$$e^{l*} - e_c^l = \frac{2bc(a_0 + h_w b) + x(a_0^2\lambda_m^2 - 8bC)}{2a_0 bC\lambda_m}.$$

It is easy to show that when the production level is at a critical level $x = x_c$, then we have $e^{l*} - e_c^l = 0$, or $e^{l*} = e_c^l$. Moreover, when $a_0^2\lambda_m^2 \leq 8bC$, then $e^{l*} - e_c^l$ is decreasing in x and $x_c > 0$. Therefore, when $x < x_c$, we have $e^{l*} > e_c^l$. The maximum of the manufacturer profit function occurs at e^{l*}, which is within region $e \geq e_c^l$.

It is also easy to show that the manufacturer profit function is continuous in e within the three regions. Furthermore, at the critical effort level e_c^u, we have $\Pi_w(p_w^u, e_c^u) = \Pi_w(p_w^o, e_c^u)$. Similarly, at the critical effort level e_c^l, we have $\Pi_w(p_w^l, e_c^l) = \Pi_w(p_w^o, e_c^l)$. Thus, the manufacturer profit function is continuous for $e \geq 0$.

We can combine the results in Cases (ia), (ib), and (ic) and conclude that the manufacturer's optimal profit function is increasing in e when $0 \leq e \leq e_c^l$ and is concave when $e \geq e_c^l$ with its maximum within that region. Thus, the optimal effort level $e^* = e^{l*}$ that results in the maximum value for the manufacturer profit will satisfy $e^{l*} > e_c^l$. According to Lemma B8, we therefore have $p_w^* = p_w^l$. Thus, if there exists a feasible first-best salary plan, then the optimal solution for the manufacturer is

$$p_w^* = \frac{a_0 + a_0\lambda_m e^* - 2x}{b}, \qquad e^* = \frac{a_0\lambda_m x}{2bC}. \qquad \blacksquare$$

B.8. Demand Promotion Model: Comparing SC2 with SC1 Under the First-Best Salary Plan

In this section, we examine the conditions under which it is more *beneficial* for the manufacturer to hire a demand promotion salesperson under the first-best salary plan than not to hire one, i.e., the manufacturer has greater profit in **SC2** than in **SC1**.

PROPOSITION B6 *In the demand promotion model under a feasible first-best salary plan, it is beneficial for the manufacturer to hire a salesperson (i.e., $\Pi_w^* \geq \Pi_{w,ns}^*$) if the salesperson's minimum utility is less than a critical threshold U_{m1}, where*

$$U_{m1} = \begin{cases} u_c^l \;=\; \frac{(a_0+h_wb)^2bC+2x^2(8bC-a_0^2\lambda_m^2)-8bCx(a_0+h_wb)}{8b^2C} & : \; if \; \{a_0^2\lambda_m^2 \leq 8bC \text{ and } x < x_c\} \\ & \quad or \; \{a_0^2\lambda_m^2 > 8bC\} \\ u_c^o \;=\; \frac{a_0^2\lambda_m^2(a_0+h_wb)^2}{8b(8bC-a_0^2\lambda^2)} & : \; if \; \{a_0^2\lambda_m^2 \leq 8bC, x \geq x_c \text{ and } R_{min} \leq R_c\} \\ u_c^u \;=\; \frac{2R_{min}(a_0^2\lambda_m^2-8Cb)+C\sqrt{R_{min}b}(8a_0+h_wb)-(a_0+h_wb)^2C}{8bC} & : \; if \; \{a_0^2\lambda_m^2 \leq 8bC, x \geq x_c \text{ and } R_{min} > R_c\} \end{cases} \tag{24}$$

Proof: It is beneficial for the manufacturer to hire a salesperson if the manufacturer profit is greater in a supply chain with a salesperson (**SC2**) than that in a supply chain without a salesperson (**SC1**). Thus, we need to compare the manufacturer profit in **SC2** with that in **SC1**. We examine the three possible manufacturer's optimal strategy in **SC2** from Proposition B5. The three cases are:

1. $p_w^* = p_w^l$ *and* $e^* = e^{l*}$
2. $p_w^* = p_w^o$ *and* $e^* = e^{o*}$
3. $p_w^* = p_w^u$ *and* $e^* = e^{u*}$

Case 1: In this case, we consider $p_w^* = p_w^l$ and $e^* = e^{l*}$. The manufacturer's profit in **SC2** is

$$\Pi_w^*(p_w^l, e^{l*}) = \frac{-4b^2CU_{min} + x^2(a_0^2\lambda_m^2 - 8bC) + 4bCx(a_0 - bc)}{4b^2C},$$

which is decreasing in U_{min}. Comparing it with the manufacturer's optimal profit in **SC1**, we find that $\Pi_w^*(p_w^l, e^{l*}) = \Pi_{w,ns}^*$ when $U_{min} = u_c^l$ where

$$u_c^l = \frac{(a_0 + h_wb)^2bC + 2x^2(8bC - a_0^2\lambda_m^2) - 8bCx(a_0 + h_wb)}{8b^2C}.$$

As a result, $\Pi_w^*(p_w^l, e^{l*}) \geq \Pi_{w,ns}^*$ when $U_{min} \leq u_c^l$.

Case 2: In this case, we consider $p_w^* = p_w^o$ and $e^* = e^{o*}$. The manufacturer's profit in **SC2** is

$$\Pi_w^*(p_w^o, e^{o*}) = \frac{(a_0^2\lambda_m^2 - 8bC)U_{min} + C(a_0 + h_wb)^2 + x(h_w + c)(a_0^2\lambda_m^2 - 8bC)}{8bC - a_0^2\lambda_m^2}.$$

We know from Proposition B5 that when the optimal solution is $(p_w^o,\ e^{o*})$, we have $a_o^2\lambda_m^2 \leq 8bC$. Hence, $\Pi_w^*(p_w^o, e^{o*})$ is decreasing in U_{min}. Comparing it with the manufacturer's optimal profit in **SC1**, we find that $\Pi_w^*(p_w^o, e^{o*}) = \Pi_{w,ns}$ when $U_{min} = u_c^o$, where

$$u_c^o = \frac{a_0^2\lambda_m^2(a_0 + h_wb)^2}{8b(8bC - a_0^2\lambda^2)}.$$

As a result, $\Pi_w^*(p_w^o, e^{o*}) \geq \Pi_{w,ns}^*$ when $U_{min} \leq u_c^o$.

Case 3: In this case, we consider $p_w^* = p_w^u$ and $e^* = e^{u*}$. The manufacturer's profit in **SC2** is

$$\Pi_w^*(p_w^u, e^{u*}) = \frac{-4bCU_{min} + R_{min}(a_0^2\lambda_m^2 - 8bC) + 4C\sqrt{R_{min}b}(a_0 + h_wb) - 4bCx(c + h_w)}{4bC},$$

which is decreasing in U_{min}. Comparing it with the manufacturer's optimal profit in **SC1**, we find that $\Pi_w^*(p_w^u, e^{u*}) = \Pi_{w,ns}^*$ when $U_{min} = u_c^u$ where

$$u_c^u = \frac{2R_{min}(a_0^2\lambda_m^2 - 8Cb) + C\sqrt{R_{min}b}(8a_0 + h_wb) - (a_0 + h_wb)^2C}{8bC}.$$

As a result, $\Pi_w^*(p_w^u, e^{u*}) \geq \Pi_{w,ns}^*$ when $U_{min} \leq u_c^u$.

In summary, $\Pi_w^* \geq \Pi_{w,ns}^*$ under the first-best salary plan when $U_{min} \leq U_{m1}$ where

$$U_{m1} = \begin{cases} u_c^l &= \frac{(a_0+h_wb)^2bC+2x^2(8bC-a_0^2\lambda_m^2)-8bCx(a_0+h_wb)}{8b^2C} & : \text{ if } p_w^* = p_w^l, e^* = e^{l*} \\ u_c^o &= \frac{a_0^2\lambda_m^2(a_0+h_wb)^2}{8b(8bC-a_0^2\lambda^2)} & : \text{ if } p_w^* = p_w^o, e^* = e^{o*} \\ u_c^u &= \frac{2R_{min}(a_0^2\lambda_m^2-8Cb)+C\sqrt{R_{min}b}(8a_0+h_wb)-(a_0+h_wb)^2C}{8bC} & : \text{ if } p_w^* = p_w^u, e^* = e^{u*} \end{cases} \tag{25}$$

Combining our knowledge of the manufacturer's optimal strategy under different input parameter settings (in Proposition B5), we can conclude that the manufacturer profit is higher in **SC2** under the first-best salary plan than in **SC1** when $U_{min} \leq U_{m1}$, where U_{m1} is given in (24). ■

THEOREM 2a *In the demand promotion salesperson model under a feasible first-best salary plan, if it is beneficial for the manufacturer to hire a salesperson, then*

(*i*) *the resulting total supply chain profit is always greater than that in a supply chain without a salesperson,*

(*ii*) *the resulting optimal retailer profit is always greater than that in a supply chain without a salesperson.*

Proof of Part (i): We prove Part (i) by showing that $\Pi_{total}^* > \Pi_{total,ns}^*$ when a demand promotion salesperson is hired under the first-best salary plan. From Proposition B5, we know that the manufacturer's three possible optimal strategies are:

1. $p_w^* = p_w^l$ *and* $e^* = e^{l*}$
2. $p_w^* = p_w^o$ *and* $e^* = e^{o*}$
3. $p_w^* = p_w^u$ *and* $e^* = e^{u*}$

Case 1: Here $p_w^* = p_w^l$ and $e^* = e^{l*}$. We compare $\Pi_{total}^*(p_w^{l*}, e^{l*})$ with $\Pi_{total,ns}^*$ and find that

$$\Pi_{total}^*(p_w^l, e^{l*}) - \Pi_{total,ns}^* \quad = \quad \frac{\left(2x(a_0^2\lambda_m^2 - 8bC) + 4bCx(a_0 + h_wb) - (a_0 + h_wb)^2bC\right)}{4bC}.$$

Moreover, we find that

$$\frac{d^2}{d^2x}\left(\Pi_{total}^*(p_w^l, e^{l*}) - \Pi_{total,ns}^*\right) = \frac{a_0^2\lambda_m^2 - 8bC}{b^2C},$$

and

$$\Pi_{total}^*(p_w^l, e^{l*}) = \Pi_{total,ns}^* \text{ when } x = x_1 \text{ or } x = x_2$$

where

$$x_1 = \frac{(a_0 + h_wb)\left(\sqrt{8bC}(a_0\lambda_m + \sqrt{8bC})\right)}{2(-a_0^2\lambda_m^2 + 8bC)} \qquad \text{and} \qquad x_2 = \frac{(a_0 + h_wb)\left(\sqrt{8bC}(a_0\lambda_m - \sqrt{8bC})\right)}{2(-a_0^2\lambda_m^2 + 8bC)}$$

- When $a_0^2\lambda_m^2 > 8bC$, then $\Pi_{total}^*(p_w^l, e^{l*}) - \Pi_{total,ns}^*$ is convex in x and $x_1 < x_2 < 0$. Hence, we can conclude that $\Pi_{total}^*(p_w^l, e^{l*}) > \Pi_{total,ns}^*$ if $x < x_1$ or $x > x_2$. Therefore, for $x \geq 0 > x_2$, we have $\Pi_{total}^*(p_w^l, e^{l*}) > \Pi_{total,ns}^*$ when $a_0^2\lambda_m^2 > 8bC$.
- When $a_0^2\lambda_m^2 \leq 8bC$, then $\Pi_{total}^*(p_w^l, e^{l*}) - \Pi_{total,ns}^*$ is concave in x and $x_2 < 0 < x_1$. Hence, we can conclude that $\Pi_{total}^*(p_w^l, e^{l*}) > \Pi_{total,ns}^*$ if $x_2 < x < x_1$. From Proposition B5, we know that, when $a_0^2\lambda_m^2 \leq 8bC$, the manufacturer's optimal strategy is (p_w^l, e^{l*}) only if $x < x_c$. Comparing x_c (Equation (19)) with x_1, we find that
$$x_1 - x_c = \frac{a_0 + h_wb}{8bC - a_0^2\lambda_m^2}(2bC + \sqrt{2}ba_0\lambda_m),$$
which is greater than zero when $a_0^2\lambda_m^2 \leq 8bC$. Hence, we can conclude that, when $a_0^2\lambda_m^2 \leq 8bC$, we have $x_c \leq x_1$. Therefore, since $x_c \geq 0$, then there exists a production level x (where $x_2 < 0 < x \leq x_c < x_1$) that results in the optimal strategy (p_w^l, e^{l*}). Thus, $\Pi_{total}^*(p_w^l, e^{l*}) > \Pi_{total,ns}^*$.

Case 2: Here $p_w^* = p_w^o$ and $e^* = e^{o*}$. We compare $\Pi_{total}^*(p_w^o, e^{o*})$ with $\Pi_{total,ns}^*$ and find that

$$\Pi_{total}^*(p_w^o, e^{o*}) - \Pi_{total,ns}^* = \frac{(a_0 + h_w)^2(a_0^4\lambda_m^4 - 8a_0^2\lambda_m^2 bC)}{4bC(a_0^2\lambda_m^2 - 8bC)^2} = \frac{(a_0 + h_w)^2 a_0^2\lambda_m^2}{4bC(a_0^2\lambda_m^2 - 8bC)}.$$

From Proposition B5, we know that when (p_w^o, e^{o*}) is the optimal strategy for the manufacturer, we have $8bC \leq a_0^2\lambda_m^2$. Thus, we know that $\Pi_{total}^*(p_w^o, e^{o*}) - \Pi_{total,ns}^* > 0$, so $\Pi_{total}^*(p_w^o, e^{o*}) > \Pi_{total,ns}^*$.

Case 3: Here $p_w^* = p_w^u$ and $e^* = e^{u*}$. We compare $\Pi_{total}^*(p_w^u, e^{u*})$ with $\Pi_{total,ns}^*$ and find that

$$\Pi_{total}^*(p_w^u, e^{u*}) - \Pi_{total,ns}^* = \frac{4(a_0 + h_w b)C\Big(\sqrt{R_{min}b}(8bC - a_0^2\lambda_m^2) - 2(a_0 + h_w b)C\Big)}{4bC},$$

which is greater than zero if $R_{min} > \frac{4(a_0+h_w b)^2C^2}{(8bC-a_0^2\lambda_m^2)^2 b}$. From Proposition B5, we know that when (p_w^u, e^{u*}) is the optimal strategy for the manufacturer, we have $R_{min} > R_c = \frac{4(a_0+h_w b)^2C^2}{(8bC-a_0^2\lambda_m^2)^2 b}$. Thus, $\Pi_{total}^*(p_w^u, e^{u*}) - \Pi_{total,ns}^* > 0$, or simply $\Pi_{total}^*(p_w^u, e^{u*}) > \Pi_{total,ns}^*$.

In summary, $\Pi_{total}^* > \Pi_{total,ns}^*$ when a demand promotion salesperson is hired under the first-best salary plan.

Proof of Part (ii): We prove part(ii) by considering the retailer profit under the three possible manufacturer's optimal solutions from Proposition B5. The three cases are:

1. $p_w^* = p_w^l$ *and* $e^* = e^{l*}$
2. $p_w^* = p_w^o$ *and* $e^* = e^{o*}$
3. $p_w^* = p_w^u$ *and* $e^* = e^{u*}$

Case 1: Since $p_w^* = p_w^o$ and $e^* = e^{o*}$, the retailer's optimal profit is

$$\Pi_r^*(p_w^o, e^{o*}) = \frac{\Big(a_0(1 + \lambda_m e) + h_w b\Big)^2}{16b} \geq \Pi_{r,ns}^* = \frac{(a_0 + h_w b)^2}{16b} \text{ when } e \geq 0.$$

Case 2: Since $p_w^* = p_w^l$ and $e^* = e^{l*}$, the retailer's optimal profit is $\Pi_r^*(p_w^l, e^{l*}) = \frac{x^2}{b}$. It is assumed that the production level $x > Q_{ns}^* = \frac{(a_0+h_w b)^2}{4}$ (otherwise, there is no need to hire a salesperson); hence,

$$\Pi_r^*(p_w^l, e^{l*}) = \frac{x^2}{b} \geq \frac{(a_0 + h_w b)^2}{16b} = \Pi_{r,ns}.$$

Case 3: Since $p_w^* = p_w^u$ and $e^* = e^{u*}$, the retailer's optimal profit is

$$\Pi_r^*(p_w^u, e^{u*}) = \frac{4(a_0 + h_w b)^2 bC^2}{(8bC - a_0^2\lambda_m^2)^2},$$

and

$$\Pi_r^* - \Pi_{r,ns}^* = \frac{(a_0 + h_w b)^2 a_0^2\lambda_m^2(16bC - a_0^2\lambda_m^2)}{16(a_0^2\lambda_m^2 - 8bC)^2 b}.$$

We know from Proposition B5 that when (p_w^u, e^{u*}) is the manufacturer's optimal strategy, we have $a_0^2\lambda_m^2 \leq 8bC$. Hence, $16bC > a_0^2\lambda_m^2$ and $\Pi_r^* - \Pi_{r,ns}^* > 0$.

In conclusion, $\Pi_r \geq \Pi_{r,ns}^*$ when a demand promotion salesperson is hired under the first-best salary plan. ■

B.9. Demand Promotion Salesperson Model Under the Quantity Based Commission Plan

To simplify notation, unless needed, we omit (α) from $e(\alpha)$, $p_w(\alpha)$, $p_r(\alpha)$; we omit (e, α, S) from $U(e, \alpha, S)$ and $W(e, \alpha, S)$; we omit (p_w, α) from $Q(p_w, \alpha)$ and $p_r(p_w, \alpha)$; and we omit (p_r, Q) from $\Pi_r(p_r, Q)$.

LEMMA B9 *In a demand promotion model, if the commission plan is feasible, the optimal responses of the retailer and the salesperson under the commission plan given a certain wholesale price p_w, commission rate α, and fixed amount S, are:*

$$\begin{aligned} e^*(p_w,\alpha) &= \frac{a_0\lambda_m\alpha}{4C}, \\ Q^*(p_w,\alpha) &= \frac{a_0^2\lambda_m^2\alpha + 4C(a_0 - p_w b)}{8C}, \\ p_r^*(p_w,\alpha) &= \frac{a_0^2\lambda_m^2\alpha + 4C(a_0 + p_w b)}{8bC}. \end{aligned} \tag{26}$$

Proof: We study the optimal responses of the retailer and the salesperson given a wholesale price, p_w, and commission plan parameters, α and S, as follows.

Salesperson's Problem: Given the wholesale price (p_w) and commission plan parameters (α and S), the salesperson's utility is:

$$U = S + \alpha Q - Ce^2 = \alpha\frac{a_0(1+\lambda_m e) - p_w b}{2} - Ce^2 + S,$$

which is maximized at

$$e^* = \frac{\alpha a_0\lambda_m}{4C}.$$

Retailer's Problem: Given a wholesale price p_w, we know that the retailer profit is maximized when $Q = D$ under any price-setting problem (Petruzzi and Dada 1999). Therefore, the retailer's profit is:

$$\Pi_r = (p_r - p_w)D = (p_r - p_w)(a - bp_r).$$

First order conditions imply that the optimal retail price is

$$p_r^*(p_w, e) = \frac{a + p_w b}{2b}.$$

Considering the salesperson's optimal effort e and base demand $a = a_0(1+\lambda_m e)$, the optimal retail price is:

$$p_r^*(p_w,\alpha) = \frac{a_0^2\lambda_m^2\alpha + 4C(a_0 + p_w b)}{8bC}.$$

The corresponding order quantity is therefore:

$$Q^*(p_w,\alpha) = D(p_r^*,\alpha) = \frac{a_0^2\lambda_m^2\alpha + 4C(a_0 - p_w b)}{8C}. \qquad \blacksquare$$

PROPOSITION B7 *In the demand promotion model, if the commission plan is feasible, the manufacturer's optimal wholesale price under the commission plan has the following structure:*

(*i*) *If $a_0^2\lambda_m^2 > 4bC$, then*

$$p_w^* = \begin{cases} p_w^u & : \text{ if } 0 \le \alpha \le \alpha_c^u \\ p_w^o & : \text{ if } \alpha_c^u \le \alpha \le \alpha_c^l \\ p_w^l & : \text{ if } \alpha_c^l \le \alpha \end{cases} \tag{27}$$

(*ii*) *If $a_0^2\lambda_m^2 \le 4bC$, then*

$$p_w^* = \begin{cases} p_w^l & : \text{ if } 0 \le \alpha \le \alpha_c^l \\ p_w^o & : \text{ if } \alpha_c^l \le \alpha \le \alpha_c^u \\ p_w^u & : \text{ if } \alpha_c^u \le \alpha \end{cases} \tag{28}$$

where

$$p_w^o = \frac{4C(a_0 - h_w b) + \alpha(\lambda_m^2 a_0^2 + 4bC)}{8bC}, \qquad p_w^l = \frac{-8xC + 4a_0C + \lambda_m^2\alpha a_0^2}{4bC},$$

$$p_w^u = \frac{4a_0C + \alpha\lambda_m^2 a_0^2 - 8C\sqrt{R_{min} b}}{4bC},$$

and

$$\alpha_c^l = \frac{4C(-a_0 - h_w b + 4x)}{a_0^2\lambda_m^2 - 4bC}, \qquad \alpha_c^u = \frac{4C(-a_0 - h_w b + 4\sqrt{R_{min}b})}{a_0^2\lambda_m^2 - 4bC}.$$

Proof:

Manufacturer's Problem: Given the optimal responses of the retailer (Q^*) and the salesperson (e^*), the manufacturer profit is

$$\Pi_w = \frac{(4a_0C + a_0^2\lambda_m^2\alpha - 4p_w bC)(p_w + h_w - \alpha)}{8C} - x(c + h_w) - S.$$

We find that, given any commission rate $\alpha \geq 0$, the manufacturer profit is concave in p_w and is maximized at

$$p_w^o = \frac{4C(a_0 - h_w b) + \alpha(\lambda_m^2 a_0^2 + 4bC)}{8bC}. \tag{29}$$

Note that p_w^o is the optimal wholesale price without taking into account of the production constraint or the retailer's IR constraint. Next we consider the effect of production and retailer's IR constraints on the wholesale price.

Production Constraint: Note that it does not make sense for the manufacturer to set wholesale price, or for the salesperson to exert effort, such that the resulting order quantity is greater than x, the manufacturer's production level. This imposes lower bound p_w^l on the wholesale price such that $Q^* \leq x$, where

$$p_w^l = arg_{p_w}\Big\{Q^* = x\Big\} = \frac{-8xC + 4a_0C + \lambda_m^2\alpha a_0^2}{4bC}. \tag{30}$$

Note that p_w^l is the lower bound on the wholesale price corresponding to the lowest wholesale price the manufacturer can charge to ensure no shortage.

Retailer's IR Constraint: Based on the argument in Lemma B9, we know that the retailer profit function based on his and the salesperson's optimal responses is:

$$\begin{aligned} \Pi_r(p_r^*, Q^*, e^*) &= p_r^*(a - bp_r^*) - p_w Q^* \\ &= \frac{(4a_0C + \lambda_m^2\alpha a_0^2 - 4p_w bC)^2}{64bC^2} \end{aligned}$$

In order to satisfy the retailer's IR constraint ($\Pi_r \geq R_{min}$), the wholesale price should be

$$p_w \geq \frac{4a_0C + \alpha\lambda_m^2 a_0^2 + 8C\sqrt{R_{min}b}}{4bC} \quad \text{or} \quad p_w \leq \frac{4a_0C + \alpha\lambda_m^2 a_0^2 - 8C\sqrt{R_{min}b}}{4bC}.$$

We find that when $p_w = \frac{4a_0C+\alpha\lambda_m^2 a_0^2+8C\sqrt{R_{min}b}}{4bC}$, the corresponding optimal order quantity is $Q^* = -\sqrt{R_{min}b} < 0$. From Equation (26), we know that Q^* is decreasing in p_w. Hence, when the first condition holds ($p_w \geq \frac{4a_0C+\alpha\lambda_m^2 a_0^2+8C\sqrt{R_{min}b}}{4bC}$), the corresponding optimal order quantity $Q^* \leq -\sqrt{R_{min}b} < 0$. Hence, we focus on the second condition

$$p_w \leq \frac{4a_0C + \alpha\lambda_m^2 a_0^2 - 8C\sqrt{R_{min}b}}{4bC},$$

and define the upper bound on p_w imposed by the retailer's IR constraint as follows:

$$p_w^u = \frac{4a_0C + \alpha\lambda_m^2 a_0^2 - 8C\sqrt{R_{min}b}}{4bC}. \tag{31}$$

Note that p_w^u is an upper bound on the wholesale price corresponding to the highest wholesale price the retailer will accept to stay in the game. Hence, p_w^u is the optimal wholesale price when the retailer's IR constraint is binding.

Let α_c^l be the commission rate that results in $p_w^o = p_w^l$. Comparing p_w^o (Equation (29)) and p_w^l (Equation (30)), we find that

$$p_w^o - p_w^l = \frac{16xC - 4a_0C - 4h_w bC + \alpha(-a_0^2\lambda_m^2 + 4bC)}{8bC},$$

and hence

$$\alpha_c^l = arg_{\alpha}\Big\{p_w^o = p_w^l\Big\} = \frac{4C(-a_0 - h_w b + 4x)}{a_0^2\lambda_m^2 - 4bC}. \tag{32}$$

- When $a_0^2\lambda_m^2 > 4bC$, then $p_w^o - p_w^l$ is decreasing in α. Hence, we know that when the commission rate $\alpha \geq \alpha_c^l$, we have $p_w^l > p_w^o$, meaning that the production constraint is binding. Moreover, when the commission rate $\alpha \leq \alpha_c^l$, $p_w^l \leq p_w^o$, meaning that the production constraint is not binding.
- When $a_0^2\lambda_m^2 \leq 4bC$, then $p_w^o - p_w^l$ is non-decreasing in α. Hence, we know that when the commission rate $\alpha < \alpha_c^l$, we have $p_w^l > p_w^o$, meaning that the production constraint is binding. Moreover, when the commission rate $\alpha \geq \alpha_c^l$, we have $p_w^l \leq p_w^o$, meaning that the production constraint is not binding.

Hence, if we neglect the effect of the retailer's IR constraint, the optimal wholesale price has the following structure:

(i) If $a_0^2\lambda_m^2 > 4bC$

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^l \\ p_w^l & : \quad \text{if } \ \alpha_c^l \leq \alpha \end{cases}$$

(ii) $a_0^2\lambda_m^2 \leq 4bC$

$$p_w^* = \begin{cases} p_w^l & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^l \\ p_w^o & : \quad \text{if } \ \alpha_c^l \leq \alpha \end{cases} \tag{33}$$

On the other hand, if α_c^u is the commission rate that results in $p_w^o = p_w^u$, then by comparing p_w^o (Equation (29)) and p_w^u (Equation (31)), we find that

$$p_w^u - p_w^o = \frac{4a_0C - 16C\sqrt{R_{min}b} + 4h_wbC + \alpha(a_0^2\lambda_m^2 - 4bC)}{8bC},$$

and hence

$$\alpha_c^u = arg_\alpha\Big\{p_w^u = p_w^o\Big\} = \frac{4C(-a_0 - h_wb + 4\sqrt{R_{min}b})}{a_0^2\lambda_m^2 - 4bC}. \tag{34}$$

- When $a_0^2\lambda_m^2 > 4bC$, then $p_w^u - p_w^o$ is increasing in α. Hence, we know that when the commission rate $\alpha < \alpha_c^u$, we have $p_w^u < p_w^o$, meaning that the retailer's IR constraint is binding. Moreover, when the commission rate $\alpha \geq \alpha_c^u$, we have $p_w^u \geq p_w^o$, meaning that the retailer's IR constraint is not binding.
- When $a_0^2\lambda_m^2 \leq 4bC$, then $p_w^u - p_w^o$ is non-increasing in α. Hence, we know that when the commission rate $\alpha \geq \alpha_c^u$, we have $p_w^u < p_w^o$, meaning that the retailer's IR constraint is binding. Moreover, when the commission rate $\alpha \leq \alpha_c^u$, we have $p_w^u \geq p_w^o$, meaning that the retailer's IR constraint is not binding.

Hence, if we neglect the effect of the production constraint, the optimal wholesale price has the following structure:

(i) If $a_0^2\lambda_m^2 > 4bC$

$$p_w^* = \begin{cases} p_w^u & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^u \\ p_w^o & : \quad \text{if } \ \alpha_c^u \leq \alpha \end{cases}$$

(ii) If $a_0^2\lambda_m^2 \leq 4bC$

$$p_w^* = \begin{cases} p_w^o & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^u \\ p_w^u & : \quad \text{if } \ \alpha_c^u \leq \alpha \end{cases} \tag{35}$$

Combining the effects of production and the retailer's IR constraints on wholesale price (Equations (33) and (35)), we can conclude that:

(i) If $a_0^2\lambda_m^2 > 4bC$

$$p_w^* = \begin{cases} p_w^u & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^u \\ p_w^o & : \quad \text{if } \ \alpha_c^u \leq \alpha \leq \alpha_c^l \\ p_w^l & : \quad \text{if } \ \alpha_c^l \leq \alpha \end{cases} \tag{36}$$

(ii) if $a_0^2\lambda_m^2 \leq 4bC$

$$p_w^* = \begin{cases} p_w^l & : \quad \text{if } \ 0 \leq \alpha \leq \alpha_c^l \\ p_w^o & : \quad \text{if } \ \alpha_c^l \leq \alpha \leq \alpha_c^u \\ p_w^u & : \quad \text{if } \ \alpha_c^u \leq \alpha \end{cases} \tag{37}$$

■

LEMMA B10 *For the demand promotion model, the commission plan is feasible when* $x \geq \sqrt{R_{min} b}$.

Proof: Recall that a compensation plan in **SC2** is feasible if both the retailer and the salesperson are willing to participate in the game. Hence, a compensation plan is feasible when, under production level x, (i) $p_w^l \leq p_w \leq p_w^u$, which reflects the retailer's IR constraint and the production constraint, and (ii) salesperson's IR constraint is met ($U \geq U_{min}$).

Retailer's IR Constraint and Production Constraint: Comparing p_w^l (Equation (30)) and p_w^u (Equation (31)), we find that $p_w^u - p_w^l = \frac{2(x-\sqrt{R_{min}b})}{b}$, which is non-negative if $x \geq \sqrt{R_{min}b}$. Hence, $x \geq \sqrt{R_{min}b}$ is a necessary condition for a commission plan to be feasible in a demand promotion model.

Salesperson's IR Constraint: We claim that, in SC2, the manufacturer can always satisfy the salesperson's IR constraint by adjusting the fixed amount S in the commission plan. We examine this claim for the following cases:

- Case 1-a: $a_0^2\lambda_m^2 > 4bC$ and $0 \leq \alpha \leq \alpha_c^u$;
- Case 1-b: $a_0^2\lambda_m^2 > 4bC$ and $\alpha_c^u \leq \alpha \leq \alpha_c^l$;
- Case 1-c: $a_0^2\lambda_m^2 > 4bC$ and $\alpha_c^l \leq \alpha$;
- Case 2-a: $a_0^2\lambda_m^2 \leq 4bC$ and $0 \leq \alpha \leq \alpha_c^l$;
- Case 2-b: $a_0^2\lambda_m^2 \leq 4bC$ and $\alpha_c^l \leq \alpha \leq \alpha_c^u$;
- Case 2-c: $a_0^2\lambda_m^2 \leq 4bC$ and $\alpha_c^u \leq \alpha$.

We only present the proof for **Case 1-a** in which $a_0^2\lambda_m^2 > 4bC$ and $0 \leq \alpha \leq \alpha_c^u$. The proofs for other cases are similar and are therefore omitted.

Case 1-a: From Proposition B7, we know that, in this case, $p_w^* = p_w^u$. The corresponding salesperson utility is

$$U(p_w^u, \alpha, S) = \frac{\alpha(8a_0C + a_0^2\lambda_m^2\alpha - 8p_w^u bC)}{16C} + S,$$

which is concave in α. We define the lower and upper bounds on the commission rate imposed by the salesperson's IR constraint (i.e., $U \geq U_{min}$) to be $\underline{\alpha}$ and $\bar{\alpha}$, respectively. Define $\tilde{U}_{min} = U_{min} - S$, we find that $U(p_w^u) \geq U_{min}$ when $\underline{\alpha}^u \leq \alpha \leq \bar{\alpha}^u$, where

$$\bar{\alpha}^u = \frac{4\left(2C\sqrt{bR_{min}} + \sqrt{4R_{min}bC^2 - a_0^2\lambda_m^2\tilde{U}_{min}C}\right)}{a_0^2\lambda_m^2}, \tag{38}$$

$$\underline{\alpha}^u = \frac{4\left(2C\sqrt{bR_{min}} - \sqrt{4R_{min}bC^2 - a_0^2\lambda_m^2\tilde{U}_{min}C}\right)}{a_0^2\lambda_m^2}. \tag{39}$$

Note that real values of $\bar{\alpha}^u$ and $\underline{\alpha}^u$ do not exist if $\tilde{U}_{min} > u_1$, where u_1 is defined as $u_1 = \frac{4bCR_{min}}{a_0^2\lambda_m^2}$, and both $\bar{\alpha}^u$ and $\underline{\alpha}^u$ are non-negative if $\tilde{U}_{min} \leq u_1$.

Given $\tilde{U}_{min} \leq u_1$, to meet the salesperson's IR constraint (i.e., $U(p_w^u) \geq U_{min}$) when $0 \leq \alpha \leq \alpha_c^u$, there should exist a commission rate α in $0 \leq \alpha \leq \alpha_c^u$ such that $\underline{\alpha}^u \leq \alpha \leq \bar{\alpha}^u$. Hence, it is necessary for $\underline{\alpha}^u \leq \alpha_c^u$, and $\bar{\alpha}^u \geq 0$. Given Equation (38), it is obvious that the second condition is always met ($\bar{\alpha}^u \geq 0$). Hence, we focus on the first condition. Comparing $\underline{\alpha}^u$ with α_c^u (Equation (34)), we find that

$$\alpha_c^u - \underline{\alpha}^u = \frac{4\left(a_0^2\lambda_m^2C(a_0 + h_w b) - 2C\sqrt{R_{min}b}(a_0^2\lambda_m^2 + 4bC)\right)}{a_0^2\lambda_m^2(a_0^2\lambda_m^2 - 4bC)} + \frac{4\sqrt{C(4CbR_{min} - a_0^2\lambda_m^2\tilde{U}_{min})}}{a_0^2\lambda_m^2}.$$

Hence, $\alpha_c^u - \underline{\alpha}^u$ is decreasing in $\tilde{U}_{min}$. Moreover, we find that $\underline{\alpha}^u \leq \alpha_c^u$ when $\tilde{U}_{min} \leq u_2$ where

$$u_2 = \frac{C\left(a_0^2\lambda_m^2(a_0 + h_w b)^2 + 64b^2R_{min}C - 4\sqrt{bR_{min}}(a_0 + h_w b)(a_0^2\lambda_m^2 + 4Cb)C\right)}{(a_0^2\lambda_m^2 - 4bC)^2}.$$

Thus, when $0 \leq \alpha \leq \alpha_c^u$, the salesperson's IR constraint is satisfied if $\tilde{U}_{min} \leq u_1$ and $\tilde{U}_{min} \leq u_2$, or simply

$$\tilde{U}_{min} \leq \min\{u_1, u_2\},$$

which can always be satisfied by adjusting the value of S in the commission plan. So we claim that the manufacturer can always satisfy the salesperson's IR constraint by adjusting the fixed amount S in the commission plan. ■

B.10. Demand Promotion Model: Comparing SC2 with SC1 Under the Commission Plan

In this section, we examine the conditions under which it is more *beneficial* for the manufacturer to hire a demand promotion salesperson under the commission plan than not to hire one, i.e., the manufacturer has larger profit in **SC2** than in **SC1**.

THEOREM 2b *In the demand promotion model under a feasible commission plan, if it is beneficial for the manufacturer to hire a salesperson, then*

(*i*) *the resulting optimal retailer profit is always greater than that in a supply chain without a salesperson.*

(*ii*) *the resulting total supply chain profit is always greater than that in a supply chain without a salesperson,*

Proof for part (i): We prove this part of the theorem by showing that, when a demand promotion salesperson is hired under the commission plan, $\Pi_r^* \geq \Pi_{r,ns}$ holds for all of the three possible optimal wholesale price cases from Proposition B7. We examine the retailer profit under the three possible optimal wholesale prices when $a_0^2\lambda_m^2 > 4bC$ in the following. The proofs of the retailer profit under the three possible optimal wholesale prices when $a_0^2\lambda_m^2 \leq 4bC$ are similar and are therefore omitted.

Case 1: In this case, we consider $p_w^* = p_w^l$, when $a_0^2\lambda_m^2 > 4bC$. We find that $\Pi_r^*(p_w^l) = \frac{x^2}{b}$. It is assumed that the production level $x > Q_{ns}^* = \frac{a+h_w b}{4}$. Hence,

$$\Pi_r^*(p_w^l) > \frac{(a + h_w b)^2}{16b} = \Pi_{r,ns}^*.$$

Case 2: In this case, we consider $p_w^* = p_w^u$, when $a_0^2\lambda_m^2 > 4bC$. From Proposition B7, we know that p_w^u is the optimal wholesale price when $a_0^2\lambda_m^2 > 4bC$ only if $\alpha \leq \alpha_c^u$. As discussed in the Case 1-(ii) of Proposition B8, we know that when $p_w^* = p_w^u$, we have $\alpha_c^u > 0$ which implies $R_{min} > \Pi_{r,ns}^* = \frac{(a_0+h_w b)^2}{16b}$. Hence,

$$\Pi_r^*(p_w^u) = R_{min} > \Pi_{r,ns}^*,$$

and the retailer profit is always greater than his profit without a salesperson when $p_w^* = p_w^u$.

Case 3: In this case, we consider $p_w^* = p_w^o$, when $a_0^2\lambda_m^2 > 4bC$. Comparing the retailer profit when $p_w^* = p_w^o$ in **SC2** with the retailer profit in **SC1**, we find that $\Pi_r^*(p_w^o) = \frac{1}{16b}\left(a_0 + h_w b + \alpha\frac{a_0^2\lambda_m^2 - 4bC}{4}\right)^2$, which is greater than $\Pi_{r,ns}^*$ when $a_0^2\lambda_m^2 > 4bC$.

In summary, $\Pi_r^* > \Pi_{r,ns}^*$ when a demand promotion salesperson is hired under the commission plan.

Proof for part (ii): We know that when a demand promotion salesperson is hired under the commission plan, $\Pi_w^* > \Pi_{w,ns}^*$ and $\Pi_r^* > \Pi_{r,ns}^*$ (see part (i) of this theorem). Since $\Pi_{total}^* = \Pi_w^* + \Pi_r^* + W$ and $W \geq 0$, we know that $\Pi_{total}^* > \Pi_{total,ns}^*$ when a demand promotion salesperson is hired under the commission plan. ■

ON-LINE APPENDIX C

Analytical Solution for Retail-Salesperson Model

C.1. The First-Best Salary Plan

Using backward induction, we determine the optimal strategies of the manufacturer, and the retailer in the retail-salesperson model under the first-best salary plan. First, we consider the optimal response of the salesperson.

Salesperson's IR Constraint: We first show that salesperson's utility is always at her minimum level by using contradiction. Suppose that, under the first-best salary plan, it is optimal for the retailer to offer a salary (W) and designate an effort level (e) to the salesperson such that the salesperson's utility is above her minimum retaining level ($U = W - Ce^2 > U_{min}$). The corresponding retailer's profit is therefore $\Pi_r = (p_r - p_w)Q(e) - W$. For the same designated effort level e, retailer's profit increases as it decreases the salesperson's salary to $\dot{W} = W - \delta$, where $\delta = U - U_{min}$. This means compensation plan (W, e) cannot be optimal, which is a contradiction.

Retailer's Problem: Given a wholesale price p_w, the retailer's profit is

$$\Pi_r(p_r, \alpha) \quad = \quad (p_r - p_w)\big(a_0(1 + \lambda_m e) - p_r b\big) - U_{min} - Ce^2.$$

We first consider the optimal value of retail price assuming a fixed effort level. Under this assumption, the retailer's profit is maximized at

$$p_r^* = \frac{a_0(1 + \lambda_m e) + p_w b}{2b}.$$

Substituting p_r^* into the retailer's profit function, we get

$$\Pi_r(p_r^*, e) = \frac{(a_0 + a_0\lambda_m e - p_w b)^2}{2b} - 4U_{min}b - 4Ce^2b.$$

Manufacturer's Problem: We now consider manufacturer's optimal wholesale price given p_r^* and e. After some algebra, the manufacturer's profit becomes

$$\Pi_w(p_w) \quad = \quad \frac{2bC(p_w + h_w)(-a_0 + p_w b)}{a_0^2\lambda_m^2 - 4bC} - x(c + h_w),$$

which is concave in p_w and is maximized at $p_w^* = \frac{a_0 - h_w b}{2b}$.

Similar to the analysis for the wholesale-salesperson, the retailer's IR constraint imposes an upper bound on the wholesale price

$$p_w^u = \frac{4a_0 + \sqrt{(-a_0^2\lambda_m^2 + 4Cb)\big(Cb(a_0 + h_w b)^2 + 16CU_{min}b^2\big)}}{4b^2C},$$

while the production constraint imposes a lower bound on the wholesale price

$$p_w^l = \frac{2a_0Cb + xa_0^2\lambda_m^2 - 4xCb}{2b^2C}.$$

Note that if $a_0^2\lambda_m^2 > 4bC$, then p_w^u does not exist and the retailer's IR constraint is always met (i.e., $\Pi_r^* \geq R_{min}$). Hence, the manufacturer's optimal wholesale price is

$$p_w^* = \begin{cases} max\Big\{p_w^o, p_w^l\Big\} & : \quad \text{if } \ a_0^2\lambda_m^2 > 4bC \\ max\Big\{min\{p_w^o, p_w^u\}, p_w^l\Big\} & : \quad \text{if } \ a_0^2\lambda_m^2 \leq 4bC \end{cases}$$

C.2. The Quantity Based Commission Plan

We determine the optimal strategies for the manufacturer, retailer and the salesperson using backward induction. First, we consider the optimal response of the salesperson.

Salesperson's Problem: Given a wholesale price p_w and a commission rate α, the retailer's problem in this model is the same as a standard price-setting newsvendor problem. Hence, the retailer profit is maximized with no shortage or leftover inventory $Q = D$ (Petruzzi and Dada 1999). Given a retail price p_r and a commission plan with commission rate α and a fixed amount S, the salesperson's utility becomes

$$\begin{aligned} U(e) &= (S + \alpha D) - Ce^2 \\ &= \alpha\big(a_0(1+\lambda_m e) - bp_r\big) - Ce^2 + S \end{aligned}$$

which is concave in e and is maximized at

$$e^* = \frac{\alpha a_0 \lambda_m}{2C}.$$

Retailer's Problem: Given a wholesale price p_w and the salesperson's optimal effort level e^*, the retailer profit becomes

$$\begin{aligned} \Pi_r(p_r, \alpha) &= (p_r - p_w)D \\ &= \frac{(p_r - p_w - \alpha)(2a_0C + \lambda_m^2 \alpha a_0^2 - 2bp_rC)}{2C}. \end{aligned}$$

We first consider the optimal value of retail price assuming a fixed commission rate. In this case, the retailer's profit is maximized at

$$p_r^* = \frac{2a_0C + \lambda_m^2 \alpha a_0^2 + 2bCp_w + 2bC\alpha}{4bC}.$$

Substituting p_r^* into the retailer's profit function, we get

$$\Pi_r(p_r^*, \alpha) = \frac{2a_0C + \lambda_m^2 \alpha a_0^2 - 2bCp_w - 2bC\alpha}{16bC^2},$$

which is convex in α ($\frac{d^2\Pi_r(p_r^*,\alpha)}{d\alpha^2} > 0$) and is minimized at $\alpha_{min} = \frac{2C(a_0 - bp_w)}{\lambda_m^2 a_0^2 - 2bC}$.

Salesperson's IR Constraint: Given the salesperson's optimal response of effort level e^* and retailer's optimal retail price p_r^*, the salesperson's utility function becomes

$$U(e^*) = \frac{\alpha(a_0 - bp_w - b\alpha)}{2} + S,$$

which is concave in α. In general, $U(e^*) \geq U_{min}$ when

$$\frac{a_0 - bp_w - \sqrt{(a_0 - bp_w)^2 - 8b\tilde{U}_{min}}}{2b} \leq \alpha \leq \frac{a_0 - bp_w + \sqrt{(a_0 - bp_w)^2 - 8b\tilde{U}_{min}}}{2b},$$

where $\tilde{U}_{min} = U_{min} - S$.

We define

$$\underline{\alpha} = \frac{a_0 - bp_w - \sqrt{(a_0 - bp_w)^2 - 8b\tilde{U}_{min}}}{2b} \qquad \text{and} \qquad \bar{\alpha} = \frac{a_0 - bp_w + \sqrt{(a_0 - bp_w)^2 - 8b\tilde{U}_{min}}}{2b}.$$

- For cases where $a_0^2\lambda_m^2 > 2bC$, we have $\alpha_{min} < 0$. Hence, $\Pi_r(p_r^*, \alpha)$ is increasing in non-negative α and $\alpha^* = \bar{\alpha}$.
- For cases where $a_0^2\lambda_m^2 \leq 2bC$, we have $\alpha_{min} > 0$. Hence, $\alpha^* = max_\alpha\Big\{\Pi_r(p_r^*, \underline{\alpha}), \Pi_r(p_r^*, \bar{\alpha})\Big\}$.

Manufacturer's Problem: We now consider manufacturer's optimal wholesale price given e^*, p_r^*, and α^*.

When $\alpha^* = \bar{\alpha}$, the manufacturer's profit becomes

$$\begin{aligned} \Pi_w(p_w) = \frac{1}{8bC}\Big(& a_0h_w(\lambda_m^2 a_0^2 + 2bC) - 8xbC(c + h_w) + p_w(-bp_w - h_wb + a_0)(\lambda_m^2 a_0^2 + 2bC) \\ &+ (p_w + h_w)(\lambda_m^2 a_0^2 - 2bC)\sqrt{(a_0 - bp_w)^2 - 8b\tilde{U}_{min}}\Big) - S. \end{aligned}$$

When $\alpha^* = \underline{\alpha}$, the manufacturer's profit becomes

$$\begin{aligned}\Pi_w(p_w) &= \frac{1}{8bC}\Big(a_0 h_w(\lambda_m^2 a_0^2 + 2bC) - 8xbC(c + h_w) + p_w(-bp_w - h_w b + a_0)(\lambda_m^2 a_0^2 + 2bC) \\ &\qquad - (p_w + h_w)(\lambda_m^2 a_0^2 - 2bC)\sqrt{(a_0 - bp_w)^2 - 8b\tilde{U}_{min}}\Big) - S.\end{aligned}$$

For both cases, the corresponding optimal wholesale price cannot be expressed in closed form. Numerically, one can first examine the shape of the manufacturer's profit function and then determined the wholesale price p_w^o that maximizes $\Pi_w(p_w)$ accordingly.

Similar to the analysis for the wholesale-salesperson, the retailer's IR constraint imposes an upper bound on the wholesale price p_w^u, while the production constraint imposes a lower bound on the wholesale price p_w^l. Hence, the manufacturer's optimal wholesale price is

$$p_w^* = max\Big\{min\{p_w^o, p_w^u\}, p_w^l\Big\}.$$