Strategic Risk from Supply Chain Disruptions

Wallace J. Hopp
Stephen M. Ross School of Business, University of Michigan, Ann Arbor, MI

Seyed M. R. Iravani and Zigeng Liu
Department of Industrial Engineering and Management Science,
Northwestern University, Evanston, IL

Abstract

Supply chain disruptions can lead to both tactical (e.g., loss of short term sales) and strategic (e.g., loss of long term market share) consequences. In this paper, we model the impact of regional supply disruptions on competing supply chains. We describe generic strategies that consist of two stages: (*i*) *preparation,* which involves investment prior to a disruption in measures that facilitate quick detection of a problem, and (*ii*) *response,* which involves post-disruption purchase of backup capacity for a component whose availability has been compromised. Using expected loss of profit due to lack of preparedness as a measure of risk, we find that the products that pose the greatest risk are those with valuable market share, low customer loyalty, and relatively limited backup capacity. Furthermore, we show that a dominant firm in the market should focus primarily on protecting its market share, while a weaker firm should focus on being ready to take advantage of a supply disruption to gain market share. In either case, we demonstrate that firms can identify the components in their bill-of-material where investment in preparedness is most valuable by evaluating only a small number of descriptive parameters.

Keywords: supply chain disruption, risk, preparedness

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1 Introduction

High profile incidents of global terrorism have elevated concerns about risks in all aspects of private and public life, including management of business operations. However, while terrorist activities are particularly newsworthy, other sources of risk, such as natural disasters and political/economic shocks, may present a more pressing challenge to managers. Regardless of the source, it is clear that preparation for and management of major disruptions is an important part of modern operations management.

In this paper, we focus on the risk of supply disruption, which is inherent in global supply chains. We divide this risk into two categories: (1) *tactical risk*, which characterizes events that result in only a short term loss of sales revenue, and (2) *strategic risk*, which is associated with events that result in a long term loss of market share, in addition to a loss of sales.

An example of a strategic risk event occurred on March 17, 2000, when a ten-minute fire at a Royal Philips Electronics semiconductor plant in Albuquerque, New Mexico, "touched off a corporate crisis that shifted the balance of power between two of Europe's biggest electronics companies..." (Wall Street Journal, January 29, 2001). This occurred because, besides directly destroying several thousand chips for mobile phones, the fire contaminated the clean room environment in the semiconductor plant, effectively shutting it down for weeks. At the time, both Nokia and Ericsson were sourcing microchips from the

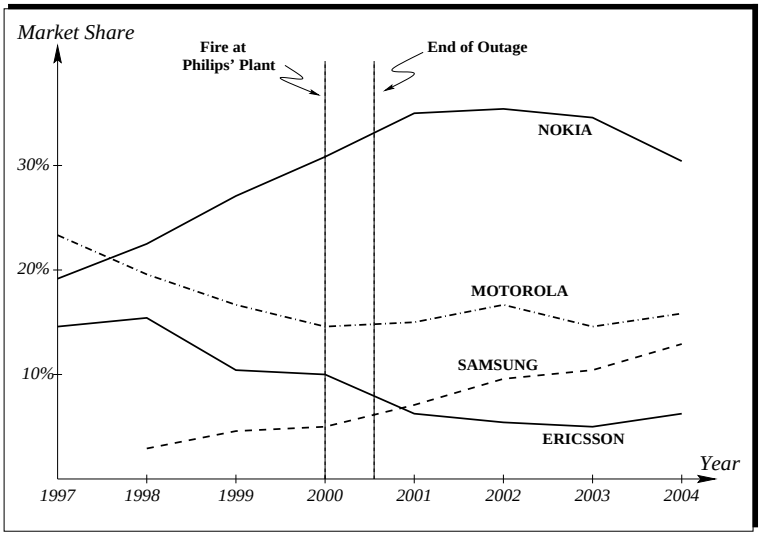

Figure 1: Global Mobile Market Share from 1997 to 2004 (Gartner 2006).

Philips plant. However, while Nokia was able to quickly shift production to other Philips plants and some Japanese and American suppliers, Ericsson was trapped by its sole source dependence on the Philips plant. Consequently, Ericsson had no way to make a rapid response to the disruption, and wound up losing around $400 million in sales by the end of the first disruption-impacted quarter (Elgin 2003). Even more damaging, six months after the fire, Ericsson's market share of the global handset market had fallen by 3%, its stock price had decreased by 12%, and Ericsson's mobile phone division reported a $2.34 billion loss for 2000 (Sheffi 2005). Ericsson never recovered the ground it lost during this crisis and withdrew from the cell phone market in 2001.

Such dramatic consequences from a seemingly minor disruption seem to be the rule rather than the exception. Hendricks and Singhal (2005) considered a sample of 827 disruptions (ranging from a shipment delay to a key part shortage) announced between 1989 and 2000 and examined stock prices over a three-year period beginning one-year prior to the announcement of a disruption and ending two years after it. They found that firms who reported supply chain disruptions reported stock returns over the three-year period that were nearly 40 percent lower than comparable firms that did not report disruptions. Evidently, supply chain disruptions are major business events that have lasting effects.

However, not all of the consequences of supply chain disruptions are negative. By the end of 2001, Nokia and Samsung reported large increases in market share, while Motorola showed a steady of market share, in contrast to declines in the three previous years (see Figure 1). Apparently, Ericsson's loss was these other firms' gain, since some customers who were unsupplied by Ericsson shifted their purchases to other brands. Hence, supply chain disruptions can pose opportunities for strategic gain, as well as loss.

To understand why the Philips disruption having such an unbalanced impact on Nokia and Ericsson,

we must go back to the year 1995. At that time, due to poor delivery performance and a relatively weak product line, Nokia was experiencing an imbalance between supply and demand. To remedy the situation, Nokia installed a monitoring process, which enabled them to spot the disruption quickly (even before Philips officially notified them of the problem) and to initiate a dialogue with Philips about alternate supplies. Once Nokia's component-purchasing manager knew about the fire, he quickly reported it to his upper-level managers, in keeping with Nokia's culture of encouraging dissemination of bad news. Nokia decided to scrutinize the situation and call Philips daily to monitor the event. When Nokia realized the shortage due to the fire would last for more than month, it assembled a task force to investigate all possible alternative sources of supply (Sheffi 2005). This included capacity of other Philips facilities, as well as that of other suppliers. The ability to use non-Philips chips was facilitated by an earlier decision by Nokia to redesign their phones to accommodate a wider range of chips that could be supplied by other suppliers in Japan and the United States. In contrast, Ericsson did not realize the severity of the disruption until Nokia had locked up all available supplies of the original chips and they did not have product flexibility to allow use of other more widely available chips.

A major reason that a small-scale disruption resulted in a huge loss for Ericsson is that it resulted in leaving customers because they cannot be satisfied (Stauffer 2003). In hindsight, it is clear that Ericsson grossly underestimated the impact of what they thought would be a one-week delay of their chip supply. When Philips called two weeks after the fire to explain that the delay would be much longer, Ericsson realized too late that they would have thousands of seriously disappointed customers who would not return. In recognition of this type of customer dynamics, Zsidisin et al. (2000) concluded that firms implementing high efficiency supply management techniques (e.g., single sourcing, just-in-time deliveries for production) must be aware that such practices may increase their exposure to risk. Stauffer (2003) further pointed out that it is no longer feasible to try to cover losses from supply chain disruptions through insurance, since the 9/11 terrorist attack and other major events have altered the policies of insurance companies to limit coverage.

Although there has been a great deal of research devoted to the design, coordination and improvement of supply chains (see, e.g., Fisher et al. (1997), Fine (2000), Lee (2003), Graves and Willems (2003), Gan et al. (2005)), relatively little of this has focused on supply chain risks. Much of what has been done is descriptive in nature. For example, Johnson (2001) divided supply chain risks into two categories: (a) *demand risks*, including seasonal imbalances, fab volatility, and new product adoption, and (b) *supply risks*, such as, manufacturing and logistics capacity limitations, currency fluctuations, and supply disruptions from political issues. Chopra and Sodhi (2004) further refined these into a taxonomy

of risks faced in supply chains and qualitatively discussed the different strategies for mitigating them. Kleindorfer and Saad (2005) discussed the implications of designing supply chain systems to deal with disruption risks, based on a conceptual framework, which they termed the "SAM-SAC" Framework, and generated several empirical results from a data set of the U.S. Chemical Industry. Christopher and Peck (2004) also discussed how to design resilient supply chains in qualitative terms, with an emphasis on fully recognizing the nature of supply chain risks. In this same vein, Christopher and Lee (2004) suggested that improved end-to-end visibility is a key element for mitigating supply chain risk.

Some research considered supply chain disruptions within multi-echelon settings. Snyder and Shen (2006) investigated both supply uncertainty and demand uncertainty in simple multi-echelon supply chains using simulation. They pointed out that these two types of uncertainties require opposite responses and therefore firms should consider both types simultaneously to identify an optimal strategy. Hopp and Yin (2006) developed an analytical model for striking a balance between the costs of inventory and/or capacity protection and the costs of lost sales in an arborescent assembly network subject to disruption. They showed that, under certain conditions, it is optimal locate inventory or capacity protection at no more than one node along each path to the customer. In a related vein, Tomlin (2006) considered a firm that has two suppliers: one inexpensive and unreliable and the other expensive and reliable with volume flexibility. They described management strategies for using the two types of supplier to mitigate disruptions under various environmental assumptions. Tomlin and Snyder (2006) further considered disruption mitigation strategies for the situation in which disruptions are stochastic and the firm gets some advance warning of the disruptions. In Tomlin and Tang (2008), by studying five stylized models (which correspond to five types of flexible strategies), Tomlin and Tang suggested that flexibility can be used as a powerful defensive protection mechanism to mitigate supply chain risks.

While the above research is beginning to reveal principles of supply chain risk mitigation, to our knowledge, there is not yet a comprehensive modeling framework with which to evaluate both tactical and strategic risks to a firm from supply disruptions. In this paper, we develop such a framework and use it to analyze both how firms in competitive environments should prepare for potential disruptions and how they should respond to disruptions once they occur.

The remainder of this paper is structured as follows: In Section 2, we introduce a duopoly model with a third party, in which two firms compete for a common backup supply after a disruption of the primary supply of a key component, but the third party, which produces a competitive product, is not affected by the disruption. Note that this matches the Philips-Nokia-Ericsson scenario, in which Nokia and Ericsson constituted the duopoly and firms like Samsung and Motorola collectively constituted the outside firm.

Section 2.1 analyzes the Backup Capacity Competition (BC Competition) within this model in order to characterize the optimal plan of action to be followed by firms after a disruption. In Section 2.2 we study the Advanced Preparedness Competition (AP Competition), in which firms compete by investing in strategies for mitigating supply chain risks in advance of a disruption. In Section 2.3 we perform a sensitivity analysis of factors that influence the AP Competition. In Section 3 we investigate the factors that influence the degree of strategic risk faced by firms and provide guidance on firms' preparation strategies. Section 4 summarizes our conclusions and points to future research directions.

2 Model Formulation

To create a framework within which to evaluate the strategic impact of a supply disruption, we begin by considering a market consisting of two firms, A and B, that sell competing products. Both products require a key component that is purchased from a common supplier or region that is subject to disruption. We assume that the key component is an essential part of the product, and therefore neither firm can produce their product without it. In addition to the two firms affected by the disruption, there is a third firm C, which offers a substitute for the products offered by firms A and B. However, because the third firm C's product is sufficiently different from those of firms A and B to allow it to rely on different components, it is not affected by the supply disruptions under consideration. Because of this, a disruption presents an opportunity for Firm C to steal customers from Firms A and B, but not for Firms A and B to steal customers from Firm C.

We model the disruption as a random event that completely stops the supply of the key component and consists of two periods: (i) a fixed lead-time T between the time of the disruption and the time when the backup capacity is able to begin supplying the key component, and (ii) a random duration of time after the lead-time that extends to the end of the outage, which we assume follows a cumulative probability distribution $F(\cdot)$ with an average of μ units of time (see Figure 2).

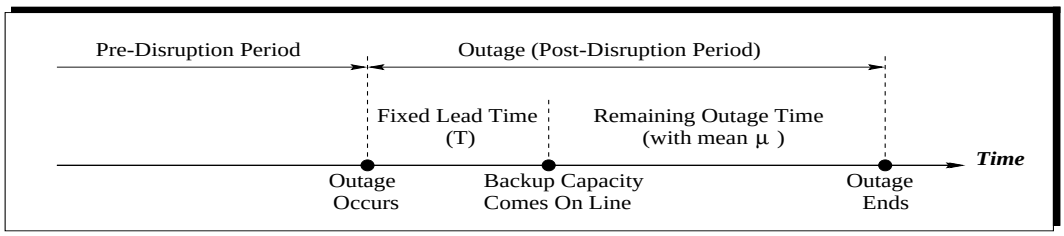

Figure 2: Time sequence of events after a disruption.

To create a model, we suppose Firm i ($i = A, B$) has capacity to produce K_i products per unit time and sells its products at a profit margin of r_i per product. Each product requires one unit of the key

component. Before a disruption, demand for Firm i's product is $\hat{d}_i^0$ per unit time. To allow us to consider cases where the firm purchases more backup capacity for the key component than it actually needs, we define the "holding cost" for one unit unused backup capacity to be h_i per unit time.

We assume that, during the outage, customers of Firm i who are unable to purchase from Firm i will buy the product from the other firm, Firm j, as long as Firm j can provide substitute products. If there is no alternate supply with which to replace the disrupted parts, then neither firm will be able to meet demand. We further assume that, the third party, Firm C, offers a less-than-perfect substitute for the products offered by Firms A and B, which customers may turn to if neither Firm A nor Firm B have supply available.

We focus primarily on the situation where a backup supplier exists that could provide an alternate supply of the disrupted parts. However, if the backup capacity is limited, as we would expect it to be, the question of how it is allocated is critical. Several factors could influence which firm (A or B) has an advantage in securing the backup capacity. One firm might have a prior relationship with the backup supplier that gives them an edge. Or a firm's size or reputation might make them a more desirable customer. Or, as seems to have been the case in the competition between Nokia and Ericsson, it might simply be a matter of who monitors the situation more carefully and therefore asks first. Our framework will accommodate any of these, but we will focus on the first-come-first-serve rule as the most interesting and most likely scenario.

We assume a business-to-business (OEM) relationship between the firms and their customers. That is, the firms have contracts with their existing customers and are therefore obligated to fill orders from these customers before filling orders from new customers. In contrast, in a business-to-consumer environment, firms cannot control which customers–existing or new–have their orders filled first. (For example, Kellogg cannot ensure that their corn flakes go to loyal Kellogg customers, as opposed to Post customers who are unable to buy their usual brand.) Under the business-to-business conditions, a firm will supply its own customers first. But if the firm is able to secure a backup capacity of the disrupted parts beyond those needed for its own customers (and it has ample supplies of other parts and the necessary production capacity), then it could meet demand from the competitor's customers. We assume that the firm will take advantage of the situation to do this if the competitor firm is unable to satisfy its own customers during the outage. If one firm makes sales to the other firm's customers, then the second firm will lose revenue in the short term. Furthermore, if some of these customers shift their future sales to the new firm, then the second firm will also lose long term market share.

Our assumption that, during the outage, customers of a firm unable to meet demand will switch

without hesitation to another firm, is motivated by the fact that these customers are themselves firms. If these firms make use of lean practices, they will have limited supplies of components on hand. Since shutting down production is very costly, they will seek alternate supplies almost immediately in the face of a disruption. For example, cellular services companies unable to obtain Ericsson handsets were quick to satisfy their customers with alternate brands. However, whether a customer will switch permanently depends on his/her brand loyalty. We model brand loyalty of Firm i's ($i = A, B$) customer by the length of time that customers of Firm i wait during the disruption before they permanently switch to the other firm's product. Specifically, we assume that a customer of Firm i who uses Firm j's ($j \neq i,\ j = A, B, C$) product during an outage, waits for an exponentially distributed amount of time with an average of γ_{ij} before permanently switching to Firm j's product. We define m_i as the (finite) net present value of the long term market share that Firm j gains from one customer who permanently switches from Firm i to Firm j.

In situations where the first firm (either A or B) to detect the disruption and contact the backup supplier will have the option to buy as much of the backup capacity as it chooses, there is incentive to invest in early detection capabilities. Indeed, this is precisely what Nokia did when it installed its new monitoring system. Hence, we include a preparedness period, in which firms invest in monitoring technology, research potential backup suppliers and take other measures to enable them to be first to secure the backup capacity in the event of a disruption. Specifically, we consider the following two periods:

- *Pre-Disruption Period:* Before a disruption occurs, both Firm A and Firm B serve their own customers. Each firm estimates the likelihood of a disruption of the key component, which may differ from the true likelihood of the disruption (p_0). After a disruption occurs, firms will seek available backup capacity for the key component. There are two types of backup capacity: (a) *dedicated capacity* that is committed to a specific firm (e.g., due to a contractual or informal relationship), and (b) *shared capacity*, for which the two firms compete. It is easy to see that if the dedicated backup capacity is not enough to cover the shortage, then firms will compete for the shared backup capacity. In this paper we focus on the shared capacity, since this is the situation that presents the firms with risk that can be mitigated through preparedness. To do this, we denote the amount of Firm i's sales ($i = A, B$) that cannot be covered by the dedicated capacity as d_i^0. Note that, since we are interested in the competition for the shared backup capacity, from now on, d_i^0 is treated as Firm i's original demand before the disruption, and we simply call the shared backup capacity the "backup capacity", which we denote by S. Both firms know that if the disruption occurs, the firm that detects the disruption and moves first has the advantage of

securing some or all of the total available backup capacity for the key component. Therefore, each firm must decide *how to invest in preparedness* in order to be able to detect the disruption first. This results in an *Advanced Preparedness Competition (AP Competition)*, in which the two firms invest in preparedness and thereby determine their respective probabilities of being first to detect the problem in the event of a disruption.

- *Post-Disruption Period:* After the disruption occurs, the firm (either A or B) that detects the disruption must decide *how much of the total available backup capacity of S to buy* in order to maximize its short- and long term profits. This results in a *Backup Capacity Competition (BC Competition)*, in which one of the firms (i.e., the winner of the AP competition) is designated as the "Leader" and gets first chance at purchasing the backup capacity. After the first firm has made its purchase, the other firm, the "Follower", then decides how much of the leftover backup capacity to purchase. Once both firms have made their purchase amount decisions, there is a lead-time period required to bring the backup capacity on-line. During this period, both Firm A and Firm B lose all of their short term sales, as well as some long term market share, to Firm C. The amount of the market share of Firm i ($i = A, B$) that is lost to Firm C is $d_i^0 - d_i$, where d_i is the demand for Firm i's product at the end of the lead-time. Since the profit loss during the lead-time is unavoidable for both firms, we can treat the moment immediately after the lead-time as the beginning of the post-disruption period. Furthermore, we assume that, after the lead-time, if the available backup capacity is not enough to cover the total market, then all unsatisfied customers will buy substitute products from Firm C during the outage. The number of customers that permanently switch to Firm C, depends on the customer brand loyalty.

It is clear that if ample backup capacity of the key component existed (i.e., S is infinite), then there would be no AP Competition or BC Competition between Firm A and Firm B because there would be no shortage to allow an opportunity to "poach" customers from the other firm. Hence, Firm i will only purchase d_i units of the capacity per unit time to satisfy its own customers ($i, j = A, B$). However, when the backup capacity is limited, then both the AP Competition and the BC Competition have an important impact on firms' short- and long-term profits and market shares. We analyze these two competitions (games) in reverse order, using a profit maximization for the objective in the BC Competition and a Nash equilibrium to define the outcome in the AP Competition.

2.1 Backup Capacity Competition (BC Competition)

In this section we consider the beginning of the post-disruption period when the winner of the AP competition (Firm A or B), that we call the *Leader*, has detected the disruption before the other firm, that we call the *Follower*. The question that the Leader faces is how much of the backup capacity it should buy, while the Follower can only purchase from whatever backup capacity is left after the Leader has made its purchase. Both firms will seek to maximize profit from both short term sales and long term sales.

One factor that both firms must consider when making their purchases of backup capacity is their production capacity, which corresponds to the maximum number of products that they can produce given ample supplies of raw materials. If, for instance, Firm A has only enough production capacity to meet demand from its existing customers, then it cannot make sales to the customers of Firm B, regardless of how much backup capacity it purchases.

We define $Y_{i,j}$ as the amount of backup capacity that Firm i ($i = A, B$) buys, when Firm i is in position j ($j = L, F$, where L = Leader, F = Follower). Let c_i ($i = A, B$) be the premium cost of one unit of the backup capacity for Firm i, and consider $\Pi_{i,j}$ (for $i = A, B$ and $j = L, F$) as the sum of the total expected profit of Firm i, given it is in position j during the outage. The total expected profit includes both the short term sales profit during the outage and also the long term sales profit due to market share gained from the competitor. Without loss of generality, suppose Firm A is the Leader and Firm B is the Follower. Then we have:

$$\begin{aligned}
\Pi_{A,L}(Y_{A,L}, Y_{B,F}) \;=\; & r_A(\text{Average number sold to A's customers during the outage}) \\
& +r_A(\text{Average number sold to B's customers during the outage}) \\
& +m_B(\text{Average number of customers (i.e., market share) gained from B}) \\
& -m_A(\text{Average number of customers (i.e., market share) lost to B}) \\
& -m_A(\text{Average number of customers (i.e., market share) lost to C}) \\
& -c_A(\text{Average backup capacity bought during the outage}) \\
& -h_A(\text{Average “inventory” of unused backup capacity during the outage}).
\end{aligned}$$

Since it is clear that the Follower will not purchase backup capacity in excess of its own production

capacity (because it cannot use it), we can express the expected profit of the Leader as:

$$\begin{aligned}
\Pi_{A,L}(Y_{A,L},Y_{B,F}) \;=\;& r_A\Big(\int_0^\infty \min\Big\{Y_{A,L},d_A\Big\}tdF_0(t)\Big)\\
&+r_A\Big(\int_0^\infty \min\Big\{[d_B-Y_{B,F}]^+,\min\big\{[Y_{A,L}-d_A]^+,K_A-d_A\big\}\Big\}tdF_0(t)\Big)\\
&+m_B\Big(\int_0^\infty \min\Big\{[d_B-Y_{B,F}]^+,\min\big\{[Y_{A,L}-d_A]^+,K_A-d_A\big\}\Big\}\\
&\qquad\times(1-e^{-\frac{t}{\gamma_{BA}}})dF_0(t)\Big)\\
&-m_A\Big(\int_0^\infty \min\Big\{[Y_{B,F}-d_B]^+,[d_A-Y_{A,L}]^+\Big\}(1-e^{-\frac{t}{\gamma_{AB}}})dF_0(t)\Big)\\
&-m_A\Big(\int_0^\infty \Big[[d_A-Y_{A,L}]^+-[Y_{B,F}-d_B]^+\Big]^+(1-e^{-\frac{t}{\gamma_{AC}}})dF_0(t)\Big)\\
&-c_A\Big(\int_0^\infty Y_{A,L}tdF_0(t)\Big)\\
&-h_A\Big(\int_0^\infty \Big[Y_{A,L}-\min\{Y_{A,L},d_A\}-\min\big\{[d_B-Y_{B,F}]^+,\\
&\qquad\min\{[Y_{A,L}-d_A]^+,K_A-d_A\}\big\}\Big]^+tdF_0(t)\Big).
\end{aligned}$$

After some algebra, we can reduce this to:

$$\begin{aligned}
\Pi_{A,L}(Y_{A,L},Y_{B,F}) \;=\;& r_A\min\Big\{Y_{A,L},d_A\Big\}\mu\\
&+\psi_A\min\Big\{[d_B-Y_{B,F}]^+,\min\big\{[Y_{A,L}-d_A]^+,K_A-d_A\big\}\Big\}\\
&-m_A\xi_{AB}\min\Big\{[Y_{B,F}-d_B]^+,[d_A-Y_{A,L}]^+\Big\}\\
&-m_A\xi_{AC}\Big[[d_A-Y_{A,L}]^+-[Y_{B,F}-d_B]^+\Big]^+\\
&-c_AY_{A,L}\mu\\
&-h_A\Big[Y_{A,L}-\min\{Y_{A,L},d_A\}-\\
&\qquad\min\Big\{[d_B-Y_{B,F}]^+,\min\big\{[Y_{A,L}-d_A]^+,K_A-d_A\big\}\Big\}\Big]^+\mu, \qquad (1)
\end{aligned}$$

where $\psi_i = r_i\mu + m_j\xi_{ji}$, and

$$\xi_{ij}=\int_0^\infty(1-e^{-\frac{t}{\gamma_{ij}}})dF_o(t) \qquad : \qquad j\neq i, \quad i=A,B, \quad j=A,B,C$$

is defined as the probability that a customer of Firm i permanently switches to Firm j after the outage.

It is clear that it is not beneficial for Firm B (the Follower) to hold inventory. Therefore, we get:

$$\begin{aligned}
\Pi_{B,F}(Y_{A,L},Y_{B,F}) \;=\;& r_B\min\Big\{Y_{B,F},d_B\Big\}\mu+\psi_B\min\Big\{[d_A-Y_{A,L}]^+,[Y_{B,F}-d_B]^+\Big\}\\
&-m_B\xi_{BA}\min\Big\{\min\big\{[Y_{A,L}-d_A]^+,K_A-d_A\big\},[d_B-Y_{B,F}]^+\Big\}\\
&-m_B\xi_{BC}\Big[[d_B-Y_{B,F}]^+-\min\big\{[Y_{A,L}-d_A]^+,K_A-d_A\big\}\Big]^+-c_BY_{B,F}\mu. \qquad (2)
\end{aligned}$$

Hence, the problem of finding the optimal backup capacity purchase for Firm A, the Leader, reduces to the following optimization problem:

$$\textbf{Problem PA:} \quad \max_{\forall\, Y_{B,F}} \quad \Pi_{A,L}(Y_{A,L}, Y_{B,F})$$

Subject to:

$$0 \le Y_{A,L} \le S, \quad 0 \le Y_{B,F} \le \min\{S,\ K_B\}$$

The optimal backup capacity purchase for the Follower can also be obtained from the optimization problem as follows:

$$\textbf{Problem PB:} \quad \max_{\forall\, Y_{A,L}} \quad \Pi_{B,F}(Y_{A,L}, Y_{B,F})$$

Subject to:

$$0 \le Y_{B,F} \le \max\{S - Y_{A,L}, 0\}, \quad 0 \le Y_{B,F} \le \min\{S,\ K_B\}$$

Proposition 1 shows that the optimal backup capacity purchases for both firms are restricted to a small number of possible values, and shows how these values depend on the Leader's production capacity (K_A) and the amount of available backup capacity (S). The proof of Proposition 1 and all other analytical results can be found in the Online Appendix.

PROPOSITION 1:

(Large Capacity Leader:) *If the Leader's production capacity is greater than the total market share of both firms (i.e., $K_A \ge d_A + d_B$), and*

(1) *if the backup capacity is less then the total market share of both firms (i.e., $S \le d_A + d_B$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$(Y^*_{A,L},\ Y^*_{B,F}) \in \Big\{(0,0),\ (0,d_B),\ \big(0,\min\{K_B,S\}\big),\ (S-d_B,0),\ (S-d_B,d_B),\ (d_A,0),\ (d_A,S-d_A),\ (S,0)\Big\};$$

(2) *if the backup capacity is larger than the total market share of both firms (i.e., $S > d_A + d_B$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$(Y^*_{A,L},\ Y^*_{B,F}) \in \Big\{(0,0),\ (0,d_B),\ \big(0,\min\{K_B,\ d_A+d_B\}\big),\ (d_A,0),\ (d_A,d_B),\ (d_A+d_B,0),\ (S,0)\Big\}.$$

(Small Capacity Leader:) *If the Leader's production capacity cannot cover the total market shares of both firms (i.e., $K_A < d_A + d_B$), and*

(1) *if the backup capacity is less than the Leader's market share (i.e., $S \leq K_A$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$(Y^*_{A,L},\ Y^*_{B,F}) \in \Big\{(0,0),\ (0,d_B),\ (0,\min\{K_B,\ S\}),\ (S-d_B,0),\ (S-d_B,d_B),\ (d_A,0),\ (d_A,S-d_A),\ (S,\ 0)\Big\};$$

(2) *if the backup capacity is larger than the Leader's market share, but it is smaller than the total market share (i.e., $K_A < S \leq d_A + d_B$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$(Y^*_{A,L},\ Y^*_{B,F}) \in \Big\{(0,0),\ (0,d_B),\ (0,\min\{K_B,\ S\}),\ (S-d_B,0),\ (S-d_B,d_B),\ (d_A,0),\ (d_A,S-d_A),\ (K_A,0),\ (K_A,S-K_A)\Big\};$$

(3) *if the backup capacity can cover the total market share of both firms (i.e., $S > d_A + d_B$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$(Y^*_{A,L},\ Y^*_{B,F}) \in \Big\{(0,0),\ (0,d_B),\ (0,\min\{K_B,\ d_A+d_B\}),\ (d_A,0),\ (d_A,d_B),\ (K_A,0),\ \big(S-(d_A+d_B)+K_A,0\big),\ \big(S-(d_A+d_B)+K_A,(d_A+d_B)-K_A\big)\Big\}.$$

Proposition 1 presents a set of solutions that dominates all feasible solutions of the BC Competition, and therefore includes all candidates for the optimal solution. Using the results of this proposition, we characterize the structure of the optimal solution of the BC Competition in Proposition 2.

PROPOSITION 2: *The optimal policy of the Backup Capacity Competition has a five-region structure.*

The thresholds that describe the five regions of the optimal policies for both the Leader and the Follower are presented in Table 1 and Table 2 in the Online Appendix. Based on the system parameters, the optimal policy for the BC Competition results in one of the following five scenarios: (i) Firms A and B Forfeit, (ii) Firm A is aggressive, (iii) Firm A protects, (iv) Firm A forfeits to Firm B, and (v) Firm B forfeits to Firm A.

To further describe this, we present an example of the optimal policy in Figure 3, in which $\max\{d_A,\ d_B\} \leq S \leq K_A < d_A + d_B$. As Figure 3 shows, the optimal solution results in five different scenarios:

(i) Firms A and B Forfeit: When $r_A \leq c^I_A$ and $r_B \leq c^1_B$, the optimal strategies for the Leader and Follower are $(Y^*_{A,L}, Y^*_{B,F}) = (0,0)$. This scenario presents situations in which neither Firm A nor Firm B is willing to buy any backup capacity due to its high premium cost[1]. Consequently, all unsatisfied customers will buy from Firm C, and depending on their customer loyalty, some of them may permanently switch to Firm C after the outage.

[1]Note that c^I_A and c^1_B are increasing functions of c_A and c_B, respectively, (see Table 8 in the Online Appendix B), so the larger c_A, the larger c^I_A, and the larger c_B, the larger c^1_B.

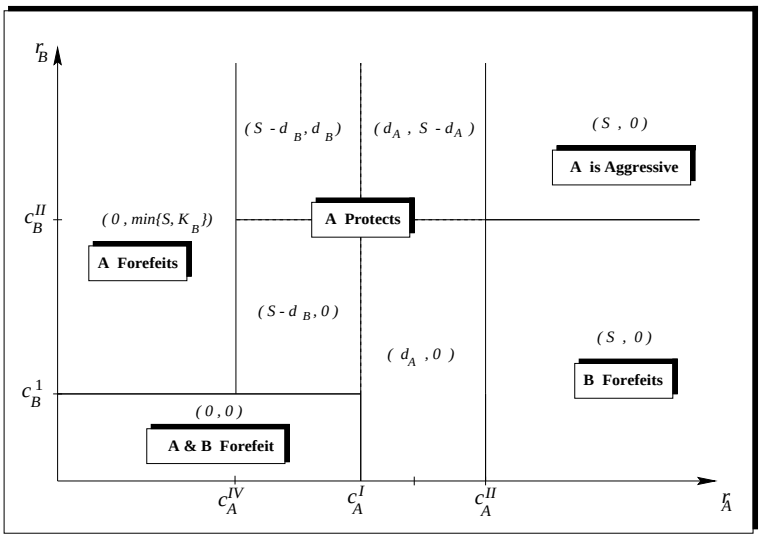

Figure 3: Five-region structure for optimal BC Competition strategy when $(\max\{d_A,\ d_B\} \leq) S \leq K_A < d_A + d_B$.

(ii) Firm A is Aggressive: This corresponds to $(Y^*_{A,F}, Y^*_{B,F}) = \big(\min\{S,\ K_A\},\ [S - K_A]^+\big)$, where the Leader (Firm A) buys the minimum of the entire backup capacity and its full production capacity. The reason is one of the following: (i) it is profitable for Firm A to satisfy its own customers and Firm B's customers using backup capacity, and (ii) although it is not profitable to satisfy Firm A's customers due to the high premium cost, it is still profitable for Firm A to satisfy Firm B's customers. (Recall that, we are considering a business-to-business environment, in which before Firm A satisfies any of Firm B's customers, it must satisfy all its own customers.) Thus, under either of the two cases, Firm A buys either the entire backup capacity or up to its full production capacity. Notice that, although there may be still some remaining backup capacity for Firm B to purchase, it is guaranteed that the unsatisfied customers of Firm B are enough for Firm A to use up all of its production capacity.

(iii) Firm A Protects: When the premium cost of the backup capacity is high, it is not profitable for Firm A to buy the backup capacity and serve its customers; however, the firm may need to protect itself from losing customers to Firms B and C. There are two types of protection strategies for Firm A (the Leader):

- $(Y^*_{A,L} = S - d_B)$, which occurs when Firm A protects itself from losing market share to Firms B. To prevent Firm B from poaching Firm A's customers, Firm A purchases some amount that leaves only d_B units of backup capacity for Firm B to satisfy its own customers. Also, in this case, since Firm A's customer loyalty to Firm C is high, Firm A will lose relatively few unsatisfied customers to Firm C.

- $(Y^*_{A,L} = d_A)$, which occurs when Firm A protects itself from losing market share to both Firms B

and Firm C. Note that, in this case, since Firm A's customer loyalty to Firm C is low, compared to the high premium cost of the backup capacity, it is economical for Firm A to buy the backup capacity and prevent Firm C from poaching its customers.

(iv) Firm A Forfeits: In this case $Y^*_{A,F} = 0$ (but $Y^*_{B,F} \neq 0$), which corresponds to the situation where the premium cost of the backup capacity is too expensive for Firm A to purchase for any purpose; therefore, Firm A does not buy any capacity.

(v) Firm B Forfeits: This corresponds to $(Y^*_{A,F}, Y^*_{B,F}) = \big(\min\{S,\ d_A + d_B,\ K_A\},\ 0\big)$. In this case, the premium cost of the backup capacity is too expensive for Firm B to use it to satisfy its own customers. Therefore, Firm B does not buy any capacity.

2.2 Advanced Preparedness Competition (AP Competition)

As we explained in the previous section, the allocation of backup capacity is entirely determined by which firm approaches the backup supplier first. But we recognize that supplier behavior may be influenced by other factors, such as: (1) *firm size:* the largest firm may receive the highest priority; (2) *willingness to pay:* the firm who is willing to pay more may get the backup capacity; (3) *firm preparedness:* the firm may affect the outcome by making more effort to carefully monitor the situation and detect the disruption first, enabling it to secure the desired backup capacity; (4) *business history:* the firm perceived by the supplier as a better customer may receive higher priority. In practice, the outcome may well be determined by a combination of these factors.

It is clear that, in the first and fourth cases, a firm has less ability to influence the outcome; in the second case, the outcome is relatively direct and the key factor is the maximum that a firm is willing to bid for capacity. Therefore, our analysis focuses on the third scenario, the firms' preparedness, which seems to have been the case in the competition between Nokia and Ericsson. We consider a case in which the firm can affect the outcome by making more effort to carefully monitor the situation and detect the disruption first. This results in the Advanced Preparedness Competition (AP Competition).

Suppose x_i $(i = A, B)$ is the effort that Firm i spends in preparedness activities such as monitoring and detection. We assume that, π_i, the probability that Firm i detects the disruption first and therefore becomes the Leader in the BC Competition is proportional to the preparedness effort of Firm i as a fraction of total preparedness effort. Specifically, we assume:

$$\pi_i = \frac{x_i}{x_i + x_j} \qquad : \quad j \neq i, \quad \text{and} \quad i = A, B, \quad j = A, B,$$

and $\pi_j = 1 - \pi_i, i \neq j$.

We defined p_0 as the true likelihood of a disruption. However, it is often the case that firms do not know p_0 and only have an estimate for it. Hence, we define:

- p_{i0} is Firm i's estimate of the likelihood of a disruption;
- $p_{j0,i}$ is Firm i's belief of Firm j's estimate of the likelihood of a disruption.

In the following, we define the *AP Competition for Firm i* ($i = A, B$) as the game in which Firm i plays the AP Competition based on its estimate of the likelihood of a disruption (i.e., p_{i0}) and its belief of Firm j's estimate of the likelihood of a disruption (i.e., $p_{j0,i}$).

We use C_e to denote the cost per unit of effort spent on preparedness activity, and Π_i^{AP} is defined as Firm i's total expected profit in the *AP Competition for Firm i*, which is given by:

$$\Pi_i^{AP} = p_{i0}\Big(\pi_i \Pi_{i,L}(Y_{i,L}^*, Y_{j,F}^*) + \pi_j \Pi_{i,F}(Y_{i,F}^*, Y_{j,L}^*) - m_i(d_i^0 - d_i)\Big) + (1 - p_{i0})\Big(r_i d_i^0(\mu + T)\Big) - C_e x_i + m_i d_i^0,$$

where $r_i d_i^0(\mu+T)$ is the total regular sales profit of Firm i during the $(\mu+T)$-day interval if no disruption occurs and $m_i d_i^0$ is Firm i's discounted future sales profit.

At the same time, Firm i believes that Firm j uses $p_{j0,i}$ as its estimate for the likelihood of a disruption, and hence, believes that Firm j is maximizing the following:

$$\Pi_{j,i}^{AP} = p_{j0,i}\Big(\pi_j \Pi_{j,L}(Y_{j,L}^*, Y_{i,F}^*) + \pi_i \Pi_{j,F}(Y_{j,F}^*, Y_{i,L}^*) - m_j(d_j^0 - d_j)\Big) + (1 - p_{j0,i})\Big(r_j d_j^0(\mu + T)\Big) - C_e x_j + m_j d_j^0.$$

Symmetrically, Firm j solves its own AP competition problem, in which Firm j maximizes Π_j^{AP} and believes that Firm i maximizes $\Pi_{i,j}^{AP}$, in which the probability of a disruption is thought to be $p_{i0,j}$.

To simplify notation we use $\Pi_{i,L}^*$ and $\Pi_{i,F}^*$ instead of $\Pi_{i,L}(Y_{i,L}^*, Y_{j,F}^*)$ and $\Pi_{i,F}(Y_{i,F}^*, Y_{j,L}^*)$, respectively. Hence, the AP Competition for Firm i is to find x_i that maximizes Π_i^{AP}, the firm's total expected profit:

$$\max_{x_i} \Pi_i^{AP} = p_{i0}\Big(\pi_i \Pi_{i,L}^* + \pi_j \Pi_{i,F}^* - m_i(d_i^0 - d_i)\Big) + (1 - p_{i0})\Big(r_i d_i^0(\mu + T)\Big) - C_e x_i + m_i d_i^0 \qquad j \neq i,$$

where $\pi_i = x_i/(x_i + x_j)$ and $\pi_j = 1 - \pi_i$.

Note that both firms would like to increase their chance (π_i) to become the Leader in the BC Competition. This can be achieved by increasing preparedness effort (i.e., x_i). However, increasing x_i does not necessarily guarantee an increase in the probability of being first, since this probability also depends on the amount of effort by the other firm. This results in a competition in preparedness efforts between the two firms (Firm A and Firm B).

PROPOSITION 3: *A unique Nash Equilibrium exists for each firm's Advanced Preparedness Competition.*

By Proposition 3, there is a unique Nash Equilibrium for the AP Competition for Firm i, i.e., (x_i^*, x_j^*), and there is also a unique Nash Equilibrium for the AP Competition of Firm j, i.e., (w_j^*, w_i^*). That is, x_i^* is the actual preparedness effort Firm i spends given it believes Firm j's preparedness effort x_j^*, while, w_j^* is the actual preparedness effort Firm j spends given it believes Firm i's preparedness effort w_i^*. Therefore, the preparedness investment by Firms A and B in the AP Competition is given by (x_i^*, w_j^*). Consequently, Firm i's probability of becoming the Leader in the BC Competition is,

$$\pi_i^* = \frac{x_i^*}{x_i^* + w_j^*}, \quad \pi_j^* = 1 - \pi_i^*.$$

Note that both Firms A and B find their optimal effort levels based on their estimates of the probability of a disruption, which might be different from the actual probability of disruption. Considering the actual probability of a disruption, p_0, the *actual expected profit* of Firm i, Π_i under the effort levels (x_i^*, w_j^*) is

$$\Pi_i = p_0\Big(\frac{x_i^*}{x_i^* + w_j^*}\Pi_{i,L}^* + \frac{w_j^*}{x_i^* + w_j^*}\Pi_{i,F}^* - m_i(d_i^0 - d_i)\Big) + (1 - p_0)\Big(r_i d_i^0(\mu + T)\Big) - C_e x_i^* + m_i d_i^0.$$

2.3 Sensitivity Analysis

To better understand the AP Competition, we start with the easiest version, which (for convenience) we call *complete information* case. In this case,

- both firms know the other firm's estimate of the likelihood of a disruption (i.e., $p_{i0,j} = p_{i0}, i = A, B$);
- both firms know the actual likelihood of a disruption (i.e., $p_{i0} = p_0, i = A, B$).

Proposition 4 presents a monotonicity property of the optimal effort level of firms with respect to their cost structure and customer loyalty.

PROPOSITION 4: *Under complete information, given a fixed Firm j's preparedness effect, the optimal preparedness effort of Firm i (i.e., x_i) is nondecreasing in m_i, m_j, and r_i, and is nonincreasing in γ_{ij}, γ_{iC}, γ_{ji}, c_i, and h_i, for $i = A, B$ and $j \neq i$, $j = A, B$.*

We now consider what we call *interfirm information* case, where

- both firms know the other firm's estimate of the likelihood of a disruption;
- each firm's estimate of the likelihood of a disruption may not be accurate.

Before we present Proposition 5, we define Firm A to be an *"aggressive" firm* if its optimal backup capacity purchasing policy corresponds to the "A is Aggressive" area. That is, if it is the Leader, Firm A will buy all available backup capacity in an attempt to steal sales from Firm B.

PROPOSITION 5: *Under interfirm information, if both firms are "aggressive" and are identical except for their size (i.e., $d_A \neq d_B$), then both firms spend the same amount of effort on preparedness (i.e., $x_A = x_B$), and therefore will have the same chance (50%) to become the Leader in the BC Competition.*

Intuitively, the larger firm faces a small market share gain (from the competitor) as the Leader, but faces a large market share loss (to the competitor) as the Follower in the BC Competition. In contrast, the smaller firm faces a large market share gain as the Leader, but a small market share loss as the Follower. Since the two firms are identical except for their size, the opportunity cost (i.e., the gap between the expected profit from being the Leader and that of being the Follower) is exactly the same for both firms. Hence, both firms will invest the same amount for preparedness in the AP Competition.

3 Numerical Study

Characterizing the optimal policies in the BC Competition and the AP Competition gives us a basic understanding of the behavior of competing firms in the face of supply chain disruptions. But from a management perspective, the most valuable outcome of our framework is the insight it can provide into the conditions that present the greatest risk from supply chain disruptions.

To qualitatively and quantitatively investigate the factors that influence the consequences of a disruption, we use our framework to examine the sensitivity of a specific risk measure to a variety of factors. The measure of risk we use is Firm i's *loss due to lack of preparedness*, which is defined as:

$$\Omega_i \quad = \quad \Pi_i - \Big(p_0\big(\Pi^*_{i,F} - m_i(d_i^0 - d_i)\big) + (1-p_0)\big(r_i d_i^0(\mu + T)\big) + m_i d_i^0\Big). \tag{3}$$

This measure represents the difference between the expected profit to Firm i if it made strategic preparation in the AP Competition and if it did not. We assume that in both cases, the rival firm prepares optimally. Therefore, when Firm i does not prepare, it usually winds up being the Follower in the BC Competition. Hence, Ω_i characterizes risk as the opportunity cost of not preparing for a disruption. Note that, it is possible for Ω_i to be negative in extreme cases where Firm i would "over-prepare" because its estimate on the likelihood of a disruption is much larger than the true likelihood. In such cases, not preparing at all is economically preferred to such over-preparation. Without loss of generality, we focus on the loss due to lack of preparation for Firm A, Ω_A.

Although we have an analytic relation between Ω and the various parameters of the model, it is so complex that, at most, we can perform single parameter sensitivity analysis using it. The expression itself cannot show us how the model factors interact and compare with each other. Therefore, we make use of regression analysis to generate a statistical relation between loss due to lack of preparedness (Ω_A) and the various factors in the model. The results of this regression show us which factors are most important in identifying high risk situations (i.e., products).

3.1 Design of Numerical Experiments

Our numerical study is based on a test suite including $(3^{22} \times 2 \times 4$ cases in total, created using different values for the model parameters. The values are chosen to cover the majority of scenarios we could observe in practice. For example, We chose three values of \$60, \$90, and \$120, for the profit margin r_i for Firm i. These numbers are chosen to reflect the cell phone market with a 50% gross margin. Another example is the values for p_0, which are 0.01, 0.05, 0.2, and 0.4. Note that, p_0 is the true likelihood that the disruption occurs this year, where for instance, $p_0 = 0.05$ means that such a failure will happen on average once every 20 years; to motivate our choices of p_0, we note that the annual frequency of earthquakes in the U.S. with magnitude larger than seven is 0.03 events per year (Mathewson 1999), while the rate in Taiwan is five to ten times higher than this. For details of the values for all parameters see Online Appendix A.

From a managerial perspective, we are interested in identifying the most important factors that affect the firm's risk exposure from a given component, so that management can target their preparedness efforts on the products where they will have the greatest impact. Since it is not reasonable to expect a firm with thousands of components to estimate all of the model parameters for every part in their portfolio (e.g., because some of these, such as customer loyalty coefficients may require significant analysis), our hope is that a simpler model consisting of only a few (i.e., less than five) important factors can reliably identify the high risk components.

To find the top five factors that have the most impact on the strategic risk for each firm, we performed a stepwise regression analysis in which, Ω_i, the strategic risk for firm i (i.e., loss due to lack of preparedness) was the dependent variables. We considered a set of 421 independent variables corresponding to the 8 system parameters, 5 profit factors, and 15 normalized factors. Specifically, we considered eight parameters, 20 linear factors, 15 squared terms, and 378 interaction terms (the combination of the 28 predictor variables). The value of α for entering or removing a variable in our step-wise regression is 0.05. Our step-wise regression is performed to find the best regression model with 1, 2, . . . , and 5 independent variables that best explain the variation in Ω_i (i.e., have the highest R-square).

The five profit factors in our regression models include the following:

- Firm A's worst-case *short-term* loss:

$$SL_{Total} = T \times r_A \times d_A^0 + \mu \times r_A \times d_A$$

  which is the loss of profit during the lead-time and outage, if Firm A cannot satisfy any of its customers.

Number of Independent Variables in the Model	R^2	Variables in the Regression Model				
		$p_0 \times LLC$	$p_0 \times LLB$	$\frac{d_A}{S}$	$\frac{p_{B0}-p_0}{p_0}$	$\frac{K_B-d_B}{d_A}$
1	42.1	+				
2	46.8	+	+			
3	47.6	+	+	+		
4	48.3	+	+	+	−	
5	48.7	+	+	+	−	+

Table 1: Regression models with the five most important factor(s) under the *ABC Model* when Firm A is larger. Sign + or − corresponds to the sign of coefficients in the regression model.

- Firm A's worst-case *long-term* loss to Firm B (i.e., LLB), or to Firm C (i.e., LLC):

$$LLB = m_A \times \frac{\mu}{\gamma_{AB}} \times d_A \qquad LLC = m_A \times \frac{T}{\gamma_{AC}} \times d_A^0 + m_A \times \frac{\mu}{\gamma_{AC}} \times d_A$$

  These long terms losses corresponds the worst case that Firm A cannot satisfy any of its own customers.

- Firm A's best-case *short-term* gain (i.e., SG) and *long-term* gain (i.e., LG):

$$SG = \mu \times r_A \times d_B \qquad LG = m_B \times \frac{\mu}{\gamma_{BA}} \times d_B$$

  which correspond to the case where Firm B cannot satisfy any of its own customers.

To get a sense of how different competitive conditions affect the factors that characterize supply chain risk, we consider the both the ABC and AB environments in our numerical studies.

3.2 ABC Model

In this section, our regression studies are based on the *ABC Model*, which considers two competitors (Firms A and B) subject to disruption and a third firm (Firm C) not vulnerable to disruption.

In Table 1 we display the best fitting regression models with one to five parameters under the condition that Firm A is larger than Firm B. The results of Table 1 show that:

Top 1 Factor: If only one factor were included in the regression model, it would be Firm A's expected worst-case long term loss to the third party C (i.e., $p_0 \times LLC$), which is clearly essential in quantifying risk. As expected, this term has a positive coefficient. While we have seen expected short term loss used as a gauge of supply chain risk in industry, we are not aware of any firms that evaluate long term risk. So this observation is of practical significance. It says that, under the range of conditions descried by our numerical examples, expected long term loss is a better predictor of overall risk than expected short term loss.

Top 2 Factors: The best two-factor model adds Firm A's expected worst-case long term loss to Firm B (i.e., $p_0 \times LLB$) to Firm A's expected worst-case long term loss to the third party C. Again this factor

Number of Independent Variables in the Model	R^2	Variables in the Regression Model				
		$p_0 \times LLC$	$p_0 \times LG$	$p_0 \times LLB$	$\frac{K_A - d_A}{d_B}$	$p_0 \times SG$
1	27.2	+				
2	35.1	+	+			
3	39.6	+	+	+		
4	41.1	+	+	+	+	
5	42.1	+	+	+	+	+

Table 2: Regression models with the five most important factor(s) under the *ABC Model* when Firm A is smaller. Sign + or − corresponds to the sign of coefficients in the regression model.

appears in the model with a positive coefficient, as expected. Along with expected long term loss to Firm C, this factor characterizes total expected long term loss.

Top 5 Factors: The regression model with five factors has an R^2 of about 50%, and, in addition to the two long term loss factors, includes (1) Firm A's sales relative to the total available backup capacity, which implies that Firm A faces greater risk when backup capacity is limited, (2) the error in Firm B's estimate of the likelihood of a disruption, and (3) Firm B's "poaching potential". All of these have positive coefficients, except the error in Firm B's estimate of the likelihood of a disruption, which implies that more error by Firm B in estimating the likelihood of a disruption results in less risk to Firm A.

In Table 2 we display the best regression models with one to five parameters under the condition that Firm A is smaller than Firm B. From the results of Table 2, we can conclude the following:

Top 1 Factor: The best one factor model again uses Firm A's expected worst-case long term loss to the third party C (i.e., $p_0 \times LLC$).

Top 2 Factors: The best two-factor model adds Firm A's expected best-case long term *gain* (i.e., $p_0 \times LG$). This is an interesting and potentially significant result, since it suggests that the smaller firm in a market should consider the possibility of gaining market share during a supply chain disruption. Failure to do this can substantially decrease expected profit.

Top 5 Factors: The regression model with five factors has an R^2 of about 42%, and, in addition to the two factors already mentioned, includes (1) Firm A's expected worst-case long term loss to Firm B, (2) Firm A's "poaching potential", and (3) Firm A's expected best-case short term gain. Thus, the most important factors for predicting risk when Firm A is smaller than Firm B are similar to those for the case where Firm A is larger than Firm B, except that they also include some factors related to potential market share gains during a disruption.

To gain more insight into these results, we classify factors related to a firm's sales profit and/or market

share losses as *loss factors* and factors related to a firm's capability to capture sales and/or market share from the competitor firm as *gain factors.* Any factors that are in neither of these categories are classified as *neutral factors.* For instance, in Table 1, $p_0 \times LLC$ (Firm A's expected worst-case long term loss to the third party C) is a "loss factor" from Firm A's perspective. Note that none of the factors listed in Table 1 are gain factors for Firm A. The factor $p_0 \times LG$ (Firm A's expected best-case long term gain) in Table 2 is a "gain factor" from Firm A's perspective. Based on the above results, we conclude that,

Observation 1:

- *When Firm A is larger than Firm B, loss factors (e.g., expected long term loss, "poaching potential" of competition, ...) are the primary determinants of the level of strategic risk faced by Firm A.*
- *When Firm A is smaller than Firm B, gain factors (e.g., expected long term gain, Firm A's "poaching potential", ...) become important determinants of the level of strategic risk that faced by Firm A.*

The structural reason behind the first conclusion of Observation 1 follows from the fact that in our sample space, fewer outcomes lead to gains than to losses. For instance, when $\frac{d_A}{S}$ is large (bigger than one, i.e., Firm A's sales is in excess of the total available backup capacity), then even if Firm A is the Leader in the BC Competition, it will still lose some customers to the third party C. On the other hand, when $\frac{d_A}{S}$ is quite small (i.e., total backup capacity is well in excess of Firm A's sales), then, although it is profitable for Firm A to purchase supply sufficient to cover its own customers, it is too expensive for Firm A to corner the backup capacity market in order to steal customers from Firm B. Only when backup capacity is in a relatively narrow vicinity of $d_A + d_B$ does Firm A have a good opportunity to gain sales and market share from Firm B. The fact that "gain scenarios" tend to require specialized conditions implies that Firm A will be more likely to find products where the value of preparation is in the prevention of loss than in the generation of gain. This observation is consistent with the widely used financial investment strategy known as *dollar-cost averaging (DCA)*, which is based on the Sharpe ratio and is also known as the reward-to-variability ratio. This strategy attempts to reduce the risk of investing too much at the "wrong" time and also too little at the "right" time by balancing the upside potential and the downside risk and striving to create a "margin of safety". The DCA strategy weights downside risk more heavily than upside potential, even though this may result in investors losing opportunities to profit from the upside potential. Despite some recent research questioning the use of the Sharpe ratio as the preferred measure (e.g., Leggio and Lien 2003), researchers still find that the DCA ranking based on different performance measures (e.g., the Sortino ratio) tends to outperform alternative investment strategies, such as lump-sum investing.

The intuition behind the second conclusion of Observation 1 is that when Firm A is the smaller player in the market (i.e., $d_A < d_B$), it has incentive to be more aggressive and hence consider more "gain factors" in selecting components to focus on. Therefore, when a firm has market sales that are smaller than those of the competitor, the firm should consider the important factors listed in Table 2 in selecting components to prioritize in its preparation strategy.

Observation 2 summarizes cases where a firm faces great strategic risk.

Observation 2: *Firm will face great risk when (1) the likelihood of a disruption is large, (2) its market share is valuable, (3) its customers loyalty (especially relative to a third party unaffected by a disruption) is low, (4) its market sales exceed the available total backup capacity, and (5) total backup capacity exceeds sales of the competition and the competitor firm has a high "poaching potential".*

As Tables 1 and 2 show,

- the most important factor affecting the risk of Firm A is $p_0 LLC$, its expected worst-case long term loss to the third party C, which can be written out as:

$$p_0 \times LLC = \frac{p_0 \times m_A}{\gamma_{AC}} \times (T \times d_A^0 + \mu \times d_A).$$

  Consistent with Observation 2, this factor clearly increases in p_0 (the likelihood of a disruption) and m_A (the value of a unit of market share), and decreases in γ_{AC} (customer loyalty relative to Firm C).

- Firm A's expected worst-case long term loss to Firm B is also a very important factor in determining Firm A's risk exposure. Writing our this factor as

$$p_0 \times LLB = p_0 \times m_A \times \frac{\mu}{\gamma_{AB}} \times d_A,$$

  also shows that it increases in p_0 and m_A, and decreases in γ_{AB}, which is again consistent with Observation 2.

- The third factor in Table 1 is Firm A's sales relative to the total available backup capacity, and it has a positive sign. This implies that the more Firm A's market sales exceed the total available backup capacity, the more risk Firm A faces. Intuitively, when the total available backup capacity is so limited that it cannot even satisfy all customers of Firm A, then Firm A faces an increased risk of losing some customers to Firm C.

- The fifth factor in Table 1 is Firm B's *poaching potential*. Note that the numerator is Firm B's remaining production capacity after satisfying its own customers, which can be used to poach Firm A's customers. Therefore, Firm A's risk increases when Firm B has sufficient poaching potential,

and the available backup capacity is much higher than what is needed to satisfy all of Firm B's customers.

Observation 3: *Under our assumptions that the third party: (1) is not affected by the disruption, (2) provides a less-than-perfect substitute for the products offered by Firms A and B, which customers only turn to if neither Firm A nor Firm B has product available, and (3) has enough production capacity, a firm's customer loyalty to the third party is a more important factor than its customer loyalty to the competitor.*

This observation comes from the fact that the third party is always ready to poach any unsatisfied customers, regardless of whether the firm is the Leader or the Follower in the AB Competition. More specifically,

- When the firm is the Follower, the competitor can poach the firm's customer only after satisfying its own customers. So, even when the (shared) backup capacity is limited ($S \leq$ the competitor's sales) and thus the firm faces no danger of losing customers to the competitor, it can still lose customers to the third party.

- When the firm is the Leader, although the competition cannot poach the firm's customer, the firm can still lose customers to the third party (during the lead time T).

As an example, consider three different firms, Huawei, Cisco and Juniper, that manufacture routers. Although Huawei's routers are not yet in the same class as the high-end models offered by Cisco and Juniper, it represents a threatening challenger to the Cisco-Juniper duopoly because its cut-rate pricing has a huge appeal to smaller-scale telecommunication firms and it has deep-pocketed support from the Chinese government. Therefore, it is logical to assume that Huawei is waiting for chances to take business from the Cisco-Juniper duopoly. Both Cisco and Juniper source microchips mainly from Taiwan, while Huawei sources from both Taiwan and Shanghai, so if a disruption of Taiwan were to affect a Cisco product and the competing Juniper product, then a less-than-perfect Huawei substitute might pick up demand because it could remain available while Cisco and Juniper are unable to meet demand. As long as the Huawei product is perceived as a sufficiently acceptable substitute by some customers, then the disruption delays may drive some Cisco and Juniper customers to Huawei.

3.3 AB Model

The above results show that supply chain risk is influenced strongly by the presence of a third party (Firm C), which is not subject to a supply chain disruption. This is not surprising, since, unless customer

Number of Independent Variables in the Model	R^2	Variables in the Regression Model				
		$p_0 \times LLB$	$p_0 \times LG$	$\frac{d_A}{S}$	$\frac{p_{B0}-p_0}{p_0}$	$\frac{K_A-d_A}{d_B}$
1	33.1	+				
2	40.8	+	+			
3	43.1	+	+	−		
4	44.5	+	+	−	−	
5	45.8	+	+	−	−	+

Table 3: Regression models (without the third party) with the top 1–5 most important factor(s) under the *AB Model* when Firm A is larger.

Number of Independent Variables in the Model	R^2	Variables in the Regression Model				
		$p_0 \times LG$	$\frac{d_B}{S}$	$p_0 \times SG$	$\frac{K_A-d_A}{d_B}$	$\frac{p_{B0}-p_0}{p_0}$
1	31.7	+				
2	34.3	+	−			
3	37.1	+	−	+		
4	38.8	+	−	+	+	
5	39.5	+	−	+	+	−

Table 4: Regression models (without the third party) with the top 1–5 most important factor(s) under the *AB Model* when Firm A is smaller.

loyalty is extremely high, a disruption event is an excellent opportunity for the third party to steal market share from the disrupted firms.

To get a deeper understanding of the drivers of risk, we now consider the situation where there is no third party. That is, the market consists of a duopoly in which both firms are subject to a supply chain disruption. More specifically, we assume that products by competitors who do not rely on the vulnerable supplier are not close substitutes for the products produced by the duopoly firms.

We repeated our regression analysis under these conditions, which resulted in the following (as Tables 3 and 4 show):

Observation 4: *Gain factors have more impact on Firm A's strategic risk in the AB Model than in the ABC Model.*

As Table 3 shows, when Firm A is larger than Firm B, There is only one loss factor among the five. i.e., Firm A's expected worst-case long term loss ($p_0 \times LLB$). On the other hand, when Firm A is smaller than Firm B, there is no loss factor among the five factors listed in Table 4. As stated in Observation 1, loss factors are the primary determinants of the strategic risks.

Intuitively, this isolated market in the AB Model (involving only Firms A and B) makes firms more aggressive, since they are not at risk of losing customers to a third party, Firm C. In a sense, this

duopolistic market presents a zero sum situation, in which Firms A and B compete for each others' customers.

Observation 5: *When Firm A is the smaller firm in a duopolistic market (i.e., the AB Model), Firm A's strategic risk becomes lower as the market size of the competitor increases relative to the available backup capacity.*

Note that, as Table 4 shows, Firm B's sales relative to the total available backup capacity (i.e., $\frac{d_B}{S}$) is the second most important factor that affects Firm A's risk when Firm A is smaller than Firm B. This factor was not a critical factor in the ABC model, where a third party exists. The reason is that in the AB model, Firm A can only lose customers to Firm B, but if Firm B's sales is larger than the total available backup capacity, then Firm B will not be able to satisfy all of its own customers even if it purchases all available backup capacity. Therefore, regardless of the position (Leader or the Follower) of Firm B is in the BC Competition, it will have no opportunity to poach any of Firm A's customers, and hence Firm A faces no risk of losing customer. This was not the case in the ABC Model, since Firm A was also at risk of losing its customers to Firm C, which is not dependent on the backup capacity S.

3.4 Predictive Power of the First Important Factor

As we noted previously, some firms in industry (including a firm with whom we have worked, which is widely considered to be among the most sophisticated about managing supply chain risks) use expected short term loss as a measure of supply chain risk. We call this the *expected short-term loss heuristic.* Of course, the five factor models summarized in Tables 1-4 provide a more accurate characterization of risk. But they also require considerably more data. Therefore, to get a sense of how well a single factor risk metric can work, we suppose that Firm A considers its specific environment (under ABC, or AB; larger or smaller than Firm B), and uses the single most important factor from the regression model to identify the products in their portfolio for which they should invest in specific preparedness activities (e.g., setting up tighter supply monitoring mechanisms, cultivating a closer relationship with the supplier, pre-qualifying backup suppliers, modifying designs to make products more flexible with regard to input components, etc.). Clearly, a single factor is not a perfect predictor of risk due to lack of preparedness (i.e., the R^2 of our single-factor regression model is not very high), but it does not need to be. The problem is not to predict actual losses (or gains) with numerical precision, but rather to accurately identify those components for which risk is greatest.

To test the utility of our model, we evaluate the ability of the most important factor (i.e., the top 1 factor) in Tables 1, 2, 3, and 4, which we call the *Top 1 Factor heuristic,* or *T1F heuristic,* separately

Difference in Performance	ABC Model		AB Model	
	A is Larger	A is Smaller	A is Larger	A is Smaller
$\mathcal{U}_{diff}^{max}$	207.5%	127.9%	229.4%	248.4%
$\overline{\mathcal{U}}_{diff}$	126.14%	71.98%	120.96%	104.77%

Table 5: Numerical study of the percent gain of Firm A's expected profit due to using the most important factor in place of the expected short term loss heuristic.

against the expected short term loss heuristic to choose top five riskiest components among 150 randomly picked items from our test suite. We did this by computing the expected short term loss heuristic:

$$p_0 \times (T + \mu) \times r_A \times d_A^0$$

for each component, and then performing the following simulation experiment:

Step 1: Randomly pick 150 components from the set of scenarios. The record for each component contains (1) the values for all of the model factors from which we can calculate the exact value of Ω_A, and (2) the value of the most important factor (which depends on the specific environment) and the value of the expected short term loss heuristic.

Step 2: Sequence (from large to small) the components according to the value of the most important factor from 1 to 150, and calculate the total risk from the top five components as $\mathcal{U}^{Factor} = \sum_{i=1}^{5} \Omega_A^{(i)}$.

Step 3: Sequence (from large to small) the components according to the value of the expected short term loss heuristic, and calculate the total risk from the top five components as $\mathcal{U}^{Heuri} = \sum_{i=1}^{5} \Omega_A^{(i)}$. (Note that this value will differ from that in Step 2 if the indices are different.)

Step 4: Record the value $\mathcal{U}_{diff} = (\mathcal{U}^{Factor} - \mathcal{U}^{Heuri})/\mathcal{U}^{Heuri}$, which represents the percent gain of profit that Firm A experiences if the firm invests in preparedness for the five items determined by the value of the most important factor, rather than investing in the five components determined by the expected short-term loss heuristic.

Step 5: Go to Step 1 until 50 replicates have been run (so that the obtained sample mean is close to the true mean). Find the maximum and mean of $\mathcal{U}_{diff}$ and denote them by $\mathcal{U}_{diff}^{max}$ and $\overline{\mathcal{U}}_{diff}$, respectively.

The results of this experiment are shown in Table 5.

Observation 6: *On average, using the most important factor (i.e., T1F heuristic) to select the riskiest components is much better than using the expected short term loss heuristic to rank components. When*

customer loyalty differs across products, then the T1F heuristic significantly outperforms the expected short term loss heuristic because it explicitly considers expected long term loss in most cases.

Observation 7: *When customer loyalty is similar across products or when customer loyalty is lower for items with higher short term loss of sales (i.e., so that short term loss is highly correlated with long term loss), then the expected short term loss heuristic is also a good predictor of total risk.*

Note that in practice, the condition in Observation 7 that, "customer loyalty is lower for items with higher short term sales loss", does not always hold. In general, we would expect high demand products to have high customer loyalty because of customers' preference and due to network externality effects. For example, comparing Apple's personal computer (PC) with Toshiba's PC, Apple's PC has high sales and high customer loyalty because of network externalities. On the other hand, Toshiba's PC is not as big a seller as Apple's PC in the U.S. market and does not have the same level of network driven loyalty (Hruska 2008). Hence, a disruption would cause a larger short term loss of sales to Apple's PC but a larger long term loss of market share to Toshiba's PC. Consequently, short term loss may not be a good predictor of long term loss.

Observation 8:

- *In both ABC and AB models, the percent gain in profit a firm experiences from using the T1F heuristic in place of the expected short term loss heuristic in evaluating Firm A's risk due to lack of preparation is very significant.*
- *For the smaller firm, the percent gain is higher in the AB model than the ABC Model.*

As Table 5 shows, the average percent gain in profit in most cases is more than 100%. This implies that firm can gain a significant increase in their profit, if they prepare for supply possible supply chain disruption based on the most important factor presented in this paper rather than the expected short term loss factor.

As also shown in Table 5, when Firm A is the smaller firm, the average and the maximum of the percent gain (104.77% and 248%) are higher in the AB model than in the ABC Model (71.98% and 127%). The reason is as follows. Comparing Table 2 and Table 4, we see that the smaller firm's most important risk factor is a "loss factor" in the ABC model, but a "gain factor" in the AB model. In contrast, the expected short-term loss heuristic uses a loss factor to evaluate the smaller firm's risk. Knowing that the smaller firm in the AB model should focus on the gain factor (which is what the T1F heuristic does), we expect the performance of the T1F heuristic to be better than that of the expected short-term loss heuristic in the AB model, but not so much better in the ABC model.

3.5 Implications for Risk Management

We can boil down our insights from the above analytic and numerical analyses to produce the following list of policies firms can pursue to mitigate the tactical and strategic consequences of supply chain disruptions.

3.5.1 Speed Polices

In scenarios where a supply disruption affects multiple firms (i.e., the ABC and AB models), earlier detection and quicker reaction make it more likely the firm will be able to secure the limited backup capacity before the competition. Therefore, we suggest firms (1) install monitoring processes on key components in order to spot disruptions quickly and to better distinguish a true disruption from day-to-day variations, and (2) build a company-wide culture of awareness and communication so that relevant parties will be made aware of the suspicious situations quickly and can therefore work together to craft a response.

In addition to promoting detection and response speed from within, firms can cultivate speed from without by strengthening relationships with suppliers. Enhancing the firm's reputation with suppliers can make it more likely that these suppliers will share information with the firm and work with it to resolve a problem. For instance, the 1997 fire at Aisin (which made 99% of Toyota's P-valves) provided an example of a successful business relationship between a manufacturer and a supplier, which led to a rapid collaborative response to a disruptive situation.

3.5.2 Customer Loyalty Polices

Under these two scenarios (ABC and AB models), the full regression model indicates that Firm A faces lower risk when it has higher customer loyalty (relative to either Firm B or Firm C). Indeed, high customer loyalty pays dividends not only during disruptive events, but also during normal operations. It has been estimated that the dollar value required to gain a new customer is six to ten times higher than the cost of maintaining a repeat customer (LeBoeuf 2000). So a firm's marketing dollars will go further if firm uses it to build, nurture, and develop its customer relationships. Our results suggest that such efforts will also make the firm more resilient to supply chain disruptions.

There are many ways a firm can cultivate greater customer loyalty, including:

- *Exceed customer expectations.* Delivering more than customers expect is one of the most powerful ways to gain customer loyalty. For instance, Lexus had a recall early in its history in the United States. They called customers, arranged to pick up their cars, and left identical cars as loaners for the day in their places. When they returned the car that evening, it was fixed, cleaned, and

had a full tank of gas and a coupon for a free oil change. Lexus wisely used this disruption as an opportunity to impress its customers and hence increase customer loyalty.

- *Pay great attention to unhappy customers.* It is impossible for a firm to keep 100% of its customers happy, so strategies are needed to identify unhappy customers who may be inclined to post negative comments (or even videos) on the web. The firm should think of giving unhappy customers a place to direct their comments where the firm has control over and hence has a chance to respond as soon and as good as possible before they go public. As an example, in 1999, Intel was notified by a mathematics professor about a floating-point-operation defect of its Pentium 486DX. When Intel failed to respond, the professor posted this problem on a mathematics web site, which quickly grabbed public attention. At that point, Intel still belittled its complaining customers and claimed that Intel would only replace the 486DX chip for users who could prove a need for using floating-point operations. Customer unhappiness was aggravated by the announcement and led to a demand for a complete recall. Intel finally gave in and recalled the 486DX, which cost more than four-hundred million dollars, and even more importantly, damaged its reputation and customer loyalty.

- *Discounts.* During normal production, discounts for repeat customers is always an effective way to make customers want to stick around. So, in a similar fashion, firms can avoid permanently losing customers who cannot be satisfied during a supply outage by providing appealing discounts to customers who are willing to return after the outage. Also, if the firm has similar product(s) available as substitute(s) to the disrupted one(s), it can discount the substitute(s) during the outage to encourage customers of the disrupted product(s) to buy them instead of competitive products.

3.5.3 Backup Capacity Polices

Our results show clearly that a firm faces the greatest risk when the shared backup supply for a component subject to disruption is limited relative to sales. So, one way to reduce risk is to take steps to increase this potential supply. There are several ways this could be done, including:

- Increase the number of firms with capability to supply the needed component. This might be accomplished by multi-sourcing a component or by otherwise allocating business to suppliers to help them develop specific capabilities.

- Contractually obligate suppliers to be able to deliver more than the contracted amount within a specified amount of time. This would ensure that a secondary supplier could be turned into a primary supplier if needed.

- Make products more flexible with regard to their constituent components. As we noted, Nokia did this prior to the Philips fire and was able to purchase chips from suppliers who were not previously suitable.

- Cultivate process and organizational flexibility. For example, the above cited case in which a host of suppliers were tapped by Toyota to produce p-valves to replace the supply disrupted by the fire at an Aisin plant is an illustration of the power of having an organizationally flexible supply chain. Working together in task force mode, the many firms in Toyota's keiretsu quickly expanded p-valve capacity that did not exist prior to the incident.

4 Conclusions and Future Work

In this paper we have developed a modeling framework that captures both the tactical consequences (short term sales loss) and the strategic consequences (long term market share shifts) of supply chain disruptions in competitive environments. The resulting models help strike a balance between the costs of preparedness and the risks of disruptions in such environments.

To gain structural insights into policies for protecting supply chains against disruptions, we separated the overall process into two periods: (1) a "pre-disruption period" in which the Advanced Preparedness (AP) Competition occurs (i.e., Firm A and Firm B invest in preparedness and thereby determine their probabilities of being first to detect a disruption), and (2) a "post-disruption period", during which the Backup Capacity (BC) Competition occurs.

For the BC Competition, we showed that, the optimal backup capacity purchasing policy is a five-region policy, which can be computed by comparing a small number of possible values. For the AP Competition, we showed a unique Nash Equilibrium exists for each firm, which describes the preparedness investment for the market.

From a managerial perspective, our numerical results suggest that a firm will face large tactical and strategic risks when (1) the likelihood of a disruption is large, (2) the firm's market share is highly valuable, (3) its customers exhibit low loyalty (especially relative to a third party), (4) its market sales exceed the available backup capacity, and (5) the backup capacity exceeds sales of the competitor firm and the competitor has a high "poaching potential". To reduce the firm's risk due to lack of preparedness, it should (a) improve its speed of detecting and reacting to a disruption, (b) cultivate greater customer loyalty, and (c) increase the amount of backup capacity (this includes both developing additional dedicated backup capacity and making it more likely that the firm will get whatever backup capacity is shared).

Since collecting data is time-consuming and expensive, it is not practical for firms to carefully estimate

all of the parameters used in this paper. We have shown that the single most important factor can make reasonable predications of which components in the firm's catalog pose the highest risks. The data collection overhead of this single factor is similar to that for a single factor method currently being used in industry, although it can predict risks much more accurately. From a qualitative standpoint, our regression studies illustrate that large firms should focus on defensive strategies, in which they invest in preparedness to protect sales and market share from disruptions, while small firms should pursue offensive strategies, in which they invest in preparedness in hopes of taking advantage of a disruption to gain market share.

While our framework offers a start toward a science of supply chain risk management, there is still considerable need for further research. We describe two of these below.

First, in this paper we have studied the scenario in which a disruption affects a single component of a single product of one firm and its competitor is affected in a symmetric fashion. However, a firm may have many final products that rely on the disrupted key component. Alternatively, a disruption (e.g., an earthquake) may affect an entire region and hence several components at once. to help guide real-world decision making, the framework needs to be extended to cover these scenarios.

Second, we have considered only business-to-business (OEM) relationships between firms and their customers. The business-to-consumer (retail) environment remains open. In a retail situation, firms have no control over which customers – existing or new – will have their orders filled first. Hence, even if a firm secures an amount of backup capacity of the disrupted component that is smaller than its sales, it may still make some sales to unsatisfied customers of the competitor. This may mean that environments with small amount of available backup capacity present greater risk than they do under the OEM assumption. To characterize risk in this setting, we first need to better understand the customer brand loyalty, so we can model the dynamics of how customers choose products and switch between brands.

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ONLINE APPENDIX A

Design of Numerical Experiments

We created a large set of scenarios ($3^{22} \times 2 \times 4$ cases in total) using realistic values for the model parameters and values chosen to cover the majority of scenarios we could observe in practice. We randomly picked one million scenarios from this large set and fit a regression model to this reduced sample.

We consider the 8 *parameters* of the model, along with five *profit factors,* and 15 *normalized factors* as follows:

Model Parameters:

We consider eight parameters in our model including p_0, μ, CV $(= \sigma/\mu)$, T, γ_{AB}, γ_{AC}, γ_{BA}, and γ_{BC}. The values of these parameters are summarized in Table 6). Note that, given the distribution of the outage time, the customer loyalty coefficients of 18, 42, 130, 260 correspond to cases in which (approximately) 80%, 50%, 20%, 10% of customers who use the competitor's (or the third party's) products during the outage will permanently switch firms, and for modeling purposes, we assume the remaining outage times are normally distributed.

Profit Factors:

We include the following five variables corresponding to the profit loss or gain for each firm, i.e., SL_{Total}, LLB, LLC, LG, and SG.

- Firm A's worst-case short term loss:

$$SL_{Total} = SL_T + SL_\mu = T \times r_A \times d_A^0 + \mu \times r_A \times d_A$$

- Firm A's worst-case long term loss (to Firm B or the third party C):

$$LLB = m_A \times \frac{\mu}{\gamma_{AB}} \times d_A$$

$$LLC = LLC_T + LLC_\mu = m_A \times \frac{T}{\gamma_{AC}} \times d_A^0 + m_A \times \frac{\mu}{\gamma_{AC}} \times d_A$$

  Note that, Firm A's worst-case long term loss to Firm B is actually expressed as $m_A \times \xi_{AB} \times d_A$, where

$$\xi_{AB} = \int_0^\infty (1 - e^{-\frac{t}{\gamma_{AB}}}) dF_o(t),$$

  and F_o is a distribution with mean μ and standard deviation σ. However when $\sigma \to 0$, we have,

$$\xi_{AB} = 1 - e^{-\frac{\mu}{\gamma_{AB}}} \simeq \frac{\mu}{\gamma_{AB}}.$$

  Therefore, to simplify, we define,

$$LLB = m_A \times \frac{\mu}{\gamma_{AB}} \times d_A.$$

  We use the same approximation to define LLC and LG.

- Firm A's best-case short term gain:

$$SG = \mu \times r_A \times d_B$$

- Firm A's best-case long term gain:

$$LG = m_B \times \frac{\mu}{\gamma_{BA}} \times d_B$$

<u>Normalized Factors:</u>

We consider the following 15 normalized factors (whose ranges are determined by the values of associated parameters and summarized in Table 6):

- $\frac{r_A}{r_B}$, which is the relative profitability of the two firms;
- $\frac{m_i}{365 r_i}$, which is the present-worth factor of Firm A based on a 15% interest factor for continuous compounding interest (Thuesen et al. 1977), $i = A, B$. So, for example, if $\frac{m_A}{365 r_A} = 0.1$ and $r_A = \$120$, then $m_A = \$4380$;
- $\frac{365 h_i}{r_i}$, which is the relative annual cost for Firm i to hold one unit of unused backup capacity, $i = A, B$;
- $\frac{c}{r_A}$, which is the relative premium Firm A pays per unit of backup capacity;
- $\frac{d_i}{S}$, which is Firm i's sales relative to the total available backup capacity, $i = A, B$;
- $\frac{K_i - d_i}{d_j}$, which is defined as the "poaching potential" of Firm i (notice that, the numerator is Firm i's remaining production capacity after satisfying its own customers), $i \neq j,\ i,\ j = A,\ B$;
- $\frac{S}{d_A + d_B}$, which is the fraction of total market sales represented by the backup capacity;
- $\frac{p_{i0} - p_0}{p_0}$, which is the error in Firm i's estimate of the likelihood of a disruption, $i = A, B$;
- $\frac{p_{j0,i} - p_{j0}}{p_{j0}}$, which is the error in Firm i's belief of Firm j's estimate of the likelihood of a disruption, $i \neq j,\ i,\ j = A,\ B$.

Unnormalized Parameters			Normalized Factors	
Parameters		Values	Factors	Ranges
μ	(day)	15, 30, 60		
$CV(\frac{\sigma}{\mu})$		$\frac{1}{6}$, $\frac{1}{3}$		
T	(day)	5, 10, 15		
γ_{AB}	(day)	18, 42, 130		
γ_{AC}	(day)	$\gamma_{AB}, \gamma_{AB}+100, 260$		
γ_{BA}	(day)	18, 42, 130		
γ_{BC}	(day)	$\gamma_{BA}, \gamma_{BA}+100, 260$		
r_A	(\$/unit)	60, 90, 120		
r_B	(\$/unit)	60, 90, 120	$\frac{r_A}{r_B}$	$0.5 \sim 2$
m_A	(\$/unit)	4380, 16425, 54750	$\frac{m_A}{365r_A}$	$0.1 \sim 2.5$
m_B	(\$/unit)	4380, 16425, 54750	$\frac{m_B}{365r_B}$	$0.1 \sim 2.5$
$(365h_A)$	(\$/unit/year)	12, 22.5, 30	$\frac{365h_A}{r_A}$	$0.1 \sim 0.5$
$(365h_B)$	(\$/unit/year)	12, 22.5, 30	$\frac{365h_B}{r_B}$	$0.1 \sim 0.5$
c	(\$/unit)	12, 90, 150	$\frac{c}{r_A}$	$0.1 \sim 2.5$
d_A	(unit)	1000, 3000, 5000	$\frac{d_A}{S}$	$0.1 \sim 10$
d_B	(unit)	1000, 3000, 5000	$\frac{d_B}{S}$	$0.1 \sim 10$
K_A	(unit)	$d_A,\ d_A+0.5d_B,\ d_A+d_B$	$\frac{K_A-d_A}{d_B}$	$0 \sim 1$
K_B	(unit)	$d_B,\ d_B+0.5d_A,\ d_A+d_B$	$\frac{K_B-d_B}{d_A}$	$0 \sim 1$
S	(unit)	500, 5000, 10000	$\frac{S}{d_A+d_B}$	$0.05 \sim 5$
p_0		0.01, 0.05, 0.2, 0.4		
p_{A0}	when $p_0 = 0.01$	0.005, 0.01, 0.05	$\frac{p_{A0}-p_0}{p_0}$	$-0.5 \sim 4$
	when $p_0 = 0.05$	0.025, 0.05, 0.1	$\frac{p_{A0}-p_0}{p_0}$	$-0.5 \sim 1$
	when $p_0 = 0.2$	0.1, 0.2, 0.3	$\frac{p_{A0}-p_0}{p_0}$	$-0.5 \sim 0.5$
	when $p_0 = 0.4$	0.2, 0.4, 0.6	$\frac{p_{A0}-p_0}{p_0}$	$-0.5 \sim 0.5$
p_{B0}	when $p_0 = 0.01$	0.005, 0.01, 0.05	$\frac{p_{B0}-p_0}{p_0}$	$-0.5 \sim 4$
	when $p_0 = 0.05$	0.025, 0.05, 0.1	$\frac{p_{B0}-p_0}{p_0}$	$-0.5 \sim 1$
	when $p_0 = 0.2$	0.1, 0.2, 0.3	$\frac{p_{B0}-p_0}{p_0}$	$-0.5 \sim 0.5$
	when $p_0 = 0.4$	0.2, 0.4, 0.6	$\frac{p_{B0}-p_0}{p_0}$	$-0.5 \sim 0.5$
$p_{B0,A}$	when $p_{B0} < 0.1$	$0.5p_{B0},\ p_{B0},\ 2p_{B0}$	$\frac{p_{B0,A}-p_{B0}}{p_{B0}}$	$-0.5 \sim 1$
	when $p_{B0} \geq 0.1$	$0.5p_{B0},\ p_{B0},\ 1.5p_{B0}$	$\frac{p_{B0,A}-p_{B0}}{p_{B0}}$	$-0.5 \sim 0.5$
$p_{A0,B}$	when $p_{A0} < 0.1$	$0.5p_{A0},\ p_{A0},\ 2p_{A0}$	$\frac{p_{A0,B}-p_{A0}}{p_{A0}}$	$-0.5 \sim 1$
	when $p_{A0} \geq 0.1$	$0.5p_{A0},\ p_{A0},\ 1.5p_{A0}$	$\frac{p_{A0,B}-p_{A0}}{p_{A0}}$	$-0.5 \sim 0.5$

Table 6: Values of input parameters and ranges of factors for numerical studies.

ONLINE APPENDIX B

Proofs of Analytical Results

PROPOSITION 1:

(Large Capacity Leader:) *If the Leader's production capacity is greater than the total market share of both firms (i.e., $K_A \geq d_A + d_B$), and*

(1) *if the backup capacity is less then the total market share of both firms (i.e., $S \leq d_A + d_B$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$\begin{aligned}(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ & \Big\{(0,0),\ (0,d_B),\ \big(0,\min\{K_B,S\}\big),\ (S-d_B,0),\ (S-d_B,d_B),\\ & (d_A,0),\ (d_A,S-d_A),\ (S,0)\Big\};\end{aligned}$$

(2) *if the backup capacity is larger than the total market share of both firms (i.e., $S > d_A + d_B$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$\begin{aligned}(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ & \Big\{(0,0),\ (0,d_B),\ \big(0,\min\{K_B,\ d_A+d_B\}\big),\ (d_A,0),\ (d_A,d_B),\\ & (d_A+d_B,0),\ (S,0)\Big\}.\end{aligned}$$

(Small Capacity Leader:) *If the Leader's production capacity cannot cover the total market shares of both firms (i.e., $K_A < d_A + d_B$), and*

(1) *if the backup capacity is less than the Leader's market share (i.e., $S \leq K_A$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$\begin{aligned}(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ & \Big\{(0,0),\ (0,d_B),\ (0,\min\{K_B,\ S\}),\ (S-d_B,0),\ (S-d_B,d_B),\\ & (d_A,0),\ (d_A,S-d_A),\ (S,\ 0)\Big\};\end{aligned}$$

(2) *if the backup capacity is larger than the Leader's market share, but it is smaller than the total market share (i.e., $K_A < S \leq d_A + d_B$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$\begin{aligned}(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ & \Big\{(0,0),\ (0,d_B),\ (0,\min\{K_B,\ S\}),\ (S-d_B,0),\ (S-d_B,d_B),\\ & (d_A,0),\ (d_A,S-d_A),\ (K_A,0),\ (K_A,S-K_A)\Big\};\end{aligned}$$

(3) *if the backup capacity can cover the total market share of both firms (i.e., $S > d_A + d_B$), then the only possible choices for the optimal backup capacity purchases for both firms are:*

$$\begin{aligned}(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ & \Big\{(0,0),\ (0,d_B),\ (0,\min\{K_B,\ d_A+d_B\}),\ (d_A,0),\ (d_A,d_B),\ (K_A,0),\\ & \big(S-(d_A+d_B)+K_A,0\big),\ \big(S-(d_A+d_B)+K_A,(d_A+d_B)-K_A\big)\Big\}.\end{aligned}$$

PROOF OF PROPOSITION 1:

We only present the proof for the case of *Large Capacity Leader*, i.e., $K_A \geq d_A + d_B$. The proof for Small Capacity Leader is similar, and is therefore omitted. To simplify notation, we let

$$\phi_{ik} = m_i \xi_{ik} \qquad : \qquad i \neq k, \quad i = A, B, \quad k = A, B$$

Since $K_A \geq d_A + d_B$, then we have $K_A - d_A \geq d_B \geq [d_B - Y^*_{B,F}(Y_{A,L})]^+$, where $Y^*_{B,F}(Y_{A,L})$ is Firm B's optimal purchase amount of backup capacity after Firm A purchases $Y_{A,L}$.

$$\begin{aligned} \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, \min\{[Y_{A,L} - d_A]^+, K_A - d_A\}\Big\} &= \\ \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, [Y_{A,L} - d_A]^+, K_A - d_A\Big\} &= \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, [Y_{A,L} - d_A]^+\Big\}. \end{aligned}$$

Therefore, equation (1) becomes:

$$\begin{aligned} \Pi_{A,L}\big(Y_{A,L}, Y^*_{B,F}(Y_{A,L})\big) &= r_A \min\Big\{Y_{A,L}, d_A\Big\}\mu \\ &\quad + \psi_A \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, [Y_{A,L} - d_A]^+\Big\} \\ &\quad - \phi_{AB} \min\Big\{[Y^*_{B,F}(Y_{A,L}) - d_B]^+, [d_A - Y_{A,L}]^+\Big\} \\ &\quad - \phi_{AC}\Big[[d_A - Y_{A,L}]^+ - [Y^*_{B,F}(Y_{A,L}) - d_B]^+\Big]^+ \\ &\quad - c_A Y_{A,L}\mu \\ &\quad - h_A\Big[Y_{A,L} - \min\{Y_{A,L}, d_A\} - \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, [Y_{A,L} - d_A]^+\Big\}\Big]^+\mu. \end{aligned} \tag{4}$$

It is easy to show that $\Pi_{A,L}$ is a piecewise linear function in $Y_{A,L}$. It is linear because equation (4) is linear in $Y_{A,L}$, and it is piecewise linear due to the existence of *Min* and *Max* functions.

To proof the results for Parts (1) and (2) of the case for "Large Capacity Leader" (i.e., $K_A \geq d_A + d_B$), we consider the following two corresponding cases:

- **Case 1:** When $S \leq d_A + d_B$;
- **Case 2:** When $S > d_A + d_B$.

CASE 1: If $S \leq d_A + d_B$, then equation (4) becomes

$$\begin{aligned} \Pi_{A,L}\big(Y_{A,L}, Y^*_{B,F}(Y_{A,L})\big) &= r_A \min\Big\{Y_{A,L}, d_A\Big\}\mu + \psi_A \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, [Y_{A,L} - d_A]^+\Big\} \\ &\quad - \phi_{AB} \min\Big\{[Y^*_{B,F}(Y_{A,L}) - d_B]^+, [d_A - Y_{A,L}]^+\Big\} \\ &\quad - \phi_{AC}\Big[[d_A - Y_{A,L}]^+ - [Y^*_{B,F}(Y_{A,L}) - d_B]^+\Big]^+ - c_A Y_{A,L}\mu. \end{aligned} \tag{5}$$

For the piecewise linear functions, it is proven that the optimal solution is one of the break points (Hager and Park 2004). Hence, the optimal value of $Y_{A,L}$ is one of four break points of equation (5), namely 0, $[S - d_B]^+$, $\min\{d_A, S\}$, and S. Below we consider each of these three cases:

Case 1-i, $Y_{A,L} = 0$: For this case, the objective function of Firm A is

$$\begin{aligned} &\Pi_{A,L}\big(0, Y^*_{B,F}(Y_{A,L})\big) \\ &= -\phi_{AB} \min\Big\{[Y^*_{B,F}(Y_{A,L} = 0) - d_B]^+, d_A\Big\} - \phi_{AC}\Big[d_A - [Y^*_{B,F}(Y_{A,L} = 0) - d_B]^+\Big]^+ \\ &= -\phi_{AB}[Y^*_{B,F}(Y_{A,L} = 0) - d_B]^+ - \phi_{AC}\Big(d_A - [Y^*_{B,F}(Y_{A,L} = 0) - d_B]^+\Big). \end{aligned} \tag{6}$$

Note that, since $S \le d_A + d_B$, we have $[Y^*_{B,F}(Y_{A,L}=0) - d_B]^+ \le S - d_B \le d_A$.

On the other hand, equation (2) is reduced to

$$\begin{aligned}\Pi_{B,F}(0, Y_{B,F}) &= r_B \min\Big\{Y_{B,F}, d_B\Big\}\mu + \psi_B \min\Big\{d_A, [Y_{B,F} - d_B]^+\Big\} - \phi_{BC}[d_B - Y_{B,F}]^+ - c_B Y_{B,F}\mu \\ &= r_B \min\{Y_{B,F}, d_B\}\mu + \psi_B [Y_{B,F} - d_B]^+ - \phi_{BC}[d_B - Y_{B,F}]^+ - c_B Y_{B,F}\mu, \end{aligned} \tag{7}$$

which is piecewise linear and has three break points: 0, $\min\{d_B,\ S\}$, and $\min\{K_B, S\}$. So, the optimal value of $Y^*_{B,F} \in \Big\{0, d_B, \min\{K_B,\ S\}\Big\}$. Consequently, the possible combination of the optimal capacity purchase for firms A and B in this case are

$$(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ \Big\{(0,0), (0, d_B), (0, \min\{K_B, S\})\Big\}.$$

Case 1-ii, $Y_{A,L} = [S - d_B]^+$: Notice that, if $[S - d_B]^+ = 0$, i.e., $S \le d_B$, then it becomes similar to Case 1-i, so here we are only interested in the case of $[S - d_B]^+ = S - d_B$.

Since $S \le d_A + d_B$, $Y_{A,L} = S - d_B \le d_A$ and $Y^*_{B,F}(Y_{A,L}) \le d_B$ must hold. Then the objective function of Firm A becomes

$$\Pi_{A,L}\big(S - d_B, Y^*_{B,F}(Y_{A,L})\big) \ = \ (r_A - c_A)(S - d_B)\mu - \phi_{AC}\big(d_A - (S - d_B)\big). \tag{8}$$

It is easy to see that the reason why $Y_{A,L} = S - d_B$ is that Firm A aims at protecting itself from loosing market share to Firm B (although sales profit is negative), i.e., if $Y_{A,L} = 0$, then $Y^*_{B,F} = S$ due to the linearity property of the profit function. Based on Firm A's protection strategy, the objective function of Firm B becomes

$$\Pi_{B,F}(S - d_B, Y_{B,F}) \ = \ (r_B - c_B)Y_{B,F}\mu - \phi_{BC}(d_B - Y_{B,F}), \tag{9}$$

which is a linear function with two end points: 0 and d_B. Thus, the optimal capacity purchase $Y^*_{B,F} \in \{0, d_B\}$. Consequently, in this case the only possible combination of the optimal capacity purchase for firms A and B are

$$(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ \Big\{(S - d_B, 0), (S - d_B, d_B)\Big\}.$$

Case 1-iii, $Y_{A,L} = \min\{d_A, S\}$: For this case, the objective function of Firm A is

$$\begin{aligned}\Pi_{A,L}\big(\min\{d_A, S\}, Y^*_{B,F}(Y_{A,L})\big) &= (r_A - c_A)\min\{d_A, S\}\mu - \phi_{AC}\big(d_A - \min\{d_A, S\}\big) \\ &= (r_A - c_A)\min\{d_A, S\}\mu - \phi_{AC}[d_A - S]^+. \end{aligned} \tag{10}$$

On the other hand, since Firm A purchases capacity to only satisfy its own customers, i.e.,$Y_{A,L} = \min\{d_A, S\}$, the objective function of Firm B, becomes

$$\Pi_{B,F}(\min\{d_A, S\}, Y_{B,F}) \ = \ (r_B - c_B)Y_{B,F}\mu - \phi_{BC}(d_B - Y_{B,F}), \tag{11}$$

which is a linear function with two end points: 0 and $[S - \min\{d_A, S\}]^+ = [S - d_A]^+$. Thus, $Y^*_{B,F} \in \{0,\ [S - \min\{d_A, S\}]^+ = [S - d_A]^+\}$. Consequently, in this case the only possible combination of the optimal capacity purchase for firms A and B are

$$(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ \Big\{(\min\{d_A, S\}, 0), (\min\{d_A, S\}, [S - d_A]^+)\Big\}.$$

Case 1-iv, $Y_{A,L} = S$: For this case, $Y^*_{B,F}(Y_{A,L}) = 0$ is the only possibility. Since $S \le d_A + d_B$, then $[S - d_A]^+ \le d_B$ must hold. Hence, the objective function of Firm A is

$$\Pi_{A,L}(S,0) \;=\; r_A \min\{S, d_A\}\mu + \psi_A[S - d_A]^+ - \phi_{AC}[d_A - S]^+ - c_A S\mu, \tag{12}$$

and the objective function of Firm B, becomes

$$\Pi_{B,F}(S,0) \;=\; -\phi_{BA} \min\left\{[S - d_A]^+, d_B\right\} - \phi_{BC}\big(d_B - [S - d_A]^+\big).$$

The only possible combination of the optimal capacity purchase for this case is $(Y^*_{A,L}, Y^*_{B,F}) \in \{(S,0)\}$.

If we combine the results for Case 1-i to Case 1-iv, we get the following possible combinations of the optimal capacity purchase for firms A and B, when $S \le d_A + d_B$

$$\begin{aligned}(Y^*_{A,L},\ Y^*_{B,F}) \;\in\; \Big\{&(0,0),\ (0,d_B), \big(0, \min\{K_B, S\}\big), (S - d_B, 0), (S - d_B, d_B),\\ &\big(\min\{d_A, S\}, 0\big), \big(\min\{d_A, S\}, [S - d_A]^+\big), (S,0)\Big\}.\end{aligned}$$

Note that, (1) the sixth possible combination $\big(\min\{d_A, S\}, 0\big)$ becomes $(d_A, 0)$, when $d_A \le S$, and reduces to the last one, $(S,0)$, when $d_A > S$; (2) the seventh possible combination $\big(\min\{d_A, S\}, [S - d_A]^+\big)$ becomes $(d_A, S - d_A)$, when $d_A \le S$, and reduces to the last one, $(S,0)$, when $d_A > S$. Thus, the total possible combinations are:

$$\begin{aligned}(Y^*_{A,L},\ Y^*_{B,F}) \;\in\; \Big\{&(0,0),\ (0,d_B), \big(0, \min\{K_B, S\}\big), (S - d_B, 0),\ \big(S - d_B, d_B\big),\\ &(d_A, 0), (d_A, S - d_A),\ (S,0)\Big\}.\end{aligned}$$

This completes the proof for Part (1) of Proposition 1 for the case with "Large Capacity Leader".

CASE 2: Similar to Case 1, it is easy to show that if $S > d_A + d_B$, then equation (1) is a piecewise linear function with four break points: 0, d_A, $d_A + d_B$ and S. We now discuss each of the four cases.

Case 2-i, $Y_{A,L} = 0$: For this case, the objective function of Firm A becomes

$$\begin{aligned}\Pi_{A,L}\big(0, Y^*_{B,F}(Y_{A,L})\big) \;=\; &-\phi_{AB} \min\left\{[Y^*_{B,F}(Y_{A,L} = 0) - d_B]^+, d_A\right\}\\ &- \phi_{AC}\big[d_A - [Y^*_{B,F}(Y_{A,L} = 0) - d_B]^+\big]^+.\end{aligned} \tag{13}$$

On the other hand, the objective function of Firm B is reduced to

$$\begin{aligned}\Pi_{B,F}(0, Y_{B,F}) \;=\; &r_B \min\Big\{Y_{B,F}, d_B\Big\}\mu + \psi_B \min\left\{d_A, [Y_{B,F} - d_B]^+\right\}\\ &- \phi_{BC}[d_B - Y_{B,F}]^+ - c_B Y_{B,F}\mu,\end{aligned} \tag{14}$$

which is a piecewise linear function with three break points: 0, d_B, and $\min\{K_B,\ d_A + d_B\}$, which constitutes the potential values for $Y^*_{B,F}$. Consequently, the possible combinations for the optimal capacity purchase for Firms A and B are

$$(Y^*_{A,L},\ Y^*_{B,F}) \;\in\; \Big\{(0,0),\ (0,d_B), \big(0, \min\{K_B,\ d_A + d_B\}\big)\Big\}.$$

Case 2-ii, $Y_{A,L} = d_A$: For this case, the objective function of Firm A becomes

$$\Pi_{A,L}\big(d_A, Y^*_{B,F}(Y_{A,L})\big) \;=\; (r_A - c_A)d_A\mu. \tag{15}$$

Due to Firm A's protection strategy, the objective function of Firm B becomes

$$\begin{aligned}\Pi_{B,F}(d_A, Y_{B,F}) &= r_B \min\Big\{Y_{B,F}, d_B\Big\}\mu - \phi_{BC}[d_B - Y_{B,F}]^+ - c_B Y_{B,F}\mu \\ &= (r_B - c_B)Y_{B,F}\mu - \phi_{BC}[d_B - Y_{B,F}]^+, \end{aligned} \tag{16}$$

which is a piecewise linear function and has two break points: 0 and d_B. Consequently, the possible combinations of the optimal capacity purchase for Firms A and B are

$$(Y^*_{A,L},\ Y^*_{B,F}) \ \in\ \big\{(d_A, 0), (d_A, d_B)\big\}.$$

Case 2-iii, $Y_{A,L} = d_A + d_B$: As shown in equation (1), the coefficient of the h is

$$\Big[Y_{A,L} - \min\{Y_{A,L}, d_A\} - \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, \min\big\{[Y_{A,L} - d_A]^+, K_A - d_A\big\}\Big\}\Big]^+\mu, \tag{17}$$

Since in this case, $Y_{A,L} = d_A + d_B$, then we have

$$Y_{A,L} - \min\{Y_{A,L}, d_A\} = (d_A + d_B) - d_A = d_B.$$

Hence, (17) becomes:

$$\Big[d_B - \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, \min\big\{d_B, K_A - d_A\big\}\Big\}\Big]^+\mu. \tag{18}$$

Note that, $K_A - d_A \geq d_B$ holds, and hence (18) reduces to

$$\Big[d_B - \min\Big\{[d_B - Y^*_{B,F}(Y_{A,L})]^+, d_B\Big\}\Big]^+\mu.$$

- If $d_B \geq Y^*_{B,F}(Y_{A,L})$, then the coefficient of the h is $Y^*_{B,F}(Y_{A,L})\mu$.
- If $d_B < Y^*_{B,F}(Y_{A,L})$, then the coefficient of the h is $d_B\mu$.

Therefore,

$$\begin{aligned}\Pi_{A,L}\big(d_A + d_B, Y^*_{B,F}(Y_{A,L})\big) &= r_A d_A\mu + \psi_A[d_B - Y^*_{B,F}(Y_{A,L})]^+ \\ &\quad - c_A(d_A + d_B)\mu - h_A \min\big\{d_B, Y^*_{B,F}(Y_{A,L} = d_A + d_B)\big\}\mu. \end{aligned} \tag{19}$$

On the other hand, the objective function of Firm B is reduced to

$$\Pi_{B,F}(d_A + d_B, Y_{B,F}) \ = \ (r_B - c_B)Y_{B,F}\mu - \phi_{BA}[d_B - Y_{B,F}]^+, \tag{20}$$

which is a piecewise linear function and has two break points: 0 and $\min\{S - d_A - d_B,\ d_B\}$. Thus, $Y^*_{B,F} = \big\{0,\ \min\{S - d_A - d_B,\ d_B\}\big\}$.

However, the combination of $\big(Y^*_{A,L} = d_A + d_B,\ Y^*_{B,F} = \min\{S - d_A - d_B,\ d_B\}\big)$ implies that only part of $Y^*_{A,L}$, which is the difference between d_B and $Y^*_{B,F}$, will be used by Firm A to gain both sales profit and market share from Firm B. But, due to the linearity property, it cannot be an optimal solution, because, in this case, if one unit of firm B's customer is profitable, then Firm A will definitely want to gain all possible units. So, if Firm A knows that B will buy all or some of the remaining backup capacity, i.e., $\min\{S - d_A - d_B,\ d_B\}$, to protect itself, the decision of $d_A + d_B$ will never be optimal for Firm A. Therefore, $Y^*_{A,L} = d_A + d_B$ can be optimal only if $Y^*_{B,F} = 0$, i.e., the only possible combinations of the optimal capacity purchase for firms A and B in this case is $(Y^*_{A,L}, Y^*_{B,F}) \in \{(d_A + d_B, 0)\}$.

Case 2-iv, $Y_{A,L} = S$: In this case, $Y^*_{B,F}(Y_{A,L} = S) = 0$ is the only possibility. Thus, $(Y^*_{A,L}, Y^*_{B,F}) \in \{(S, 0)\}$. The objective function of Firm A is

$$\Pi_{A,L}(S, 0) \ = \ r_A d_A\mu + \psi_A d_B - c_A S\mu - h_A(S - d_A - d_B)\mu. \tag{21}$$

If we combine the results for Case 2-i to Case 2-iv, we get the following possible combinations of the optimal capacity purchase for firms A and B, when $S > d_A + d_B$,

$$(Y^*_{A,L},\ Y^*_{B,F}) \ \in \ \Big\{(0,0),\ (0,d_B), \big(0, \min\{K_B,\ d_A + d_B\}\big),\ (d_A,0), (d_A,d_B),\ (d_A + d_B, 0), (S,0)\Big\}.$$

This completes the proof for Part (2) of Proposition 1 for the case of "Large Capacity Leader". □

PROPOSITION 2: *The optimal policy of the Backup Capacity Competition has a five-region structure.*

PROOF OF PROPOSITION 2:

We present the proof for the case of *Large Capacity Leader*, i.e., $K_A \geq d_A + d_B$. The proof for Small Capacity Leader is similar, and is therefore omitted.

PROOF for the Follower, Firm B:

We first determine the regions for the optimal capacity purchase for the Follower. We consider the following two corresponding cases (same as the configuration of the proof for Proposition 1):

- **Case 1:** When $S \leq d_A + d_B$. For this case, we introduce three thresholds on Firm B's unit profit margin, namely, (i) c_B^I, separating the region in which Firm B forfeits and the region in which Firm B is aggressive, (ii) c_B^{II}, separating the region in which Firm B forfeits and the region in which Firm B protects, and (iii) c_B^{III}, separating the region in which Firm B protects and the region in which Firm B is aggressive.

- **Case 2:** When $S > d_A + d_B$. For this case, we introduce four thresholds on Firm B's unit profit margin, namely, (i) c_B^i, separating the region in which Firm B forfeits and the region in which Firm B is aggressive, (ii) c_B^{II}, separating the region in which Firm B forfeits and the region in which Firm B protects, (iii) c_B^{III}, separating the region in which Firm B protects and the region in which Firm B is aggressive, and (iv) c_B^{iv}, which separates the following two responses: Given Firm A purchases the amount of $d_A + d_B$, Firm B can either buy the remaining to protect itself from losing customers to Firm A (this response implies that $d_A + d_B$ cannot be optimal for Firm A), or buy nothing (this response implies that $d_A + d_B$ is optimal for Firm A).

The summary of the thresholds introduced in this proof can be found in Table 7.

We present the proof based on the same cases that we presented in Proposition 1.

Case 1-i, $Y_{A,L} = 0$: Define

$$c_B^I = c_B - \frac{\phi_{AB}\big[\min\{K_B,\ S\} - d_B\big]^+ + \phi_{BC}\min\big\{d_B, \min\{K_B,\ S\}\big\}}{\mu \min\{K_B,\ S\}}.$$

We now show that when $r_B \geq c_B^I$, then $\Pi_{B,F}\big(0, \min\{K_B,\ S\}\big) \geq \Pi_{B,F}(0,0)$. By equation(7) we have

$$\begin{aligned}
&\Pi_{B,F}\big(0, \min\{K_B,\ S\}\big) - \Pi_{B,F}(0,0) \\
&= r_B \min\big\{\min\{K_B,\ S\}, d_B\big\}\mu + \psi_B\big[\min\{K_B,\ S\} - d_B\big]^+ \\
&\qquad - \phi_{BC}\big[d_B - \min\{K_B,\ S\}\big]^+ - c_B \min\{K_B,\ S\}\mu \\
&\quad - \Big(r_B \min\{0, d_B\}\mu + \psi_B[0 - d_B]^+ - \phi_{BC}[d_B - 0]^+ - c_B(0)\mu\Big), \\
&= r_B \min\big\{\min\{K_B,\ S\}, d_B\big\}\mu + (r_B\mu + \phi_{AB})\big[\min\{K_B,\ S\} - d_B\big]^+ \\
&\qquad - \phi_{BC}\big[d_B - \min\{K_B,\ S\}\big]^+ - c_B \min\{K_B,\ S\}\mu \\
&\quad - (-\phi_{BC} d_B), \\
&= r_B \min\{K_B,\ S\}\mu + \phi_{AB}\big[\min\{K_B,\ S\} - d_B\big]^+ \\
&\qquad + \phi_{BC} \min\big\{d_B, \min\{K_B,\ S\}\big\} - c_B \min\{K_B,\ S\}\mu, \\
&= r_B \min\{K_B,\ S\}\mu - \Big(c_B \min\{K_B,\ S\}\mu \\
&\qquad - \phi_{AB}\big[\min\{K_B,\ S\} - d_B\big]^+ - \phi_{BC} \min\big\{d_B, \min\{K_B,\ S\}\big\}\Big).
\end{aligned}$$

It is clear that if $r_B \geq c_B^I$, then the right-hand side is positive, which implies

$$\Pi_{B,F}\big(0, \min\{K_B,\ S\}\big) \geq \Pi_{B,F}(0,0).$$

Therefore, if $r_B \geq c_B^I$, then $Y_{B,F}^*(Y_{A,L} = 0) = \min\{K_B,\ S\}$; otherwise, $Y_{B,F}^*(Y_{A,L} = 0) = 0$.

Define

$$c_B^{II} = c_B - \frac{\phi_{BC}}{\mu}.$$

We now show that when $r_B \geq c_B^{II}$, then $\Pi_{B,F}(0, d_B) \geq \Pi_{B,F}(0,0)$. By equation (7) we have

$$\begin{aligned}
\Pi_{B,F}(0, d_B) - \Pi_{B,F}(0,0) &= r_B d_B \mu - c_B d_B \mu \\
&\quad - \Big(r_B \min\{0, d_B\}\mu + \psi_B[0 - d_B]^+ - \phi_{BC}[d_B - 0]^+ - c_B(0)\mu\Big), \\
&= (r_B - c_B) d_B \mu - (-\phi_{BC} d_B), \\
&= r_B d_B \mu - (c_B d_B \mu - \phi_{BC} d_B).
\end{aligned}$$

It is clear that if $r_B \geq c_B^{II}$, then the right-hand side is positive, which implies

$$\Pi_{B,F}(0, d_B) \geq \Pi_{B,F}(0,0).$$

Therefore, if $r_B \geq c_B^{II}$, then $Y_{B,F}^*(Y_{A,L} = 0) = d_B$; otherwise, $Y_{B,F}^*(Y_{A,L} = 0) = 0$.

Define

$$c_B^{III} = c_B - \frac{\phi_{AB}}{\mu}.$$

We now show that when $r_B \geq c_B^{III}$, then $\Pi_{B,F}\big(0, \min\{K_B,\ S\}\big) \geq \Pi_{B,F}(0, d_B)$. By equation (7) we have

$$\begin{aligned}
&\Pi_{B,F}\big(0, \min\{K_B,\ S\}\big) - \Pi_{B,F}(0, d_B) \\
&= r_B \min\big\{\min\{K_B,\ S\}, d_B\big\}\mu + (r_B\mu + \phi_{AB})\big[\min\{K_B,\ S\} - d_B\big]^+ \\
&\qquad - \phi_{BC}\big[d_B - \min\{K_B,\ S\}\big]^+ - c_B \min\{K_B,\ S\}\mu \\
&\quad - \Big((r_B - c_B)d_B\mu\Big), \\
&= (r_B - c_B)\min\{K_B,\ S\}\mu + \phi_{AB}\big(\min\{K_B,\ S\} - d_B\big) \\
&\quad - (r_B - c_B)d_B\mu, \\
&= r_B\big(\min\{K_B,\ S\} - d_B\big)\mu - \Big(c_B\big(\min\{K_B,\ S\} - d_B\big)\mu \\
&\qquad - \phi_{AB}\big(\min\{K_B,\ S\} - d_B\big)\Big).
\end{aligned}$$

It is clear that if $r_B \geq c_B^{III}$, then the right-hand side is positive, which implies

$$\Pi_{B,F}\big(0, \min\{K_B,\ S\}\big) \geq \Pi_{B,F}(0, d_B).$$

Therefore, if $r_B \geq c_B^{III}$, then $Y^*_{B,F}(Y_{A,L} = 0) = \min\{K_B,\ S\}$; otherwise, $Y^*_{B,F}(Y_{A,L} = 0) = d_B$.

Case 1-ii, $Y_{A,L} = S - d_B$: We now show that when $r_B \geq c_B^{II}$, then $\Pi_{B,F}(S - d_B, d_B) \geq \Pi_{B,F}(S - d_B, 0)$. By equation (9) we have

$$\begin{aligned}
\Pi_{B,F}(S - d_B, d_B) - \Pi_{B,F}(S - d_B, 0) &= (r_B - c_B)d_B\mu - \phi_{BC}(d_B - d_B) \\
&\quad -\big((r_B - c_B)(0)\mu - \phi_{BC}(d_B - 0)\big), \\
&= (r_B - c_B)d_B\mu + \phi_{BC}d_B, \\
&= r_B d_B\mu - (c_B d_B\mu - \phi_{BC}d_B).
\end{aligned}$$

It is clear that if $r_B \geq c_B^{II}$, then the right-hand side is positive, which implies

$$\Pi_{B,F}\big(S - d_B, d_B\big) \geq \Pi_{B,F}(S - d_B, 0).$$

Therefore, if $r_B \geq c_B^{II}$, then $Y^*_{B,F}(Y_{A,L} = S - d_B) = d_B$; otherwise, $Y^*_{B,F}(Y_{A,L} = S - d_B) = 0$.

Case 1-iii, $Y_{A,L} = d_A$: By equation (11), it is easy to see that, if $r_B \geq c_B^{II}$, then $Y^*_{B,F}(Y_{A,L} = d_A) = S - d_A$; otherwise, $Y^*_{B,F}(Y_{A,L} = d_A) = 0$.

Case 2-i, $Y_{A,L} = 0$: Define

$$c_B^i = c_B - \frac{\phi_{AB}\big(\min\{K_B,\ d_A + d_B\} - d_B\big) + \phi_{BC}d_B}{\mu\min\{K_B,\ d_A + d_B\}}.$$

By equation (14), we have

$$\begin{aligned}
&\Pi_{B,F}\big(0,\min\{K_B,\ d_A+d_B\}\big)-\Pi_{B,F}(0,0)\\
&=r_B\min\Big\{\min\{K_B,\ d_A+d_B\},d_B\Big\}\mu+\psi_B\big[\min\{K_B,\ d_A+d_B\}-d_B\big]^+\\
&\qquad-\phi_{BC}\big[d_B-\min\{K_B,\ d_A+d_B\}\big]^+-c_B\min\{K_B,\ d_A+d_B\}\mu\\
&\quad-\Big(r_B\min\Big\{0,d_B\Big\}\mu+\psi_B\min\big\{d_A,[0-d_B]^+\big\}-\phi_{BC}[d_B-0]^+-c_B(0)\mu\Big),\\
&=r_B\min\Big\{\min\{K_B,\ d_A+d_B\},d_B\Big\}\mu\\
&\qquad+(r_B\mu+\phi_{AB})\big[\min\{K_B,\ d_A+d_B\}-d_B\big]^+\\
&\qquad-c_B\min\{K_B,\ d_A+d_B\}\mu\\
&\quad-(-\phi_{BC}d_B),\\
&=r_B\min\{K_B,\ d_A+d_B\}\mu-\Big(c_B\min\{K_B,\ d_A+d_B\}\mu\\
&\quad-\phi_{AB}\big(\min\{K_B,\ d_A+d_B\}-d_B\big)-\phi_{BC}d_B\Big).
\end{aligned}$$

It is clear that, if $r_B\geq c_B^i$, then the right-hand-side is positive, which implies that

$$\Pi_{B,F}\big(0,\min\{K_B,\ d_A+d_B\}\big)\geq\Pi_{B,F}(0,0).$$

Therefore, if $r_B\geq c_B^i$ holds, then $Y^*_{B,F}(Y_{A,L}=0)=\min\{K_B,\ d_A+d_B\}$; otherwise, $Y^*_{B,F}(Y_{A,L}=0)=0$.

Define

$$c_B^{ii}=c_B-\frac{\phi_{BC}}{\mu}=c_B^{II}.$$

By equation (14), we have

$$\begin{aligned}
\Pi_{B,F}(0,d_B)-\Pi_{B,F}(0,0)&=(r_B-c_B)d_B\mu-(-\phi_{BC}d_B),\\
&=r_Bd_B\mu-(c_Bd_B\mu-\phi_{BC}d_B).
\end{aligned}$$

It is clear that, if $r_B\geq c_B^{II}$, then the right-hand-side is positive, which implies that

$$\Pi_{B,F}(0,d_B)\geq\Pi_{B,F}(0,0).$$

Therefore, if $r_B\geq c_B^{II}$ holds, then $Y^*_{B,F}(Y_{A,L}=0)=d_B$; otherwise, $Y^*_{B,F}(Y_{A,L}=0)=0$.

Define

$$c_B^{iii}=c_B-\frac{\phi_{AB}}{\mu}=c_B^{III}.$$

By equation (14), we have

$$\begin{aligned}
&\Pi_{B,F}\big(0,\min\{K_B,\ d_A+d_B\}\big)-\Pi_{B,F}(0,d_B)\\
&=r_B\min\Big\{\min\{K_B,\ d_A+d_B\},d_B\Big\}\mu+\psi_B\big[\min\{K_B,\ d_A+d_B\}-d_B\big]^+\\
&\qquad-\phi_{BC}\big[d_B-\min\{K_B,\ d_A+d_B\}\big]^+-c_B\min\{K_B,\ d_A+d_B\}\mu\\
&\quad-\Big((r_B-c_B)d_B\mu\Big),\\
&=r_B\min\Big\{\min\{K_B,\ d_A+d_B\},d_B\Big\}\mu\\
&\qquad+(r_B\mu+\phi_{AB})\big[\min\{K_B,\ d_A+d_B\}-d_B\big]^+\\
&\qquad-c_B\min\{K_B,\ d_A+d_B\}\mu\\
&\quad-(r_B-c_B)d_B\mu,\\
&=r_B\big(\min\{K_B,\ d_A+d_B\}-d_B\big)\mu-\Big(c_B\big(\min\{K_B,\ d_A+d_B\}-d_B\big)\mu\\
&\quad-\phi_{AB}\big(\min\{K_B,\ d_A+d_B\}-d_B\big)\Big).
\end{aligned}$$

It is clear that, if $r_B \geq c_B^{III}$, then the right-hand-side is positive, which implies that

$$\Pi_{B,F}\big(0, \min\{K_B,\ d_A + d_B\}\big) \geq \Pi_{B,F}(0, d_B).$$

Therefore, if $r_B \geq c_B^{III}$ holds, then $Y^*_{B,F}(Y_{A,L} = 0) = \min\{K_B,\ d_A + d_B\}$; otherwise, $Y^*_{B,F}(Y_{A,L} = 0) = d_B$.

Case 2-ii, $Y_{A,L} = d_A$: By equation (16), we have

$$\begin{aligned}\Pi_{B,F}(d_A, d_B) - \Pi_{B,F}(d_A, 0) &= r_B d_B \mu - c_B d_B \mu - (-\phi_{BC} d_B) \\ &= r_B d_B \mu - (c_B d_B \mu - \phi_{BC} d_B).\end{aligned}$$

It is clear that, if $r_B \geq c_B^{II}$, then the right-hand-side is positive, which implies that

$$\Pi_{B,F}(d_A, d_B) \geq \Pi_{B,F}(d_A, 0).$$

Therefore, if $r_B \geq c_B^{II}$ holds, then $Y^*_{B,F}(Y_{A,L} = d_A) = d_B$; otherwise, $Y^*_{B,F}(Y_{A,L} = d_A) = 0$.

Case 2-iii, $Y_{A,L} = d_A + d_B$: Define

$$c_B^{iv} = c_B - \frac{\phi_{BA}}{\mu},$$

and notice that, $c_B^{II} \geq c_B^{iv}$, since we have assumed that $\xi_{BA} \geq \xi_{BC}$. By equation (20)

$$\begin{aligned}&\Pi_{B,F}(d_A + d_B, \min\{S - d_A - d_B,\ d_B\}) - \Pi_{B,F}(d_A + d_B, 0) \\ &= r_B \min\Big\{\min\{S - d_A - d_B,\ d_B\}, d_B\Big\}\mu \\ &\qquad - \phi_{BA}\big[d_B - \min\{S - d_A - d_B,\ d_B\}\big]^+ \\ &\qquad - c_B \min\{S - d_A - d_B,\ d_B\}\mu \\ &\quad - (-\phi_{BA} d_B) \\ &= \big(r_B - (c_B - \frac{\phi_{BA}}{\mu})\big) \min\{S - d_A - d_B,\ d_B\}\mu.\end{aligned}$$

Therefore, if $r_B \geq c_B^{iv}$ holds, then $Y^*_{B,F}(Y_{A,L} = d_A + d_B) = \min\{S - d_A - d_B,\ d_B\}$; otherwise, $Y^*_{B,F}(Y_{A,L} = d_A + d_B) = 0$.

PROOF for the Leader, Firm A:

We now present our proofs for the regions of the optimal capacity purchasing policy for the Leader. We consider the following two corresponding cases:

- **Case 1:** When $S \leq d_A + d_B$;
- **Case 2:** When $S > d_A + d_B$.

CASE 1, $S \leq d_A + d_B$: For this case, in the following we will construct the threshold structure for optimal backup capacity competition strategy under condition $K_A \geq d_A + d_B \geq S$. We consider the following *four sub-cases,* which can occur depending on the profit margin of Firm B:

1. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:
    - Both Firm A and Firm B forfeit, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (0, 0)$, which guarantees that $Y^*_{B,F}(\forall\, Y_{A,L}) = 0$.

- Firm A protects and Firm B forfeits, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (S - d_B, 0)$ and $(Y^*_{A,L},\ Y^*_{B,F}) = (d_A, 0)$.
- Firm A buys all and Firm B forfeits, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (S, 0)$.

In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0,0)$, $\Pi_{A,L}(S-d_B,0)$, $\Pi_{A,L}(d_A,0)$, and $\Pi_{A,L}(S,0)$. In other words, we want to find the thresholds among the "A and B Forfeit" region, the "A Protects" region, and the "B Forfeits" region.

2. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:

   - Firm A forfeits and Firm B protects, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (0, d_B)$. This guarantees that when the Leader, Firm A, decides to protect itself against losing customers to Firm B (and Firm C), the Follower, Firm B, will purchase to satisfy as many of its own customers as possible to protect against losing customers to Firm C.
   - Both Firm A and Firm B protect, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (S - d_B, d_B)$ and $(Y^*_{A,L},\ Y^*_{B,F}) = (d_A, S - d_A)$.
   - Firm A is aggressive, which results in $(Y^*_{A,L},\ Y^*_{B,F}) = (S, 0)$.

   In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0, d_B)$, $\Pi_{A,L}(S - d_B, d_B)$, $\Pi_{A,L}(d_A, S - d_A)$, and $\Pi_{A,L}(S, 0)$. In other words, we want to find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

3. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:

   - Firm A forfeits and Firm B is aggressive, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (0, \min\{K_B,\ S\})$.
   - Firm A protects and Firm B forfeit, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (S - d_B, 0)$ and $(Y^*_{A,L},\ Y^*_{B,F}) = (d_A, 0)$.
   - Firm A is aggressive, which results in $(Y^*_{A,L},\ Y^*_{B,F}) = (S, 0)$.

   In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0, \min\{K_B,\ S\})$, $\Pi_{A,L}(S-d_B, 0)$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(S, 0)$. In other words, we want to find the thresholds among the the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

4. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:

   - Firm A forfeits and Firm B is aggressive, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (0, \min\{K_B,\ S\})$.
   - Both Firm A and Firm B protect, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (S - d_B, d_B)$ and $(Y^*_{A,L},\ Y^*_{B,F}) = (d_A, S - d_A)$.
   - Firm A is aggressive, which results in $(Y^*_{A,L},\ Y^*_{B,F}) = (S, 0)$.

   In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0, \min\{K_B,\ S\})$, $\Pi_{A,L}(S - d_B, d_B)$, $\Pi_{A,L}(d_A, S - d_A)$, and $\Pi_{A,L}(S, 0)$. In other words, we want to find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

CASE 1 – Subcase 1: In this sub-case, by investigating the relationship among $\Pi_{A,L}(0,0)$, $\Pi_{A,L}(S - d_B, 0)$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(S, 0)$, we find the thresholds among the "A and B Forfeit" region, the "A Protects" region, and the "B Forfeits" region.
If $r_A \geq c^I_A$, which is defined as

$$c^I_A = c_A - \frac{\phi_{AC}}{\mu},$$

then by equation (6) and equation (8), we have

$$\begin{aligned}\Pi_{A,L}(S-d_B,0)-\Pi_{A,L}(0,0) &= (r_A-c_A)(S-d_B)\mu-\phi_{AC}\big(d_A-(S-d_B)\big)-(-\phi_{AC}d_A)\\ &= \big(r_A-(c_A-\frac{\phi_{AC}}{\mu})\big)(S-d_B)\mu\geq 0;\end{aligned}$$

also, by equation (8) and equation (10), we have

$$\begin{aligned}\Pi_{A,L}(d_A,0)-\Pi_{A,L}(S-d_B,0) &= (r_A-c_A)d_A\mu-\Big((r_A-c_A)(S-d_B)\mu-\phi_{AC}\big(d_A-(S-d_B)\big)\Big)\\ &= \big(r_A-(c_A-\frac{\phi_{AC}}{\mu})\big)\big(d_A-(S-d_B)\big)\mu\geq 0.\end{aligned}$$

That is, c_A^I is the threshold between the "A and B Forfeit" region and the "A Protects" region. Also, c_A^I is the threshold between the following two scenarios: the "A Protects against only B" scenario and the "A Protects against both B and C" scenario.

- If $r_A\geq c_A^I$, then we have

  $$\Pi_{A,L}(d_A,0)\geq\Pi_{A,L}(S-d_B,0)\geq\Pi_{A,L}(0,0).$$

  Now, to check the relationship between $\Pi_{A,L}(S,0)$ and $\Pi_{A,L}(d_A,0)$, we define

  $$c_A^{II}=c_A-\frac{\phi_{BA}}{\mu}.$$

  If $r_A\geq c_A^{II}$, then by equation (10) and equation (12), we have

  $$\begin{aligned}\Pi_{A,L}(S,0)-\Pi_{A,L}(d_A,0) &= r_Ad_A\mu+\psi_A(S-d_A)-c_AS\mu-\big(r_Ad_A\mu-c_Ad_A\mu\big)\\ &= \big((r_A\mu+\phi_{BA})-c_A\mu\big)(S-d_A)\\ &= \big(r_A-(c_A-\frac{\phi_{BA}}{\mu})\big)(S-d_A)\mu\geq 0.\end{aligned}$$

  That is, c_A^{II} is the threshold between the "A Protects" region and the "B Forfeits" region.

- If $r_A<c_A^I$, then we have

  $$\Pi_{A,L}(0,0)\geq\Pi_{A,L}(S-d_B,0)\geq\Pi_{A,L}(d_A,0).$$

  Now, to check the relationship between $\Pi_{A,L}(S,0)$ and $\Pi_{A,L}(0,0)$, we define

  $$c_A^{III}=c_A-\frac{\phi_{BA}[S-d_A]^+ + \phi_{AC}\min\{d_A,S\}}{\mu S}.$$

  If $r_A\geq c_A^{III}$, then by equation (6) and equation (12), we have

  $$\begin{aligned}\Pi_{A,L}(S,0)-\Pi_{A,L}(0,0) &= r_A\min\{S,d_A\}\mu+\psi_A[S-d_A]^+-\phi_{AC}[d_A-S]^+-c_AS\mu-(-\phi_{AC}d_A)\\ &= r_AS\mu-\big(c_AS\mu-\phi_{BA}[S-d_A]^+-\phi_{AC}\min\{d_A,S\}\big)\geq 0.\end{aligned}$$

  That is, c_A^{III} is the threshold between the "A and B Forfeit" region and the "B Forfeits" region.

CASE 1 – Subcase 2: In this sub-case, by investigating the relationship among $\Pi_{A,L}(0,d_B)$, $\Pi_{A,L}(S-d_B,d_B)$, $\Pi_{A,L}(d_A,S-d_A)$, and $\Pi_{A,L}(S,0)$, we find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

By equation (6), equation (8), and equation (10), we know

- if $r_A \geq c_A^I$, then
$$\Pi_{A,L}(d_A, S - d_A) \geq \Pi_{A,L}(S - d_B, d_B) \geq \Pi_{A,L}(0, d_B).$$
That is, c_A^I is the threshold between the "A Forfeits" region and the "A Protects" region.
By equation (10) and equation (12), we know, if $r_A \geq c_A^{II}$, then
$$\Pi_{A,L}(S, 0) \geq \Pi_{A,L}(d_A, S - d_A).$$
That is, c_A^{II} is the threshold between the "A Protects" region and the "A is Aggressive" region.

- if $r_A < c_A^I$, then
$$\Pi_{A,L}(0, d_B) \geq \Pi_{A,L}(S - d_B, d_B) \geq \Pi_{A,L}(d_A, S - d_A).$$
By equation (6) and equation (12), we know, if $r_A \geq c_A^{III}$, then
$$\Pi_{A,L}(S, 0) \geq \Pi_{A,L}(0, d_B).$$
That is, c_A^{III} is the threshold between the "A Forfeits" region and the "A is Aggressive" region.

CASE 1 – Subcase 3: In this sub-case, by investigating the relationship among $\Pi_{A,L}(0, \min\{K_B,\ S\})$, $\Pi_{A,L}(S - d_B, 0)$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(S, 0)$, we find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

If $r_A \geq c_A^{IV}$, which is defined as

$$\begin{aligned} c_A^{IV} &= c_A - \frac{\phi_{AB}\big(\min\{K_B,\ S\} - d_B\big) + \phi_{AC}\Big((S - d_B) - \big(\min\{K_B,\ S\} - d_B\big)\Big)}{\mu(S - d_B)} \\ &= c_A - \frac{\phi_{AC}}{\mu} - \frac{(\phi_{AB} - \phi_{AC})\big(\min\{K_B,\ S\} - d_B\big)}{\mu(S - d_B)}, \end{aligned}$$

then by equation (6) and equation (8), we have

$$\begin{aligned} &\Pi_{A,L}(S - d_B, 0) - \Pi_{A,L}\big(0, \min\{K_B,\ S\}\big) \\ &= (r_A - c_A)(S - d_B)\mu - \phi_{AC}\big(d_A - (S - d_B)\big) \\ &\quad - \Big(- \phi_{AB}[\min\{K_B,\ S\} - d_B]^+ - \phi_{AC}\Big(d_A - \big[\min\{K_B,\ S\} - d_B\big]^+\Big)\Big) \\ &= r_A(S - d_B)\mu - \Big(c_A(S - d_B)\mu \\ &\quad - \phi_{AB}\big(\min\{K_B,\ S\} - d_B\big) - \phi_{AC}\big((S - d_B) - \big(\min\{K_B,\ S\} - d_B\big)\big)\Big) \geq 0. \end{aligned}$$

That is, c_A^{IV} is the threshold between the "A Forfeits" region and the "A Protects against only B" region.
If $r_A \geq c_A^V$, which is defined as

$$\begin{aligned} c_A^V &= c_A - \frac{\phi_{AB}\big[\min\{K_B,\ S\} - d_B\big]^+ + \phi_{AC}\Big(d_A - \big[\min\{K_B,\ S\} - d_B\big]^+\Big)}{\mu d_A} \\ &= c_A - \frac{\phi_{AC}}{\mu} - \frac{(\phi_{AB} - \phi_{AC})\big[\min\{K_B,\ S\} - d_B\big]^+}{\mu d_A}, \\ &\qquad \big(\textit{Notice that } d_A \geq [S - d_B]^+,\ \textit{so } c_A^V \geq c_A^{IV}.\big) \end{aligned}$$

then by equation (6) and equation (10), we have

$$\begin{aligned}
&\Pi_{A,L}(d_A,0) - \Pi_{A,L}\big(0,\min\{K_B,\ S\}\big)\\
&= (r_A - c_A)d_A\mu\\
&\quad - \Big(-\phi_{AB}\big[\min\{K_B,\ S\} - d_B\big]^+ - \phi_{AC}\Big(d_A - \big[\min\{K_B,\ S\} - d_B\big]^+\Big)\Big)\\
&= r_A d_A\mu - \Big(c_A d_A\mu\\
&\quad - \phi_{AB}\big[\min\{K_B,\ S\} - d_B\big]^+ - \phi_{AC}\Big(d_A - \big[\min\{K_B,\ S\} - d_B\big]^+\Big)\Big) \geq 0.
\end{aligned}$$

That is, c_A^V is the threshold between the "A Forfeits" region and the "A Protects against both B and C" region.

Recall, by equation (8) and equation (10), we have shown, if $r_A \geq c_A^I$, then

$$\Pi_{A,L}(d_A,0) - \Pi_{A,L}(S - d_B,0) \quad = \quad (r_A - c_A^I)\big(d_A - (S - d_B)\big)\mu \geq 0.$$

Therefore, (Note that $c_A^I \geq c_A^V \geq c_A^{IV}$ automatically holds),

- if $r_A \geq c_A^I$, then
$$\Pi_{A,L}(d_A,0) \geq \Pi_{A,L}(S - d_B,0) \geq \Pi_{A,L}(0,\min\{K_B,\ S\}).$$
That is, c_A^I is the threshold between the "A Forfeits" region and the "A Protects" region.

  Now, we want to check the relationship between $\Pi_{A,L}(S,0)$ and $\Pi_{A,L}(d_A,0)$. Note that, we have shown that if $r_A \geq c_A^{II}$, then
$$\Pi_{A,L}(S,0) \geq \Pi_{A,L}(d_A,0).$$
That is, c_A^{II} is the threshold between the "A Protects against both B and C" region and the "A is Aggressive" region.

- if $r_A \leq c_A^I$ but $r_A \geq c_A^V$ (so $r_A \geq c_A^{IV}$ must hold), then
$$\Pi_{A,L}(S - d_B,0) \geq \Pi_{A,L}(d_A,0) \geq \Pi_{A,L}(0,\min\{K_B,\ S\});$$
also if $r_A \leq c_A^V$ but $r_A \geq c_A^{IV}$, then
$$\Pi_{A,L}(S - d_B,0) \geq \Pi_{A,L}(0,\min\{K_B,\ S\}) \geq \Pi_{A,L}(d_A,0);$$
Now, to check the relationship between $\Pi_{A,L}(S,0)$ and $\Pi_{A,L}(S - d_B,0)$, we define
$$c_A^{VI} \quad = c_A - \frac{\phi_{BA}[S - d_A]^+ + \phi_{AC}\big(\min\{d_A,S\} - (S - d_B)\big)}{\mu d_B}.$$
If $r_A \geq c_A^{VI}$, then by equation (8) and equation (12), we have
$$\begin{aligned}
\Pi_{A,L}(S,0) - \Pi_{A,L}(S - d_B,0) \quad &= \quad r_A\min\{S,d_A\}\mu + \psi_A[S - d_A]^+ - \phi_{AC}[d_A - S]^+ - c_A S\mu\\
&\quad -\Big((r_A - c_A)(S - d_B)\mu - \phi_{AC}\big(d_A - (S - d_B)\big)\Big)\\
&= \quad r_A d_B\mu - \Big(c_A d_B\mu\\
&\quad -\phi_{BA}[S - d_A]^+ - \phi_{AC}\big(\min\{d_A,S\} - (S - d_B)\big)\Big) \geq 0.
\end{aligned}$$
That is, c_A^{VI} is the threshold between the "A Protects against only B" region and the "A is Aggressive" region.

- if $r_A \le c_A^{IV}$, then
$$\Pi_{A,L}(0, \min\{K_B,\ S\}) \ge \Pi_{A,L}(S - d_B, 0) \ge \Pi_{A,L}(d_A, 0).$$

Now, to check the relationship between $\Pi_{A,L}(S, 0)$ and $\Pi_{A,L}(0, \min\{K_B,\ S\})$, we define

$$\begin{aligned} c_A^{VII} &= c_A - \frac{\phi_{AB}[\min\{K_B,\ S\} - d_B]^+ + \phi_{BA}[S - d_A]^+}{\mu S} \\ &\quad - \frac{\phi_{AC}\Big(\min\{d_A,\ S\} - \big[\min\{K_B,\ S\} - d_B\big]^+\Big)}{\mu S}. \end{aligned}$$

If $r_A \ge c_A^{VII}$, then by equation (6) and equation (12), we have

$$\begin{aligned} &\Pi_{A,L}(S, 0) - \Pi_{A,L}\big(0, \min\{K_B,\ S\}\big) \\ &= r_A \min\{S, d_A\}\mu + \psi_A[S - d_A]^+ - \phi_{AC}[d_A - S]^+ - c_A S\mu \\ &\quad - \Big(- \phi_{AB}\big[\min\{K_B,\ S\} - d_B\big]^+ - \phi_{AC}\Big(d_A - \big[\min\{K_B,\ S\} - d_B\big]^+\Big)\Big) \\ &= r_A S\mu - \Big(c_A S\mu - \phi_{AB}\big[\min\{K_B,\ S\} - d_B\big]^+ - \phi_{BA}[S - d_A]^+ \\ &\quad - \phi_{AC}\Big(\min\{d_A,\ S\} - \big[\min\{K_B,\ S\} - d_B\big]^+\Big)\Big) \ge 0. \end{aligned}$$

That is, c_A^{VII} is the threshold between the "A Forfeits" region and the "A is Aggressive" region.

CASE 1 – Subcase 4: In this sub-case, by investigating the relationship among $\Pi_{A,L}(0, \min\{K_B,\ S\})$, $\Pi_{A,L}(S - d_B, d_B)$, $\Pi_{A,L}(d_A, S - d_A)$, and $\Pi_{A,L}(S, 0)$, we find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

By equation (6) and equation (8), we know, if $r_A \ge c_A^{IV}$, then

$$\Pi_{A,L}(S - d_B, d_B) \ge \Pi_{A,L}\big(0, \min\{K_B,\ S\}\big).$$

By equation (6) and equation (10), we know, if $r_A \ge c_A^{V}$, then

$$\Pi_{A,L}(d_A, S - d_A) \ge \Pi_{A,L}\big(0, \min\{K_B,\ S\}\big).$$

By equation (8) and equation (10), we know, if $r_A \ge c_A^{I}$, then

$$\Pi_{A,L}(d_A, S - d_A) \ge \Pi_{A,L}(S - d_B, d_B).$$

Therefore, (Not that $c_A^I \ge c_A^V \ge c_A^{IV}$ automatically holds),

- if $r_A \ge c_A^I$, then
$$\Pi_{A,L}(d_A, S - d_A) \ge \Pi_{A,L}(S - d_B, d_B) \ge \Pi_{A,L}(0, \min\{K_B,\ S\}).$$

  That is, c_A^I is the threshold between the "A Forfeits" region and the "A Protects" region.

  By equation (10) and equation (12), we know, if $r_A \ge c_A^{II}$, then
$$\Pi_{A,L}(S, 0) \ge \Pi_{A,L}(d_A, S - d_A).$$

  That is, c_A^{II} is the threshold between the "A Protects against both B and C" region and the "A is Aggressive" region.

- if $r_A \le c_A^I$ but $r_A \ge c_A^V$ (so $r_A \ge c_A^{IV}$ must hold), then

$$\Pi_{A,L}(S - d_B, d_B) \ge \Pi_{A,L}(d_A, S - d_A) \ge \Pi_{A,L}(0, \min\{K_B,\ S\}),$$

and if $r_A \le c_A^V$ but $r_A \ge c_A^{IV}$, then

$$\Pi_{A,L}(S - d_B, d_B) \ge \Pi_{A,L}(0, \min\{K_B,\ S\}) \ge \Pi_{A,L}(d_A, S - d_A).$$

By equation (8) and equation (12), we know, if $r_A \ge c_A^{VI}$, then

$$\Pi_{A,L}(S, 0) \ge \Pi_{A,L}(S - d_B, d_B).$$

That is, c_A^{VI} is the threshold between the "A Protects against only B" region and the "A is Aggressive" region.

- if $r_A \le c_A^{IV}$, then

$$\Pi_{A,L}(0, \min\{K_B,\ S\}) \ge \Pi_{A,L}(S - d_B, d_B) \ge \Pi_{A,L}(d_A, S - d_A).$$

By equation (6) and equation (12), we know, if $r_A \ge c_A^{VII}$, then

$$\Pi_{A,L}(S, 0) \ge \Pi_{A,L}\big(0, \min\{K_B,\ S\}\big).$$

That is, c_A^{VII} is the threshold between the "A Forfeits" region and the "A is Aggressive" region.

CASE 2, $S > d_A + d_B$: For this case, in the following we will construct the threshold structure for the optimal backup capacity competition strategy under condition $K_A \ge d_A + d_B$ and $S > d_A + d_B$. We consider the following *five sub-cases:*

1. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:

   - Both Firm A and Firm B forfeit, i.e., $(Y_{A,L}^*,\ Y_{B,F}^*) = (0, 0)$, which guarantees that $Y_{B,F}^*(\forall\, Y_{A,L}) = 0$.
   - Firm A protects and Firm B forfeits, i.e., $(Y_{A,L}^*,\ Y_{B,F}^*) = (d_A, 0)$.
   - Firm A buys enough to satisfy the total market and Firm B forfeits, i.e., $(Y_{A,L}^*,\ Y_{B,F}^*) = (d_A + d_B, 0)$.

   In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0, 0)$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(d_A + d_B, 0)$. In other words, we want to find the thresholds among the "A and B Forfeit" region, the "A Protects" region, and the "B Forfeits" region.

2. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:

   - Firm A forfeits and Firm B protects, i.e., $(Y_{A,L}^*,\ Y_{B,F}^*) = (0, d_B)$. This guarantees that when the Leader, Firm A, decides to protect itself against losing customers, the Follower, Firm B, will purchase to satisfy as many of its own customers as possible to protect against losing customers to Firm C.
   - Both Firm A and Firm B protect, i.e., $(Y_{A,L}^*,\ Y_{B,F}^*) = (d_A, d_B)$.
   - Firm A is aggressive, which results in $(Y_{A,L}^*,\ Y_{B,F}^*) = (S, 0)$.

   In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0, d_B)$, $\Pi_{A,L}(d_A, d_B)$, and $\Pi_{A,L}(S, 0)$. In other words, we want to find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

3. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:

   - Firm A forfeits and Firm B is aggressive, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (0, \min\{K_B,\ d_A + d_B\})$.
   - Firm A protects and Firm B forfeits, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (d_A, 0)$.
   - Firm A is aggressive and Firm B forfeits. More specifically, The Follower, Firm B, decides to buy nothing when the Leader, Firm A, purchases the amount of $d_A + d_B$ to cover the total market, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (d_A + d_B, 0)$.

   In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0, \min\{K_B,\ d_A+d_B\})$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(d_A+d_B, 0)$. In other words, we want to find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

4. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:

   - Firm A forfeits and Firm B is aggressive, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (0, \min\{K_B,\ d_A + d_B\})$.
   - Firm A protects and Firm B forfeit, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (d_A, 0)$.
   - Firm A is aggressive, which results in $(Y^*_{A,L},\ Y^*_{B,F}) = (S, 0)$. Note that, comparing with the third scenario in the above sub-case, here, instead of purchasing the amount of $d_A + d_B$, Firm A has to buy all available backup capacity to corner the whole market, because if Firm A purchases the amount of $d_A + d_B$, Firm B will buy the remaining backup capacity to protect itself against losing customers to Firm A, and hence $d_A + d_B$ is not optimal for Firm A.

   In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0, \min\{K_B,\ d_A+d_B\})$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(S, 0)$. In other words, we want to find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

5. As Firm A's profit margin increases (i.e., we move from left to right along the x-axis of the five-region structure), the following three scenarios occur:

   - Firm A forfeits and Firm B is aggressive, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (0, \min\{K_B,\ d_A + d_B\})$.
   - Both Firm A and Firm B protect, i.e., $(Y^*_{A,L},\ Y^*_{B,F}) = (d_A, d_B)$.
   - Firm A is aggressive, which results in $(Y^*_{A,L},\ Y^*_{B,F}) = (S, 0)$.

   In this sub-case, we want to construct the relationship among $\Pi_{A,L}(0, \min\{K_B,\ d_A+d_B\})$, $\Pi_{A,L}(d_A, d_B)$, and $\Pi_{A,L}(S, 0)$. In other words, we want to find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

CASE 2 – Subcase 1: In this sub-case, by investigating the relationship among $\Pi_{A,L}(0, 0)$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(d_A + d_B, 0)$, we find the thresholds among the "A and B Forfeit" region, the "A Protects" region, and the "B Forfeits" region.

First of all, it is easy to see that $\Pi_{A,L}(Y_{A,L} = d_A + d_B, Y_{B,F} = 0) \geq \Pi_{A,L}(Y_{A,L} = S, Y_{B,F} = 0)$ is always satisfied due to the saving of holding cost.

- If $r_A \geq c^i_A$, which is defined as

$$c^i_A = c_A - \frac{\phi_{AC}}{\mu} = c^I_A,$$

  then by equation (13) and equation (15), we have

$$\begin{aligned}\Pi_{A,L}(d_A, 0) - \Pi_{A,L}(0, 0) &= (r_A - c_A)d_A\mu - (-\phi_{AC}d_A) \\ &= \big(r_A - (c_A - \frac{\phi_{AC}}{\mu})\big)d_A\mu \geq 0.\end{aligned}$$

  That is, c^I_A is the threshold between the "A and B Forfeit" region and the "A Protects" region.

- If $r_A \geq c_A^{ii}$, which is defined as

$$c_A^{ii} = c_A - \frac{\phi_{BA}}{\mu} = c_A^{II},$$

then by equation (15) and equation (19), we have

$$\begin{aligned}\Pi_{A,L}(d_A + d_B, 0) - \Pi_{A,L}(d_A, 0) &= r_A d_A \mu + \psi_A d_B - c_A(d_A + d_B)\mu - (r_A - c_A)d_A\mu \\ &= \big(r_A - (c_A - \frac{\phi_{BA}}{\mu})\big)d_B\mu \geq 0.\end{aligned}$$

That is, c_A^{II} is the threshold between the "A Protects" region and the "B Forfeits" region.

- If $r_A \geq c_A^{iii}$, which is defined as

$$c_A^{iii} = c_A - \frac{\phi_{BA}d_B + \phi_{AC}d_A}{\mu(d_A + d_B)},$$

then by equation (13) and equation (19), we have

$$\begin{aligned}\Pi_{A,L}(d_A + d_B, 0) - \Pi_{A,L}(0, 0) &= r_A d_A \mu + \psi_A d_B - c_A(d_A + d_B)\mu - (-\phi_{AC}d_A) \\ &= \Big(r_A(d_A + d_B)\mu - \big(c_A(d_A + d_B)\mu - \phi_{BA}d_B - \phi_{AC}d_A\big)\Big) \geq 0.\end{aligned}$$

That is, c_A^{iii} is the threshold between the "A and B Forfeit" region and the "B Forfeits" region.

CASE 2 – Subcase 2: In this sub-case, by investigating relationship among $\Pi_{A,L}(0, d_B)$, $\Pi_{A,L}(d_A, d_B)$, and $\Pi_{A,L}(S, 0)$, we find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

- By equation (13) and equation (15), we know, if $r_A \geq c_A^i$ (*i.e.*, c_A^I), then

$$\Pi_{A,L}(d_A, d_B) \geq \Pi_{A,L}(0, d_B).$$

That is, c_A^I is the threshold between the "A Forfeits" region and the "A Protects" region.

- If $r_A \geq c_A^{iv}$, which is defined as

$$c_A^{iv} = c_A \frac{S - d_A}{d_B} + \frac{h_A(S - d_A - d_B)}{d_B} - \frac{\phi_{BA}}{\mu},$$

then by equation (15) and equation (21), we have

$$\begin{aligned}\Pi_{A,L}(S, 0) - \Pi_{A,L}(d_A, d_B) &= r_A d_A \mu + \psi_A d_B - c_A S\mu - h_A(S - d_A - d_B)\mu - \big((r_A - c_A)d_A\mu\big) \\ &= r_A d_B \mu - \Big(c_A(S - d_A)\mu + h_A(S - d_A - d_B)\mu - \phi_{BA}d_B\Big) \geq 0.\end{aligned}$$

That is, c_A^{iv} is the threshold between the "A Protects" region and the "A is Aggressive" region.

- If $r_A \geq c_A^v$, which is defined as

$$c_A^v = c_A \frac{S}{d_A + d_B} + \frac{h_A(S - d_A - d_B)}{d_A + d_B} - \frac{\phi_{BA}d_B}{\mu(d_A + d_B)} - \frac{\phi_{AC}d_A}{\mu(d_A + d_B)},$$

then by equation (13) and equation (21), we have

$$\begin{aligned}\Pi_{A,L}(S, 0) - \Pi_{A,L}(0, d_B) &= r_A d_A \mu + \psi_A d_B - c_A S\mu - h_A(S - d_A - d_B)\mu - (-\phi_{AC}d_A) \\ &= r_A(d_A + d_B)\mu - \Big(c_A S\mu + h_A(S - d_A - d_B)\mu - \phi_{BA}d_B - \phi_{AC}d_A\Big) \geq 0.\end{aligned}$$

That is, c_A^v is the threshold between the "A Forfeits" region and the "A is Aggressive" region.

CASE 2 – Subcase 3: In this sub-case, by investigating the relationship among $\Pi_{A,L}(0, \min\{K_B,\ d_A + d_B\})$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(d_A + d_B, 0)$, we find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

- If $r_A \geq c_A^{vi}$, which is defined as

$$\begin{aligned} c_A^{vi} &= c_A - \frac{\phi_{AB}\big(\min\{K_B,\ d_A + d_B\} - d_B\big)}{\mu d_A} \\ &\quad - \frac{\phi_{AC}\Big(d_A - \big(\min\{K_B,\ d_A + d_B\} - d_B\big)\Big)}{\mu d_A}, \\ &= c_A - \frac{\phi_{AC}}{\mu} - \frac{(\phi_{AB} - \phi_{AC})\big(\min\{K_B,\ d_A + d_B\} - d_B\big)}{\mu d_A}, \end{aligned}$$

then by equation (13) and equation (15), we have

$$\begin{aligned} &\Pi_{A,L}(d_A, 0) - \Pi_{A,L}\big(0, \min\{K_B,\ d_A + d_B\}\big) \\ &= (r_A - c_A) d_A \mu \\ &\quad - \Big(- \phi_{AB} \min\Big\{ \min\{K_B,\ d_A + d_B\} - d_B, d_A \Big\} - \phi_{AC}\Big[d_A - \big(\min\{K_B,\ d_A + d_B\} - d_B\big)\Big]^+\Big) \\ &= r_A d_A \mu - \Big(c_A d_A \mu \\ &\quad - \phi_{AB}\big(\min\{K_B,\ d_A + d_B\} - d_B\big) - \phi_{AC}\big(d_A - (\min\{K_B,\ d_A + d_B\} - d_B)\big)\Big) \geq 0. \end{aligned}$$

That is, c_A^{vi} is the threshold between the "A Forfeits" region and the "A Protects" region.

- We have shown that if $r_A \geq c_A^{II}$, then

$$\Pi_{A,L}(d_A + d_B, 0) \geq \Pi_{A,L}(d_A, 0).$$

That is, c_A^{II} is the threshold between the "A Protects" region and the "A is Aggressive" region.

- If $r_A \geq c_A^{vii}$, which is defined as

$$\begin{aligned} c_A^{vii} &= c_A - \frac{\phi_{AB}\big(\min\{K_B,\ d_A + d_B\} - d_B\big)}{\mu(d_A + d_B)} - \frac{\phi_{BA} d_B}{\mu(d_A + d_B)} \\ &\quad - \frac{\phi_{AC}\Big(d_A - \big(\min\{K_B,\ d_A + d_B\} - d_B\big)\Big)}{\mu(d_A + d_B)}, \end{aligned}$$

then by equation (13) and equation (19), we have

$$\begin{aligned} &\Pi_{A,L}(d_A + d_A, 0) - \Pi_{A,L}\big(0, \min\{K_B,\ d_A + d_B\}\big) \\ &= r_A d_A \mu + \psi_A d_B - c_A (d_A + d_B)\mu \\ &\quad - \Big(- \phi_{AB} \min\big\{ \min\{K_B,\ d_A + d_B\} - d_B, d_A \big\} - \phi_{AC}\Big[d_A - \big(\min\{K_B,\ d_A + d_B\} - d_B\big)\Big]^+\Big) \\ &= r_A (d_A + d_B)\mu - \Big(c_A (d_A + d_B)\mu \\ &\quad - \phi_{AB}\big(\min\{K_B,\ d_A + d_B\} - d_B\big) - \phi_{BA} d_B \\ &\quad - \phi_{AC}\Big(d_A - \big(\min\{K_B,\ d_A + d_B\} - d_B\big)\Big)\Big) \geq 0. \end{aligned}$$

That is, c_A^{vii} is the threshold between the "A Forfeits" region and the "A is Aggressive" region.

CASE 2 – Subcase 4: In this sub-case, by investigating the relationship among $\Pi_{A,L}(0, \min\{K_B,\ d_A + d_B\})$, $\Pi_{A,L}(d_A, 0)$, and $\Pi_{A,L}(S, 0)$, we find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

- We have shown that if $r_A \geq c_A^{vi}$, then

$$\Pi_{A,L}(d_A, 0) \geq \Pi_{A,L}\big(0, \min\{K_B,\ d_A + d_B\}\big).$$

  That is, c_A^{vi} is the threshold between the "A Forfeits" region and the "A Protects" region.

- If $r_A \geq c_A^{iv}$, then by equation (15) and equation (21), we have

$$\begin{aligned}
&\Pi_{A,L}(S, 0) - \Pi_{A,L}(d_A, 0) \\
&= r_A d_A \mu + \psi_A d_B - c_A S\mu - h_A(S - d_A - d_B)\mu \\
&\quad - \big((r_A - c_A) d_A \mu\big) \\
&= r_A d_B \mu - \Big(c_A(S - d_A)\mu + h_A(S - d_A - d_B)\mu - \phi_{BA} d_B\Big) \geq 0.
\end{aligned}$$

  That is, c_A^{iv} is the threshold between the "A Protects" region and the "A is Aggressive" region.

- If $r_A \geq c_A^{viii}$, which is defined as

$$\begin{aligned}
c_A^{viii} \quad = \quad & c_A \frac{S}{d_A + d_B} + \frac{h_A(S - d_A - d_B)}{d_A + d_B} - \frac{\phi_{AB}\big(\min\{K_B,\ d_A + d_B\} - d_B\big)}{\mu(d_A + d_B)} - \frac{\phi_{BA} d_B}{\mu(d_A + d_B)} \\
& - \frac{\phi_{AC}\Big(d_A - \big(\min\{K_B,\ d_A + d_B\} - d_B\big)\Big)}{\mu(d_A + d_B)},
\end{aligned}$$

  then by equation (13) and equation (21), we have

$$\begin{aligned}
&\Pi_{A,L}(S, 0) - \Pi_{A,L}(0, \min\{K_B,\ d_A + d_B\}) \\
&= r_A d_A \mu + \psi_A d_B - c_A S\mu - h_A(S - d_A - d_B)\mu \\
&\quad - \Big(- \phi_{AB} \min\big\{ \min\{K_B,\ d_A + d_B\} - d_B, d_A\big\} - \phi_{AC}\Big[d_A - \big(\min\{K_B,\ d_A + d_B\} - d_B\big)\Big]^+\Big) \\
&= r_A(d_A + d_B)\mu - \Big(c_A S\mu + h_A(S - d_A - d_B)\mu \\
&\quad - \phi_{AB}\big(\min\{K_B,\ d_A + d_B\} - d_B\big) - \phi_{BA} d_B \\
&\quad - \phi_{AC}\big(d_A - (\min\{K_B,\ d_A + d_B\} - d_B)\big)\Big) \geq 0.
\end{aligned}$$

  That is, c_A^{viii} is the threshold between the "A Forfeits" region and the "A is Aggressive" region.

CASE 2 – Subcase 5: In this sub-case, by investigating the relationship among $\Pi_{A,L}(0, \min\{K_B,\ d_A + d_B\})$, $\Pi_{A,L}(d_A, d_B)$, and $\Pi_{A,L}(S, 0)$, we find the thresholds among the "A Forfeits" region, the "A Protects" region, and the "A is Aggressive" region.

- By equation (13) and equation (15), we know, if $r_A \geq c_A^{vi}$, then

$$\Pi_{A,L}(d_A, d_B) \geq \Pi_{A,L}\big(0, \min\{K_B,\ d_A + d_B\}\big).$$

  That is, c_A^{vi} is the threshold between the "A Forfeits" region and the "A Protects" region.

- By equation (15) and equation (21), we know, if $r_A \geq c_A^{iv}$, then

$$\Pi_{A,L}(S, 0) \geq \Pi_{A,L}(d_A, d_B).$$

  That is, c_A^{iv} is the threshold between the "A Protects" region and the "A is Aggressive" region.

- By equation (13) and equation (21), we know, if $r_A \geq c_A^{viii}$, then

$$\Pi_{A,L}(S,0) \geq \Pi_{A,L}(0, \min\{K_B,\ d_A + d_B\}).$$

That is, c_A^{viii} is the threshold between the "A Forfeits" region and the "A is Aggressive" region.

This completes the proof of Proposition 2 for the "Large Capacity Leader". □

PROPOSITION 3: *A unique Nash Equilibrium exists for each firm's Advanced Preparedness Competition.*

Proof: To prove that a unique Nash Equilibrium exists for the AP Competition of Firm A, first we must show the following (Rudin 1976):

- Function Π_i^{AP} does have a unique maximum, $i = A, B$.
- Function $\Pi_{j,i}^{AP}$ does have a unique maximum, $i, j = A, B$ and $i \neq j$.
- Function $x_i(x_j)$ is continuous in x_j and strictly concave in x_j, $i, j = A, B$ and $i \neq j$.

Let $\Pi_{i,L}^*$ and $\Pi_{i,F}^*$ be the optimal expected profits for Firm i when it is the Leader and the Follower, respectively, in the BC Competition. It is easy to see that for Firm i, the profit of being the Leader in the BC Competition is, at least, as much as being the Follower, and furthermore, if $\Pi_{i,L}^* = \Pi_{i,F}^*$, then Firm i will definitely have no interest in the AP Competition, which is obviously not an interesting case. So here we assume that $\Pi_{i,L}^* > \Pi_{i,F}^*$, $i = A, B$. Also $x_i > 0$, since x_i represents Firm i's preparedness effort, and that zero implies that Firm i has no interest in the AP Competition.

To show that, for any given Firm B's preparedness effort x_B^o, function Π_i^{AP} has a unique maximum, $\hat{x}_A$, consider

$$\begin{aligned} f(x_A|x_B^o) \equiv \Pi_A^{AP} &= p_{A0}\big(\pi_A \Pi_{A,L}^* + \pi_B \Pi_{A,F}^* - m_A(d_A^0 - d_A)\big) + (1 - p_{A0}) r_A d_A^0 (\mu + T) \\ &\quad - C_e x_A + m_A d_A^0, \\ &= C_L^A\Big(\frac{x_A}{x_A + x_B^o}\Big) + C_F^A\Big(\frac{x_B^o}{x_A + x_B^o}\Big) - C_e x_A + constant, \end{aligned} \tag{22}$$

where $C_L^A = p_{A0}\Pi_{A,L}^*$, $C_F^A = p_{A0}\Pi_{A,F}^*$. Note that C_L^A, C_F^A and C_e are independent of x_i, and x_i is positive. So,

$$f'(x_A|x_B^o) = \frac{d}{d\,x_A} f(x_A|x_B^o) = \frac{(C_L^A - C_F^A)x_B^o}{(x_A + x_B^o)^2} - C_e.$$

By setting $f'(\hat{x}_A|x_B^o) = 0$, and solving for $\hat{x}_A$, we get

$$\begin{aligned} \hat{x}_A &= \sqrt{\frac{(C_L^A - C_F^A)}{C_e} x_B^o} - x_B^o, \\ &= \sqrt{\frac{p_{A0}(\Pi_{A,L}^* - \Pi_{A,F}^*)}{C_e} x_B^o} - x_B^o. \end{aligned} \tag{23}$$

Notice that, $C_L^A - C_F^A \geq 0$ always holds, since p_0 is non-negative and we have already argued that $\Pi_{A,L}^* > \Pi_{A,F}^*$. It is clear that for any fixed x_B^o, there exists a unique $\hat{x}_A$, which is a non-negative real number, i.e., the optimal solution of $f(x_A|x_B)$ does exist and it is unique.

Furthermore,

$$f''(x_A|x_B^o) = \frac{-2(C_L^A - C_F^A)x_B^o}{(x_A + x_B^o)^3} < 0,$$

Thresholds	Expression
When $S \le d_A + d_B$	
c_A^I	$c_A - \frac{\phi_{AC}}{\mu}$
c_A^{II}	$c_A - \frac{\phi_{BA}}{\mu}$
c_A^{III}	$c_A - \frac{\phi_{BA}[S-d_A]^+}{\mu S} - \frac{\phi_{AC}\min\{d_A,S\}}{\mu S}$
c_A^{IV}	$c_A - \frac{\phi_{AC}}{\mu} - \frac{(\phi_{AB}-\phi_{AC})\left(\min\{K_B,\ S\}-d_B\right)}{\mu(S-d_B)}$
c_A^{V}	$c_A - \frac{\phi_{AC}}{\mu} - \frac{(\phi_{AB}-\phi_{AC})\left[\min\{K_B,\ S\}-d_B\right]^+}{\mu d_A}$
c_A^{VI}	$c_A - \frac{\phi_{BA}[S-d_A]^+}{\mu d_B} - \frac{\phi_{AC}\left(\min\{d_A,S\}-(S-d_B)\right)}{\mu d_B}$
c_A^{VII}	$c_A - \frac{\phi_{AB}\left[\min\{K_B,\ S\}-d_B\right]^+}{\mu S} - \frac{\phi_{BA}[S-d_A]^+}{\mu S} - \frac{\phi_{AC}\left(\min\{d_A,\ S\}-\left[\min\{K_B,\ S\}-d_B\right]^+\right)}{\mu S}$
c_B^{I}	$c_B - \frac{\phi_{AB}\left[\min\{K_B,\ S\}-d_B\right]^+}{\mu\min\{K_B,\ S\}} - \frac{\phi_{BC}\min\left\{d_B,\min\{K_B,\ S\}\right\}}{\mu\min\{K_B,\ S\}}$
c_B^{II}	$c_B - \frac{\phi_{BC}}{\mu}$
c_B^{III}	$c_B - \frac{\phi_{AB}}{\mu}$
When $S > d_A + d_B$	
c_A^{I}	$c_A - \frac{\phi_{AC}}{\mu}$
c_A^{II}	$c_A - \frac{\phi_{BA}}{\mu}$
c_A^{iii}	$c_A - \frac{\phi_{BA}d_B}{\mu(d_A+d_B)} - \frac{\phi_{AC}d_A}{\mu(d_A+d_B)}$
c_A^{vi}	$c_A\frac{S-d_A}{d_B} + \frac{h_A(S-d_A-d_B)}{d_B} - \frac{\phi_{BA}}{\mu}$
c_A^{v}	$c_A\frac{S}{d_A+d_B} + \frac{h_A(S-d_A-d_B)}{d_A+d_B} - \frac{\phi_{BA}d_B}{\mu(d_A+d_B)} - \frac{\phi_{AC}d_A}{\mu(d_A+d_B)}$
c_A^{vi}	$c_A - \frac{\phi_{AC}}{\mu} - \frac{(\phi_{AB}-\phi_{AC})\left(\min\{K_B,\ d_A+d_B\}-d_B\right)}{\mu d_A}$
c_A^{vii}	$c_A - \frac{\phi_{AB}\left(\min\{K_B,\ d_A+d_B\}-d_B\right)}{\mu(d_A+d_B)} - \frac{\phi_{BA}d_B}{\mu(d_A+d_B)} - \frac{\phi_{AC}\left(d_A-\left(\min\{K_B,\ d_A+d_B\}-d_B\right)\right)}{\mu(d_A+d_B)}$
c_A^{viii}	$c_A\frac{S}{d_A+d_B} + \frac{h_A(S-d_A-d_B)}{d_A+d_B} - \frac{\phi_{AB}\left(\min\{K_B,\ d_A+d_B\}-d_B\right)}{\mu(d_A+d_B)}$ $- \frac{\phi_{BA}d_B}{\mu(d_A+d_B)} - \frac{\phi_{AC}\left(d_A-\left(\min\{K_B,\ d_A+d_B\}-d_B\right)\right)}{\mu(d_A+d_B)}$
c_B^{i}	$c_B - \frac{\phi_{AB}\left(\min\{K_B,\ d_A+d_B\}-d_B\right)}{\mu\min\{K_B,\ d_A+d_B\}} - \frac{\phi_{BC}d_B}{\mu\min\{K_B,\ d_A+d_B\}}$
c_B^{II}	$c_B - \frac{\phi_{BC}}{\mu}$
c_B^{III}	$c_B - \frac{\phi_{AB}}{\mu}$
c_B^{iv}	$c_B - \frac{\phi_{BA}}{\mu}$

Table 7: Thresholds for the "Large Capacity Leader" case.

Thresholds	Expression
c_A^I	$c_A - \frac{\phi_{AC}}{\mu}$
c_A^{II}	$c_A - \frac{\phi_{BA}}{\mu}$
c_A^{III}	$c_A - \frac{\phi_{BA}[S-d_A]^+}{\mu S} - \frac{\phi_{AC}\min\{d_A,S\}}{\mu S}$
c_A^{IV}	$c_A - \frac{\phi_{AC}}{\mu} - \frac{(\phi_{AB}-\phi_{AC})\Big(\min\{K_B,\ S\}-d_B\Big)}{\mu(S-d_B)}$
c_A^{V}	$c_A - \frac{\phi_{AC}}{\mu} - \frac{(\phi_{AB}-\phi_{AC})\Big[\min\{K_B,\ S\}-d_B\Big]^+}{\mu d_A}$
c_A^{VI}	$c_A - \frac{\phi_{BA}[S-d_A]^+}{\mu d_B} - \frac{\phi_{AC}\Big(\min\{d_A,S\}-(S-d_B)\Big)}{\mu d_B}$
c_A^{VII}	$c_A - \frac{\phi_{AB}\Big[\min\{K_B,\ S\}-d_B\Big]^+}{\mu S} - \frac{\phi_{BA}[S-d_A]^+}{\mu S} - \frac{\phi_{AC}\Big(\min\{d_A,\ S\}-\Big[\min\{K_B,\ S\}-d_B\Big]^+\Big)}{\mu S}$
c_A^{8}	$c_A - \frac{\phi_{BA}(K_A-d_A)}{\mu K_A} - \frac{\phi_{AC}d_A}{\mu K_A}$
c_A^{9}	$c_A - \frac{\phi_{BA}(K_A-d_A)}{\mu\Big(K_A-(S-d_B)\Big)} - \frac{\phi_{AC}\Big(d_A-(S-d_B)\Big)}{\mu\Big(K_A-(S-d_B)\Big)}$
c_A^{10}	$c_A - \frac{\phi_{AB}\Big[\min\{K_B,\ S\}-d_B\Big]^+}{\mu K_A} - \frac{\phi_{BA}(K_A-d_A)}{\mu K_A} - \frac{\phi_{AC}\Big(d_A-\Big[\min\{K_B,\ S\}-d_B\Big]^+\Big)}{\mu K_A}$
c_A^{11}	$c_A\frac{S-(2d_A+d_B)+K_A}{\mu(K_A-d_A)} + h_A\frac{S-(d_A+d_B)}{\mu(K_A-d_A)} - \frac{\phi_{BA}}{\mu}$
c_A^{12}	$c_A\frac{S-(d_A+d_B)+K_A}{K_A} + h_A\frac{S-(d_A+d_B)}{K_A} - \frac{\phi_{BA}(K_A-d_A)}{\mu K_A} - \frac{\phi_{AC}d_A}{\mu K_A}$
c_A^{13}	$c_A - \frac{\phi_{AB}\Big(\min\{K_B,\ d_A+d_B\}-d_B\Big)}{\mu(d_A+d_B)} - \frac{\phi_{AC}\Big(d_A-\Big(\min\{K_B,\ d_A+d_B\}-d_B\Big)\Big)}{\mu(d_A+d_B)}$
c_A^{14}	$c_A - \frac{\phi_{AB}\Big(\min\{K_B,\ d_A+d_B\}-d_B\Big)}{\mu K_A} - \frac{\phi_{BA}(K_A-d_A)}{\mu K_A} - \frac{\phi_{AC}\Big(d_A-\Big(\min\{K_B,\ d_A+d_B\}-d_B\Big)\Big)}{\mu K_A}$
c_A^{15}	$c_A\frac{S-(d_A+d_B)+K_A}{\mu K_A} + h_A\frac{S-(d_A+d_B)}{\mu K_A} - \frac{\phi_{AB}\Big(\min\{K_B,\ d_A+d_B\}-d_B\Big)}{\mu K_A}$ $- \frac{\phi_{BA}(K_A-d_A)}{\mu K_A} - \frac{\phi_{AC}\Big(d_A-\Big(\min\{K_B,\ d_A+d_B\}-d_B\Big)\Big)}{\mu K_A}$
c_B^{1}	$c_B - \frac{\phi_{AB}\Big[\min\Big\{\min\{d_A+d_B,S\},\ K_B\Big\}-d_B\Big]^+}{\mu\min\Big\{\min\{d_A+d_B,S\},\ K_B\Big\}} - \frac{\phi_{BC}\min\Big\{d_B,\min\Big\{\min\{d_A+d_B,S\},\ K_B\Big\}\Big\}}{\mu\min\Big\{\min\{d_A+d_B,S\},\ K_B\Big\}}$
c_B^{II}	$c_B - \frac{\phi_{BC}}{\mu}$
c_B^{III}	$c_B - \frac{\phi_{AB}}{\mu}$
c_B^{4}	$c_B - \frac{\phi_{BC}}{\mu} - \frac{(\phi_{BA}-\phi_{BC})(S-d_A-d_B)}{\mu(S-K_A)}$
c_B^{5}	$c_B - \frac{\phi_{BC}}{\mu} - \frac{(\phi_{BA}-\phi_{BC})(K_A-d_A)}{\mu d_B}$

Table 8: Thresholds for the "Small Capacity Leader" case.

which implies that the optimal solution of $f(x_A|x_B)$ is a maximum.

Similarly, we can show that, for any given Firm A's preparedness effort x_A^o, function $\Pi_{B,A}^{AP}$ has a unique maximum, $\hat{x}_B$. Consider,

$$\begin{aligned} f(x_B|x_A^o) \equiv \Pi_{B,A}^{AP} &= p_{B0,A}\big(\pi_B\Pi_{B,L}^* + \pi_A\Pi_{B,F}^* - m_B(d_B^0 - d_B)\big) + (1-p_{B0,A})r_B d_B^0(\mu+T) \\ &\quad -C_e x_B + m_B d_B^0 \\ &= p_{B0,A}\Pi_{B,L}^*\Big(\frac{x_B}{x_A^o + x_B}\Big) + p_{B0,A}\Pi_{B,F}^*\Big(\frac{x_A^o}{x_A^o + x_B}\Big) - C_e x_B + constant, \end{aligned} \tag{24}$$

and,

$$\hat{x}_B = \sqrt{\frac{p_{B0,A}(\Pi_{B,L}^* - \Pi_{B,F}^*)}{C_e}x_A^o} - x_A^o. \tag{25}$$

We now show that function $x_A(x_B)$ is continuous in x_B and strictly concave in x_B. Letting $H = \sqrt{\frac{C_L^A - C_F^A}{C_e}}$, which is a non-negative constant, from equation (23) we obtain

$$x_A(x_B) = H \times \sqrt{x_B} - x_B + constant.$$

First of all, it is easy to see that the above expression of $x_A(x_B)$ is continuous by the form of its function. Secondly, it is strictly concave because

$$\frac{d^2}{dx_B^2}x_A(x_B) = -\frac{1}{4}H \times (x_B)^{-3/2} < 0$$

is satisfied. Similarly, $x_B(x_A)$ is continuous and strictly concave.

Having the above results, we now show that Nash Equilibrium exists and is unique. Since we have already shown that $x_A(x_B)$ is strictly concave, to prove that the Nash Equilibrium exists, we need to show that at the very beginning of the range of $x_B \in (0,\infty)$, we must have $\frac{d}{dx_B}x_A(x_B) > 1$ (Rudin 1976). Intuitively, this condition guarantees $x_A(x_B)$ will intersect with the line, $x_A = x_B$, and the similar condition of $x_B(x_A)$ will guarantee its intersection with $x_A = x_B$. Therefore, a unique Nash Equilibrium does exist.

We now check whether there exists a x_B, which is within the very beginning of the range $(0,\infty)$, such that $\frac{H}{2}(x_B)^{-1/2} - 1 > 1$.

$$\begin{aligned} \frac{d}{dx_B}x_A(x_B) = \frac{H}{2}(x_B)^{-1/2} - 1 &> 1 \\ \Leftrightarrow\ x_B^{1/2} &< \tfrac{H}{4} \\ \Leftrightarrow\ x_B &< \tfrac{C_L^A - C_F^A}{16C_e}, \end{aligned}$$

which obviously can be easily satisfied by some x_B within the very beginning of the range $(0,\infty)$, since the right-hand-side is just a positive constant. Similar arguments hold for $x_B(x_A)$. This completes the proof of Proposition 3. □

PROPOSITION 4: *Under complete information, given a fixed Firm j's preparedness effect, the optimal preparedness effort of Firm i (i.e., x_i) is nondecreasing in m_i, m_j, and r_i, and is nonincreasing in γ_{ij}, γ_{iC}, γ_{ji}, c_i, and h_i, for $i = A, B$ and $j \neq i$, $j = A, B$.*

Proof: In the proof of Proposition 3, we have already shown that, for any given Firm B's preparedness effort x_B^o, Firm A's optimal preparedness effort is

$$\hat{x}_A = \sqrt{\frac{C_L^A - C_F^A}{C_e} x_B^o} - x_B^o.$$

Thus, to show that $\hat{x}_A$ is nondecreasing (or nonincreasing) in say parameter α, we only need to show that $C_L^A - C_F^A$ is nondecreasing (or nonincreasing) in α. According to equations (1) and (2), under complete information, we have

$$\begin{aligned} C_L^A \;=\; & p_o\Big(r_A \min\{Y_{A,L}^*, d_A\}\mu + \psi_A \min\Big\{[d_B - Y_{B,F}^*]^+, \min\{[Y_{A,L}^* - d_A]^+, K_A - d_A\}\Big\} \\ & - m_A \xi_{AB} \min\Big\{\min\{[Y_{B,F}^* - d_B]^+, K_B - d_B\}, [d_A - Y_{A,L}^*]^+\Big\} \\ & - m_A \xi_{AC}\Big[[d_A - Y_{A,L}]^+ - \min\{[Y_{B,F}^* - d_B]^+, K_B - d_B\}\Big]^+ \\ & - c_A Y_{A,L}^* \mu - h_A(\text{some non-negative terms})\Big), \end{aligned}$$

and

$$\begin{aligned} C_F^A \;=\; & p_o\Big(r_A \min\{Y_{A,F}^*, d_A\}\mu + \psi_A \min\Big\{[d_B - Y_{B,L}^*]^+, \min\{[Y_{A,F}^* - d_A]^+, K_A - d_A\}\Big\} \\ & - m_A \xi_{AB} \min\Big\{\min\{[Y_{B,L}^* - d_B]^+, K_B - d_B\}, [d_A - Y_{A,F}^*]^+\Big\} \\ & - m_A \xi_{AC}\Big[[d_A - Y_{A,F}^*]^+ - \min\{[Y_{B,L}^* - d_B]^+, K_B - d_B\}\Big]^+ \\ & - c_A Y_{A,F}^* \mu\Big). \end{aligned}$$

Therefore,

$$\begin{aligned} C_L^A - C_F^A \;=\; & p_o\Big(r_A\big(\min\{Y_{A,L}^*, d_A\} - \min\{Y_{A,F}^*, d_A\}\big)\mu \\ & - c_A(Y_{A,L}^* - Y_{A,F}^*)\mu - h_A(\text{some non-negative terms}) \\ & + \psi_A \min\Big\{[d_B - Y_{B,F}^*]^+, \min\{[Y_{A,L}^* - d_A]^+, K_A - d_A\}\Big\} \\ & - \psi_A \min\Big\{[d_B - Y_{B,L}^*]^+, \min\{[Y_{A,F}^* - d_A]^+, K_A - d_A\}\Big\} \\ & - m_A \xi_{AB} \min\Big\{\min\{[Y_{B,F}^* - d_B]^+, K_B - d_B\}, [d_A - Y_{A,L}^*]^+\Big\} \\ & + m_A \xi_{AB} \min\Big\{\min\{[Y_{B,L}^* - d_B]^+, K_B - d_B\}, [d_A - Y_{A,F}^*]^+\Big\} \\ & - m_A \xi_{AC}\Big[[d_A - Y_{A,L}^*]^+ - \min\{[Y_{B,F}^* - d_B]^+, K_B - d_B\}\Big]^+ \\ & + m_A \xi_{AC}\Big[[d_A - Y_{A,F}^*]^+ - \min\{[Y_{B,L}^* - d_B]^+, K_B - d_B\}\Big]^+\Big). \end{aligned} \tag{26}$$

It is easy to see that for Firm i, the purchasing amount when the firm is the Leader is at least as many as that when the firm is the Follower. The reason is the when the firm is the Leader it has access to a larger backup capacity than that when it is the Follower. Thus, $Y_{A,L}^* \geq Y_{A,F}^*$ and $Y_{B,L}^* \geq Y_{B,F}^*$. We have

- With respect to the coefficient of ψ_A, i.e., the third and the fourth lines on the right-hand-side of equation (26), we have $[d_B - Y_{B,F}^*]^+ \geq [d_B - Y_{B,L}^*]^+$ and $[Y_{A,L}^* - d_A]^+ \geq [Y_{A,F}^* - d_A]^+$, so

$$\min\Big\{[d_B - Y_{B,F}^*]^+, \min\{[Y_{A,L}^* - d_A]^+, K_A - d_A\}\Big\} - \min\Big\{[d_B - Y_{B,L}^*]^+, \min\{[Y_{A,F}^* - d_A]^+, K_A - d_A\}\Big\} \geq 0,$$

i.e., the coefficient of ψ_A in equation (26) is non-negative. Furthermore, we know

$$\begin{aligned}\psi_A &= r_A\mu + m_B\xi_{BA}\\ &= r_A\mu + m_B\int_0^\infty (1-e^{-\frac{t}{\gamma_{BA}}})dF_o(t).\end{aligned}$$

So, equation (26) (and therefore $\hat{x}_A$) is nondecreasing in m_B and nonincreasing in γ_{BA}. Furthermore, the summation of the third and the fourth lines on the right-hand-side of equation (26) is also nondecreasing in r_A.

- With respect to the coefficient of r_A in the first line on the right-hand-side of equation (26), we have

$$\min\{Y^*_{A,L}, d_A\} - \min\{Y^*_{A,F}, d_A\} \geq 0$$

since $Y^*_{A,L} \geq Y^*_{A,F}$. Therefore, we can conclude that the coefficient of r_A in the first line on the right-hand-side of equation (26) is non-negative, and therefore equation (26) (and therefore $\hat{x}_A$) is nondecreasing in r_A.

- With respect to the coefficient of $m_A\xi_{AB}$, i.e., the fifth and the sixth lines on the right-hand-side of equation (26), we know $[Y^*_{B,L}-d_B]^+ \geq [Y^*_{B,F}-d_B]^+$ and $[d_A-Y^*_{A,F}]^+ \geq [d_A-Y^*_{A,L}]^+$, so the coefficient of $m_A\xi_{AB}$ in equation (26) is non-negative. So, equation (26) (and therefore $\hat{x}_A$) is nonincreasing in γ_{AB}. Furthermore, the coefficient of m_A (i.e., the fifth and the sixth lines in on the right-hand-side of equation (26)) is non-negative. Therefore, that part of equation (26) is non-decreasing in m_A.

- With respect to the other coefficient of $m_A\xi_{AC}$, i.e., the last two lines on the right-hand-side of in equation (26), we will show that

$$\begin{aligned}&\Big[[d_A - Y^*_{A,L}]^+ - \min\big\{[Y^*_{B,F} - d_B]^+, K_B - d_B\big\}\Big]^+\\ &\quad - \Big[[d_A - Y^*_{A,F}]^+ - \min\big\{[Y^*_{B,L} - d_B]^+, K_B - d_B\big\}\Big]^+ \leq 0.\end{aligned} \tag{27}$$

It is easy to see that,

  - if $[d_A - Y^*_{A,L}]^+ = 0$, then equation (27) is smaller than or equal to zero. This is because $\delta \equiv \min\big\{[Y^*_{B,F} - d_B]^+, K_B - d_B\big\} \geq 0$ since both $[Y^*_{B,F} - d_B]^+ \geq 0$ and $K_B - d_B \geq 0$. Therefore, the first term of equation (27) becomes $[0-\delta]^+ = 0$. Furthermore, the second term of equation (27) is non-negative.
  - if $d_A - Y^*_{A,L} > 0$, then by Proposition 1, we know, $Y^*_{A,L} \in \{0,\ S - d_B\}$. Hence,
    - if $Y^*_{A,L} = 0$, then $Y^*_{A,F} = 0$ and $Y^*_{B,L} = Y^*_{B,F}$ must hold. This is because $Y^*_{A,F} \leq Y^*_{A,L}$ always holds, so $Y^*_{A,L} = 0$ implies that $Y^*_{A,F} = 0$. Thus, Firm A always purchases nothing, i.e., Firm A's position in the BC Competition has no effect on Firm B's purchase decision, and hence $Y^*_{B,L} = Y^*_{B,F}$. As a result, equation (27) is equal to zero.
    - if $Y^*_{A,L} = S - d_B$, then it is easy to see that, $Y^*_{B,L} = \min\{K_B, S\}$ and $Y^*_{B,F} \in \{0,\ d_B\}$ must hold. Because $Y^*_{A,L} = S - d_B$ means that to be the Leader Firm A protects, which by Proposition 2 implies that Firm B must be aggressive to steal Firm A's customers. So $Y^*_{B,L} = \min\{K_B, S\}$, and $Y^*_{B,F} \in \{0,\ d_B\}$ can be obtained directly from Proposition 1. As a result, equation (27) becomes

$$\begin{aligned}&\big(d_A - (S - d_B)\big) - \Big(d_A - \min\{S - d_B, K_B - d_B\}\Big)\\ &= \min\{S - d_B, K_B - d_B\} - (S - d_B) \leq 0.\end{aligned}$$

Therefore, equation (27) is non-positive, i.e., the coefficient of $m_A\xi_{AC}$ in equation (26) is non-negative. Consequently, equation (26) (and therefore $\hat{x}_A$) is nonincreasing in γ_{AC}. Furthermore, we can conclude that equation (26) (and therefore $\hat{x}_A$) is nondecreasing in m_A.

Finally, it is clear that equation (26) (and therefore $\hat{x}_A$) is nonincreasing in c_A and h_A. This completes the proof of Proposition 4. □

PROPOSITION 5: *Under interfirm information, if both firms are "aggressive" and are identical except for their size (i.e., $d_A \neq d_B$), then both firms spend the same amount of effort on preparedness (i.e., $x_A = x_B$), and therefore will have the same chance (50%) to become the Leader in the BC Competition.*

Proof: We know that the probability for Firm i to be the Leader in the BC Competition is,

$$\pi_i^* = \frac{x_i^*}{x_i^* + w_j^*}, \quad \pi_j^* = 1 - \pi_i^*,$$

where x_i^* is the preparedness effort Firm i spends in the AP Competition and w_j^* is the preparedness effort Firm j spends in the AP Competition.

Based on the proof of Proposition 3, we have the expression of x_A^* from equations (23) and (25). Recall that x_i^* is Firm A's actual preparedness effort Firm i given it believes Firm j's preparedness effort is x_j^*. Plugging the unique Nash Equilibrium for the AP Competition for Firm i, i.e., (x_A^*, x_B^*), into equations (23) and (25), under interfirm information, we have

$$x_A^* = \sqrt{\frac{p_{A0}(\Pi_{A,L}^* - \Pi_{A,F}^*)}{C_e} x_B^*} - x_B^*,$$

and

$$x_B^* = \sqrt{\frac{p_{A0}(\Pi_{B,L}^* - \Pi_{B,F}^*)}{C_e} x_A^*} - x_A^*.$$

Note that *interfirm information* indicates $p_{i0,j} = p_{i0}, i, j = A, B$, and *identical firms* implies $p_{A0} = p_{B0}$.

We can write out the preparedness investment by Firm A in the AP Competition, x_A^*, as follows:

$$x_A^* = \frac{p_{A0}}{C_e} \frac{(\Delta\Pi_A^*)^2 \Delta\Pi_B^*}{(\Delta\Pi_A^* + \Delta\Pi_B^*)^2}, \tag{28}$$

where $\Delta\Pi_i^* = \Pi_{i,L}^* - \Pi_{i,F}^*, i = A, B$.

Similarly, we obtain w_B^* based on equations (23) and (25) as follows:

$$w_B^* = \frac{p_{A0}}{C_e} \frac{(\Delta\Pi_B^*)^2 \Delta\Pi_A^*}{(\Delta\Pi_B^* + \Delta\Pi_A^*)^2}. \tag{29}$$

Now, by the Equations (28) and (29), the probability for Firm i to be the Leader in the BC Competition becomes,

$$\pi_i^* = \frac{x_i^*}{x_i^* + w_j^*} = \frac{\Delta\Pi_i^*}{\Delta\Pi_i^* + \Delta\Pi_j^*}.$$

Therefore, to show $\pi_i^* = 50\%$, we only need to show that $\Delta\Pi_i^* = \Delta\Pi_j^*$.

In the following, since two firms are identical except the firm size, we omit the subscript of all parameters except the firm size that indicates Firm i or Firm j (e.g., $K \equiv K_A = K_B$). Also, without loss of generality, we assume $d_A > d_B$.

When $d_A > d_B$, based on the relationship among d_A, d_B, K, and $d_A + d_B$ we have the following two different cases: $d_B \leq d_A \leq K \leq d_A + d_B$ and $d_B \leq d_A \leq d_A + d_B \leq K$. Furthermore, under each case, we also need to discuss the relationship among the backup capacity S, firms' sales, the total market sales, and their production capacities, so for each case, we consider five mutually exclusive and collectively exhaustive sub-cases:

- **Case 1:** $d_B \le d_A \le K \le d_A + d_B$:
  - $S \le d_B$;
  - $d_B \le S \le d_A$;
  - $d_A \le S \le K$;
  - $K \le S \le d_A + d_B$;
  - $d_A + d_B \le S$.
- **Case 2:** $d_B \le d_A \le d_A + d_B \le K$:
  - $S \le d_B$;
  - $d_B \le S \le d_A$;
  - $d_A \le S \le d_A + d_B$;
  - $d_A + d_B \le S \le K$;
  - $K \le S$.

Since the proof procedure is similar, we will just give one detailed example. In the following, we show that the proposition hold under the situation where $d_B \le d_A \le K \le S \le d_A + d_B$.

When $K \le S \le d_A + d_B$, based on Proposition 1, since both firms are "aggressive", i.e., given the chance, a firm will buy all available backup capacity in an attempt to steal sales from the competitor, it is easy to see that the only two possible optimal purchases combinations are:

$$(Y^*_{i,L},\ Y^*_{j,F}; Y^*_{j,L},\ Y^*_{i,F}) \quad \in \quad \Big\{(K, S-K; K, S-K),\ (K, 0; K, 0)\Big\}.$$

- If $(Y^*_{i,L},\ Y^*_{j,F}; Y^*_{j,L},\ Y^*_{i,F}) = (K, S-K; K, S-K)$, then we have,

$$\begin{aligned}\Delta\Pi^*_i &= r\mu\big(d_i - (S-K)\big) + (r\mu + m\xi)(K - d_i) + m\xi(K - d_j) + m\xi_C\big((d_i + d_j) - S\big) \\ &\quad -c\mu\big(K - (S-K)\big), \\ &= (r-c)\mu(2K - S) + m\xi\big(2K - (d_i + d_j)\big) + m\xi_C\big((d_i + d_j) - S\big),\end{aligned}$$

  which clearly depends on the total duopoly market size, $d_i + d_j$.

- If $(Y^*_{i,L},\ Y^*_{j,F}; Y^*_{j,L},\ Y^*_{i,F}) = (K, 0; K, 0)$, then we have,

$$\Delta\Pi^*_i \quad = \quad (r-c)\mu K + m\xi\big(2K - (d_i + d_j)\big) + m\xi_C\big((d_i + d_j) - K\big),$$

  which clearly depends on the total duopoly market size, $d_i + d_j$.

So, when $d_B \le d_A \le K \le S \le d_A + d_B$, $\Delta\Pi^*_i = \Delta\Pi^*_j$ holds, and hence each firm has a probability of 50% to win the BC Competition. This completes the proof for this case. The proofs of other cases are similar, and therefore omitted. □