Ira Gerhardt: Research Statement

Many real-world systems can be modeled as queueing networks; candidates for this include manufacturing and transportation systems, call centers, emergency rooms, and internet traffic. However, queueing models designed to analyze the behavior of these systems typically utilize assumptions, such as independent arrivals or interarrival and service time distributions with time-stationary parameters, that may be inappropriate. Studies of internet protocol traffic indicate that connection rates are affected by time of day and day of week, while interconnection times typically are not mutually independent, as connections often occur in bursts. Failing to account for nonstationarity (e.g., by using only the average arrival rate over a day) may lead to underestimation of key performance measures due to unidentified system congestion. Therefore, utilizing nonstationary models may yield more accurate results. This motivation has led me to focus on developing methods to study the behavior of nonstationary queueing networks, through both numerical methods and simulation.

Typically, queueing models utilize stationary processes and provide techniques to calculate long-run performance measures such as the limiting queue-length and wait-time distributions (and their respective moments, such as mean and variance) as time extends to infinity. In nonstationary queueing models, such long-run performance measures are not applicable; instead, transient results must be provided in the form of time-dependent performance measures.

In my research, I utilize numerically tractable nodal models with nonstationary arrival and service processes that are Markovian, in the sense that interarrival and service times are characterized by the time to absorption of a finite-state continuous-time Markov chain. This typically makes it possible to derive a finite system of Moment Differential Equations (MDEs) that describe the instantaneous rate of change of the time-dependent moments of the node size (i.e., the number of entities at the node) in terms of the moments themselves. My research involves extending the MDE approach for nonstationary queueing nodes to nonstationary queueing networks. I derive a finite system of Departure-moment Differential Equations (DDEs) to calculate time-dependent moments of the departure counting process from each node in the network. Since the departure process from one node in a queueing network may serve as the arrival process to another node (either directly or through superposition or splitting), the departure count moments at one node may yield information useful in calculating the arrival count moments at another.

Notice that the departure process from a nonstationary queueing node typically is non-Markovian. This implies that arrival processes to nodes within the network may be non-Markovian as well, and the resulting nodal models may be numerically intractable. I provide techniques to specify a Markovian process with the appropriate count moments to approximate the true arrival process at each node within the network. This yields tractable nodal models from which approximations for the time-dependent moments of the respective node size may be calculated. Simulation may be used to assess the accuracy of the approximated nodal moments.

The simulation of nonstationary processes is another research focus of mine, with an emphasis on generating (through simulation) nonstationary arrival processes that are either more or less variable than Poisson. At present, there are no well-known techniques for accomplishing this, while several techniques exist for generating a Non-Stationary Poisson Process (NSPP) (i.e., a Poisson process with non-constant arrival rate). One approach I have investigated involves extending techniques that generate a NSPP by transforming a stationary Poisson process, to generate Non-Stationary, Non-Poisson (NSNP) arrivals by transforming a stationary renewal process (i.e., a generalized Poisson process where the independent and identically distributed interarrival times may not be exponentially distributed). These extended techniques are successful in that they preserve through transformation some measure of the “non-Poissonness” of the base process (i.e., if the stationary
renewal process is more or less variable than Poisson, then the generated NSNP process is more or less variable than a NSPP, respectively).

The numerical methods and simulation techniques described in this statement are only a few of the possible tools that I would like to develop in order to analyze nonstationary queueing networks. The applicability of such networks in modeling many real-world systems ensures that future directions for this research abound. One of my goals is to provide a comprehensive technique for specifying a nonstationary Markovian process to capture both the moments and the dependence in the arrival process at each node in a nonstationary network. Future projects for simulating nonstationary processes include developing techniques for transforming a stationary nonrenewal process (i.e., where the interarrival times are not mutually independent) to target some measure of dependence in the resulting nonstationary process. I am also interested in providing techniques for specifying the base process to be transformed when presented properties of, or data from, the nonstationary arrival process, as well as statistical tools for validating these techniques.

I believe these future projects present a great opportunity to involve both undergraduate and graduate students in my research. For example, nonstationary queueing models may be useful tools in real-world consulting projects. Students involved in these would gain exposure to actual business practices while applying their knowledge of stochastic models, applied probability, and statistical methods to real-world problems. Taking part in projects in my research areas would be a worthwhile experience for any student interested in mathematics, applied mathematics, statistics, computer science, and operations research, as well as for those students considering careers in these fields.