DECISION ANALYSIS
Vol. 4, No. 1, March 2007, pp. 3–16
ISSN 1545-8490 | EISSN 1545-8504 | 07 | 0401 | 0003

DOI 10.1287/deca.1060.0082
© 2007 INFORMS

# Adding Extrinsic Goals to the Quality-Adjusted Life Year Model

Gordon B. Hazen
Department of Industrial Engineering and Management Sciences, Northwestern University,
Evanston, Illinois 60208-3119, gbh305@lulu.it.northwestern.edu

Methods for evaluating health quality are central to medical decision analyses. The most important such method is the quality-adjusted life year (QALY), in which a patient's length of life is given weight proportional to his/her quality of health. QALYs have become ubiquitous in medical cost-effectiveness as a measure of preference for health outcomes. However, numerous studies have demonstrated that the correlation between measured QALYs and a patient's current health is at best modest. Moreover, it is known that individuals may trade lifetime for improved health quality when remaining lifetime is long, but not when it is short; and those with poor health quality may prefer to survive only until important life milestones and no longer. These behaviors are inconsistent with the QALY model. To address these concerns, we examine methods for including life goals in health preference models. The QALY model already captures *ongoing* goals such as minimizing chronic pain or maintaining physical mobility, goals whose achievement has impact proportional to length of life. However, other goals, termed *extrinsic* goals, such as completing an important project or seeing a child graduate from college, are qualitatively different—their achievement has impact that is independent of length of life, and therefore cannot be captured using QALYs. In this paper, we present a generalization of the QALY model that incorporates both ongoing goals and extrinsic goals. The new model allows the plausible behaviors mentioned above but forbidden by the QALY model.

*Key words*: quality-adjusted life years; quality of life; maximum endurable time; time trade-offs; the zero condition; generalized utility independence; expected utility; medical decision analysis
*History*: Received on June 28, 2006. Accepted by Robert T. Clemen and Don N. Kleinmuntz on November 30, 2006, after 2 revisions.

## Introduction

Methods for evaluating health quality are central to medical decision analyses and cost-effectiveness analyses. The most important such method is the quality-adjusted life year (QALY) model, in which a patient's survival duration is given weight proportional to the quality of health the patient experiences. The recommendation of the Panel on Cost Effectiveness in Health and Medicine (Gold et al. 1996) is that medical cost-effectiveness studies should incorporate morbidity and mortality consequences into a single measure using QALYs. QALYs have indeed become ubiquitous in these and other analyses: A Medline search on quality-adjusted life years for the five-year period ending December 2002 produced 1,070 articles, and the identical search for the five-year period 2001–2005 produced 1,639 results.

In practice, QALYs are elicited by subjectively querying patients, physicians, or community members. A variety of methods are available for this purpose (e.g., Gold et al. 1996, Hunink et al. 2001). These include rating scale, time trade-off, standard gamble, and multiattribute health indexes such as the Health Utilities Index (HUI; e.g., Feeny et al. 1995) or the EuroQol (e.g., Dolan 1997).

However, as Tsevat (2000) points out, numerous studies have demonstrated that the correlation between one's current health and the time trade-off or standard gamble utility for that health state is at best modest. Willingness to trade away time or take a gamble is often much less than the general public, health care professionals, and even family members believe. As a partial explanation, Tsevat suggests that a person's willingness or unwillingness to trade away life years or accept a gamble involving life years is more a function of how the person values *quality of life* than *quality of health*.

What is the distinction between quality of life and quality of health? Health quality is the familiar component of medical decision analyses captured by

QALYs and reflecting direct impacts on health. Quality of life, however, is a much more nebulous concept that includes life states that are unrelated to health as well as those that are related to health. For this reason, it is usually not treated explicitly in medical decision analyses.

What issues are potentially relevant to quality of life? It is a truism to state that striving to attain goals gives purpose to life. We focus here on goals that arise in contexts such as the following:

- an author might want to complete a book
- an athlete might want to play on a championship team
- an artist might struggle to complete a major work
- an engineer or architect might endeavor to see a project to completion
- a politician might strive to achieve higher office
- many individuals seek to have children and raise families
- individuals seek the financial and social welfare of their families

Like these quality-of-life issues, issues related to quality of *health* can also be recast in terms of goals. For example, mobility, chronic pain, and emotional stress all affect quality of health, and the corresponding goals are to increase mobility, to reduce pain, to decrease emotional stress. We call these *ongoing goals*, and distinguish them from the goals just listed by noting that the importance of achieving ongoing health-related goals is *modulated by life duration*. The overall impact of pain, stress, or lack of mobility depends on the duration for which these are endured. In contrast, for goals such as those listed above, which we term *extrinsic goals*, the level of goal achievement has importance that is *unrelated to life duration*. We take this as the defining distinction between extrinsic and ongoing goals. In a recent telephone survey by Schwartz et al. (2007), participants revealed 232 extrinsic goals involving education, family, health and fitness, personal fulfillment, professional issues, travel, and wealth.

Note that it is the quality-of-life *impact* of extrinsic goal achievement that is not time modulated. In contrast, it may well be that an individual's *ability to achieve* an extrinsic goal is heavily time modulated—certainly the achievement of the extrinsic goals listed above requires significant time commitments. The value an individual assigns to life duration may therefore be due not only to life duration per se, but also to the associated ability to achieve extrinsic goals, goals whose *importance* is independent of life duration—a subtle but significant distinction on which we will comment further below.

In the QALY model, quality of health is given weight proportional to health duration. It follows that the QALY model cannot directly account for extrinsic goals, whose importance is by definition independent of duration. In QALY assessment, the presence of extrinsic goals might account for subjects' lack of willingness to trade away time or take a gamble that might shorten life (Miyamoto and Eraker 1988): Why, for example, should an author trade away or risk time she needs to complete an important work, for health quality improvements that do not affect her ability to write? Moreover, extrinsic-goal issues may impact not only simple utility assessments, but also the value structure appropriate for decision and cost-effectiveness analyses. For instance, prophylactic oophorectomy (surgical removal of the ovaries) is an option for women at high risk for ovarian cancer; but this course of action negatively impacts the extrinsic goal of bearing children. This extrinsic goal is difficult or impossible to model adequately under the QALY format.

Our thesis is, therefore, that when there are multiple objectives involving quality of health *and* extrinsic goals, the conventional QALY model is inadequate. In what follows, we describe mathematically how this occurs and specify an alternate multiattribute utility function that is capable of surmounting these difficulties.

## Assumptions Underlying the QALY Model

The QALY model assigns utility $U(q, t)$ to a duration-$t$ sojourn in health state $q$, where

$$U(q, t) = U_Q(q)U_T(t). \tag{1}$$

Here, $U_Q(q)$ is the utility assigned to health state $q$, usually normalized so that $U_Q(q^*) = 1$ for full health $q^*$; and $U_T(t)$ is the utility assigned to a duration-$t$ sojourn in $q^*$. It is often assumed that $U_T(t)$

is discounted future time, that is, if the discount rate is $r \geq 0$, then

$$U_T(t) = \int_0^t e^{-rs}\,ds = \begin{cases} t & \text{if } r = 0 \\ \frac{1}{r}(1 - e^{-rt}) & \text{if } r > 0. \end{cases} \tag{2}$$

Assumptions on preferences under which von Neuman-Morgenstern utility $U(q, t)$ takes the QALY form (1) are given by Pliskin et al. (1980), Miyamoto et al. (1998), and Miyamoto (1999). The following assumptions are closely related to those in the latter two references, and are particularly simple.

Assumption A1 (The Zero Condition). *Strict preference between states of health disappears when survival duration is zero, that is, for all states $q$, $q'$ of health*:

$$(q, t_0) \sim (q', t_0).$$

Here we denote zero survival duration by $t_0$, and "$\sim$" denotes indifference. In the sequel, "$\succeq$" will denote weak preference (strict preference or indifference).

Assumption A2 (Generalized Utility Independence (GUI) for Lifetime). *If $\succeq_q$ is a conditional preference relation over lifetime gambles $\tilde{t}$ when the health state is $q$, then for any two states $q$, $q'$ not equivalent to death, the relations $\succeq_q$ and $\succeq_{q'}$ are either identical or reversed*.

Here health state $q$ is *equivalent to death* if $(q, t) \sim (q, t_0)$ for all survival durations $t$.

Assumptions A1 and A2 guarantee the QALY representation (1) for von Neumann-Morgenstern utility $U$. Generalized utility independence was first introduced by Fishburn and Keeney (1975). In place of the GUI Assumption A2, Miyamoto et al. (1998) use an apparently weaker assumption called *standard gamble independence*. The latter actually implies GUI in this context. We use GUI assumption here, as its implications are more explicit.

## Utility Models for Extrinsic Goals and Quality of Life

In this section we present plausible utility models that are sensitive not only to quality and quantity of life, but also to some measure of extrinsic goal achievement. We assume that extrinsic goal achievement can be captured by some attribute $G$ ranging from no achievement to full achievement. For some extrinsic goals, it may be that these are the only possible levels of achievement (e.g., a politician either achieves higher office or does not), but in general we allow for intermediate levels as well.

We seek a von Neumann-Morgenstern utility function $U(g, q, t)$, where $g$ is some level of an extrinsic goal achievement attribute $G$; $q$ is some level of a health quality attribute $Q$; and $t$ is life duration, a level of the life duration attribute $T$. What assumptions on preference might be plausible over these three attributes?

Consider first the zero condition (Assumption A1 above). Although it seems evident that strict preference over health quality should disappear when life duration is zero, this fails for extrinsic goals by their very definition. We account for this by formulating a *conditional zero condition*, as follows.

Assumption B1 (Conditional Zero Condition). *For each particular level $g$ of extrinsic goal achievement, strict preference for health quality disappears when life duration is zero, that is, for all goal achievement levels $g$ and all health states $q$, $q'$*,

$$(g, q, t_0) \sim (g, q', t_0).$$

Again, $t_0$ denotes zero survival duration. Consider next the generalized utility independence Condition A2. It seems natural to extend this in the following way:

Assumption B2 (Generalized Utility Independence (GUI) of $T$ from $(G, Q)$). *If $\succeq_{g,q}$ is a conditional preference relation over lifetime gambles $\tilde{t}$ when goal achievement is $g$ and health status is $q$, then for any two health states $q$, $q'$ not equivalent to death, and any two levels $g$, $g'$ of goal achievement, the relations $\succeq_{g,q}$ and $\succeq_{g',q'}$ are either identical or reversed*.

Next, some assumptions about how extrinsic goal attainment interacts with health quality and survival would be useful. One analytically convenient assumption is the following.

Assumption B3 (Marginality Between $G$ and $(Q, T)$). *Preference over gambles $(\tilde{g}, \tilde{q}, \tilde{t})$ depends only on the marginal distribution of $\tilde{g}$ and of $(\tilde{q}, \tilde{t})$, and not on their joint distribution. That is, for any $(\tilde{g}_1, \tilde{q}_1, \tilde{t}_1)$ and*

$(\tilde{g}_2, \tilde{q}_2, \tilde{t}_2)$, *if* $\tilde{g}_1$ *and* $\tilde{g}_2$ *have the same probability distribution, and* $(\tilde{q}_1, \tilde{t}_1)$ *and* $(\tilde{q}_2, \tilde{t}_2)$ *have the same joint probability distribution, then*

$$(\tilde{g}_1, \tilde{q}_1, \tilde{t}_1) \sim (\tilde{g}_2, \tilde{q}_2, \tilde{t}_2)$$

*regardless of the joint distribution of either* $(\tilde{g}_1, \tilde{q}_1, \tilde{t}_1)$ *or* $(\tilde{g}_2, \tilde{q}_2, \tilde{t}_2)$.

Here we treat gambles $\tilde{g}$, $\tilde{q}$, $\tilde{t}$ as random variables, that is, functions on a shared outcome space $\Omega$. Therefore, writing $(\tilde{g}, \tilde{q}, \tilde{t})$ does not necessarily denote three independent gambles.

It is possible to imagine situations in which B3 fails, e.g., a patient might not care about health quality if extrinsic goals are unachieved. Nevertheless, B3 may be accurate or a useful approximation in many situations. Here we regard it as a convenient assumption to be refined by further research. Marginality was first introduced by Fishburn (1965), and is discussed by Keeney and Raiffa (1976) under the name *additive independence*.

These three assumptions are sufficient to give a reasonably tractable form for the utility function $U(g, q, t)$.

Theorem 1. *Suppose there is a von Neumann-Morgenstern utility function* $U(g, q, t)$ *whose expectation represents preference over gambles on* $G \times Q \times T$, *and there are levels* $g_0, g^* \in G$, *a level* $q^* \in Q$, *and levels* $t_0, t^* \in T$ *such that*

$$g^* \succ g_0 \quad \text{given } Q = q^*, \ \ T = t^*,$$
$$t^* \succ t_0 \quad \text{given } Q = q^*, \ \ G = g^*.$$

Then, the conjunction of the conditions:

Condition B1. The conditional zero-condition;

Condition B2. $T$ is generalized utility independent of $(Q, G)$;

Condition B3. Marginality between $G$ and $(Q, T)$ is equivalent to the existence of utility functions $U_T(t)$, $U_Q(q)$, $U_G(g)$, and a weight $k_G > 0$ such that

$$U(g, q, t) \sim U_Q(q)U_T(t) + k_G U_G(g) \tag{3}$$

where

$$U_G(g_0) = 0 \qquad U_G(g^*) = 1$$
$$U_Q(q^*) = 1$$
$$U_T(t_0) = 0 \qquad U_T(t^*) = 1.$$

The proof of this result is given in the appendix. The symbol "$\sim$" in (3) denotes equivalence up to positive linear transformation. The utility function (3) consists of the usual QALY form $U_Q(q)U_T(t)$ plus a term $U_G(g)$ representing extrinsic goal achievement. This is a simple representation, but one advantage of deriving it from Assumptions B1 to B3, instead of simply postulating it, is that we know then that the representation is reasonable (if we believe the assumptions are reasonable). Moreover, we can be reassured that no form consistent with the assumptions and more general than (3) exists. The representation (3) is a good candidate for practical use, as it is simple enough to allow reasonably straightforward utility assessment (which we will discuss below) and is computationally tractable. For convenience, we will sometimes rewrite (3) in the equivalent form

$$U(g, q, t) \sim U_Q(q)U_T(t) - k_G(1 - U_G(g)), \tag{4}$$

in which utility reduces directly to the QALY model when the goal $g = g^*$ is achieved.

Although we have taken the domain of the utility function $U(g, q, t)$ to be the Cartesian product $G \times Q \times T$, it is quite possible that for a particular decision problem, the set of possible outcomes is not all of $G \times Q \times T$. Essentially, this is because goal achievement may interact with either health quality or survival.

For instance, if an athlete's extrinsic goal is to play on a championship team, then for an athlete who has no immediate prospect of doing so, the combinations $(g = \text{Success}, q, t = 0)$ or $(g = \text{Success}, q = \text{Disabled}, t)$ are infeasible, as immediate death or permanent disability would preclude athletic participation of any kind. Nevertheless, one may still consider these combinations as hypothetical scenarios in which the goal has *already* been achieved and is followed by immediate death or permanent disability. Including such hypothetical scenarios may facilitate utility assessment (see below).

We are unaware of examples in which $(g, q, t)$ combinations are *logically* infeasible, that is, infeasible even as hypothetical outcomes. If such examples exist and are important, follow-up research would be needed to derive utility representations over subsets of $G \times Q \times T$.

## A Survival-Target Proxy for Extrinsic Goal Achievement

As we have noted, the importance of extrinsic goal achievement is, by definition, duration independent, but the achievement of such goals may nevertheless require time commitment. When the goal achievement $g$ is binary (either the goal is achieved or it is not), one natural proxy attribute for $g$ is $s_G =$ *whether* (*yes or no*) *survival time is sufficient for goal achievement.* (See Keeney and Raiffa 1976, pp. 55–63, for a discussion of proxy attributes.) Suppose there is an estimate $t_G$ for the required time commitment. Then for a simple health profile $h = (q, t)$, we can express $s_G$ as

$$s_G = [t \geq t_G] = \begin{cases} 1 & \text{if } t \geq t_G \\ 0 & \text{if } t < t_G. \end{cases} \tag{5}$$

We call this the *survival-target* proxy and $t_G$ the *survival target*. The survival target $t_G$ may, of course, be uncertain, in which case it would be enough to provide a probability distribution for $t_G$. If $t_G$ is uncertain, then for any simple health profile $h = (q, t)$, the corresponding level of goal achievement $g$ is uncertain. We assume here that given $h$, the uncertainty in $g$ is *entirely* due to the uncertainty in $t_G$, that is, $g = g^*$ when $t \geq t_G$ and $g = g_0$ when $t < t_G$. It follows that $s_G$ is perfectly predictive of $g$. Therefore, if $F_G(t)$ is the distribution function of $t_G$, the goal achievement utility induced by $h$ is

$$\begin{aligned} \mathrm{E}[U_G(g) \mid h] &= \mathrm{E}[U_G(g) \mid s_G = 1, h] F_G(t) \\ &\quad + \mathrm{E}[U_G(g) \mid s_G = 1, h](1 - F_G(t)) \\ &= 1 \cdot F_G(t) + 0 \cdot (1 - F_G(t)) \\ &= F_G(t). \end{aligned} \tag{6}$$

Here we have assumed that $t_G$ is probabilistically independent of $h$.

Distinguishing the survival-target proxy $s_G$ from the underlying extrinsic goal $G$ is conceptually useful. The survival target $s_G$ would be employed in any decision analysis via its link to survival duration. We also use it predictively below to illustrate how gaps in the QALY model may be accounted for. However, utility assessment would take place on the more fundamental underlying goal $G$. We comment further on assessment below.

## Filling Extrinsic-Goal Gaps in the QALY Model

It is well known that the QALY model fails to account for certain types of persistent preference behavior. We discuss these next, and illustrate how adding an extrinsic goal achievement attribute to the utility function can account for these types of preference behavior.

### Maximum Endurable Time

The phenomenon of *maximum endurable time* was discovered by Sutherland et al. (1982), and has also been discussed by Dolan (1996), Miyamoto et al. (1998), Stalmeier et al. (2001), and Dolan and Stalmeier (2003). The phenomenon occurs when subjects indicate they can tolerate no more than a particular time in an undesirable health state, beyond which each additional increment of time decreases overall utility. As an example, Miyamoto et al. relate an instance of a patient who regarded his health state as almost intolerable, but who wanted to live at least five more years to see his son graduate from high school. This behavior, in conjunction with the unconditional preference for longer healthy lifetime, is incompatible with the QALY model (1), in which the utility $U_Q(q)U_T(t)$ for survival duration $t$, if strictly increasing for the healthy state $q = q^*$, must either strictly increase, strictly decrease, or remain constant for all health states $q$.

However, maximum endurable time is readily modeled by utility functions such as (3) containing an extrinsic-goal achievement attribute modeled by a survival-target proxy (5). In (3), take for simplicity $U_T(t) = t$, and consider a health profile $h = (q, t)$ in which a state with negative health quality $u_Q = U_Q(q) < 0$ is occupied for a lifetime $t$. Then, using (6), the overall utility induced by $h =$ is

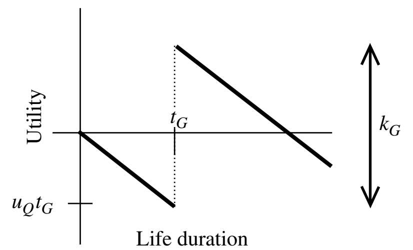
$$\mathrm{E}[U \mid h] = \mathrm{E}[U(g, q, t) \mid h] = u_Q t + k_G \cdot F_G(t). \tag{7}$$

In the simple case in which there is no uncertainty in the survival target $t_G$, we obtain

$$\mathrm{E}[U \mid h] = \begin{cases} u_Q t & \text{if } t \leq t_G \\ u_Q t + k_G & \text{if } t > t_G. \end{cases}$$

In Figure 1 we graph $\mathrm{E}[U \mid h]$ as a function of life duration $t$. If $k_G + u_Q t_G > 0$, as depicted, then the utility-maximizing choice of life duration is $t = t_G$.

**Figure 1** **Health Profile $h$ Occupies a Worse-than-Death Health State Having Quality Coefficient $u_Q < 0$ for Life Duration $t$**




*Notes.* Here we see overall utility as a function of life duration $t$ when there is a survival target $t_G$. Because the health state is worse than death, living longer is always worse, with the exception of the point $t_G$ in time at which the survival goal is achieved and the utility increment $k_G$ is earned. The optimal life duration is $t = t_G$, and $t_G$ is the maximum endurable time in this health state.

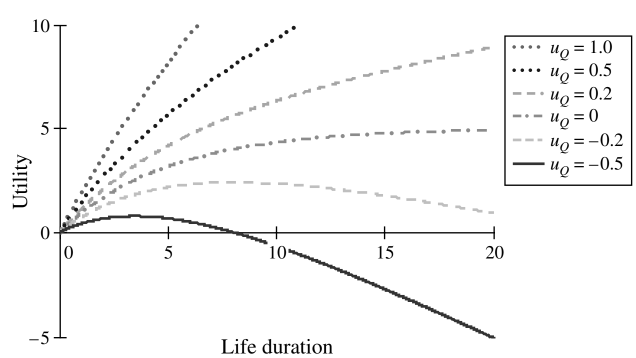
Therefore, at any time $t$ prior to $t_G$, an individual with this utility function would prefer to keep living long enough to reach lifetime $t_G$, but afterwards would prefer to die. Maximum endurable time preference is thereby exhibited.

Figure 1 differs from the typical portrayal of maximum endurable time preference (e.g., Miyamoto et al. 1998, Stalmeier et al. 2001, Stalmeier et al. 2005), where overall utility *increases* from $t = 0$ (in Figure 1 it *decreases*) until some critical point beyond which it decreases and eventually becomes negative. This qualitative behavior can be captured in our model if we assume that the survival target $t_G$ is uncertain. In this case, (7) yields the graphs of induced utility versus life duration in Figure 2 when $F_G(t) = 1 - e^{-t/\beta}$ ($t \geq 0$), an exponential distribution with mean $\beta =$ 5 years. This matches the conventional depiction of maximum endurable time preference.

**Indifference to Health Quality at Short Durations**
Miyamoto and Eraker (1988) found that subjects might accept a trade-off of life duration for improved health quality when remaining lifetime was long, but decline such trade-offs if remaining lifetime was short. Regardless of the utility model, this behavior means that the overall utility induced by health profile $h = (q, t)$ does not depend on health state $q$ when $t$ is short. The QALY model (1) cannot accommodate this behavior, which would require, if the prospective improvement in health is from $q_0$ to $q^*$, that $U(q^*, t)$ and $U(q_0, t)$ be identical for small $t$, but diverge for large $t$ (as, for example, in Figure 6 in Miyamoto et al. 1998).

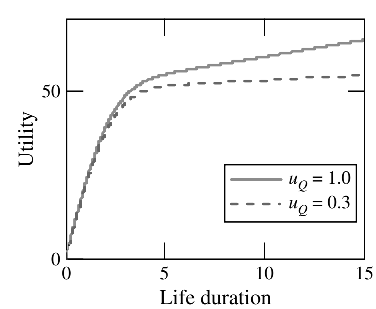
There are, however, versions of the extrinsic-goal model that behave in this way, at least approximately. Suppose we use a survival-target proxy (5) for goal achievement, as above, and take the target $t_G$ to be uncertain with a half-normal distribution around zero given by $F_G(t) = 2\Phi(t/\sigma) - 1$ for $t \geq 0$, where $\Phi$ is the standard normal distribution. The resulting graphs of induced utility $\mathrm{E}[U \mid q^*, t]$ and $\mathrm{E}[U \mid q_0, t]$ versus $t$, computed using (7), are displayed in Figure 3. Here the two curves are nearly identical for life duration $t$ smaller than roughly two years, but diverge for larger $t$. Therefore, indifference to health quality at short durations is plausibly modeled by an extrinsic goal model. In this particular model with $k_G = 50$,

**Figure 2** **Utility as a Function of Life Duration when the Survival Target $t_G$ for Goal Achievement Is Uncertain, for Several Different Quality Levels $u_Q$**



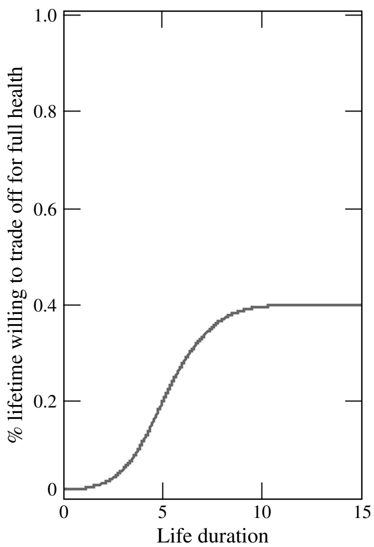

*Notes.* Here $t_G$ has an exponential distribution with mean five years, and the trade-off weight $k_G$ is 5. This matches the conventional depiction of maximum endurable time preference.

**Figure 3** **Expected Utility as a Function of Life Duration for Two Different Qualities of Life $u_Q$ in an Extrinsic Goal Model with Survival Target $t_G$ for Extrinsic Goal Achievement Uncertain with a Half-Normal ($\mu = 0$, $\sigma = 1.8$) Distribution, and Trade-off Weight for Goal Achievement $k_G = 50$**




*Notes.* The two curves are virtually identical for life durations of two years or less, but diverge for larger durations. This model closely approximates indifference to health quality at short durations.

**Figure 4** **Percentage of Life Duration an Extrinsic-Goal Decision Maker Would Be Willing to Trade Off, as a Function of Life Duration, in Order to Raise Health Quality from $u_Q = 0.6$ to 1.0, Assuming a Survival Target for Extrinsic Goal Achievement as Described in Figure 3**




*Notes.* The QALY model predicts that this percentage would be constant at 40% regardless of life duration, but an extrinsic-goal decision maker would become reluctant to trade off virtually any portion of short remaining life durations under two years.

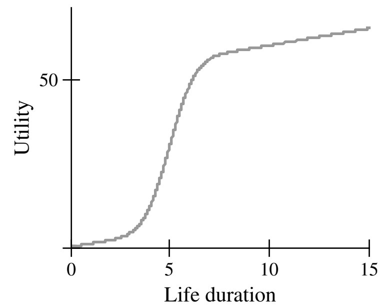
goal achievement is quite important—50 times more important than an additional year of life.

What are the resulting time trade-offs under this extrinsic-goal utility function? In Figure 4 we graph life duration versus the percentage of lifetime a decision maker would willing to trade off to improve health quality from $u_Q = 0.6$ to 1.0. In the absence of an extrinsic goal (that is, under the QALY model), this percentage would be a constant 40% regardless of life duration. A decision maker with the uncertain survival target of the preceding paragraph would also trade off 40% of lifetime for lifetimes longer than five years. However, he would trade off only 3% of a two-year lifetime, and the trade-off approaches a limiting value of 1.7% as lifetime approaches zero.

## Risk Attitudes for Length of Life

In the QALY model, the utility function $U_T(t)$ for length of life is, of course, a von Neuman-Morgenstern utility function (vNM utility for short). Although widely accepted for prescriptive purposes, vNM utility has widely documented failures as a descriptive model of preference (e.g., Tversky and Kahneman 1981). Verhoef et al. (1994) provide evidence that prospect theory (Kahneman and Tversky 1979), a *purely descriptive* extension of vNM utility, can better describe subjects' preferences over risky lifetimes.

**Figure 5** **Cumulative Utility as a Function of Life Duration Under an Extrinsic Goal Utility Model Where Survival Target $t_G$ for Extrinsic Goal Achievement Is Uncertain with a Truncated Normal ($\mu = 5$, $\sigma = 1$) Distribution, and Trade-off Weight for Goal Achievement Is $k_G = 50$**




*Note.* The S-Shape is compatible with the prospect theory preference data reported in Verhoef et al. 1994.

A key component of the prospect theory model is the *aspiration level* (or *reference point*). Outcomes above this level are subjectively regarded as *gains*, and outcomes below this level are regarded as *losses*. Because subjects are typically risk averse for gains, and risk seeking for losses, the resulting utility curves over cumulative lifetime are S-shaped.

However, as Nease (1994) points out in a response, the aspiration levels found by Verhoef et al. (1994) might well be caused by extrinsic goals of the very type we discuss here—those with survival-target proxies. A survival-target preference model can in fact produce S-shaped cumulative utility curves with inflection points comparable to the aspiration levels reported by Verhoef et al. (see Figure 5). It is significant that an extrinsic-goal preference model—itself a vNM utility function, but with an additional goal achievement attribute—can accommodate these observed violations of the QALY model and still retain its prescriptive status.

## Prescriptive vs. Descriptive Perspectives

Note that we are not here proposing extrinsic-goal utility models such as (3) as *descriptive* models of preference behavior, which is complex and many faceted. The utility models we discuss are meant for *prescriptive* purposes. However, the ability to accommodate observed behaviors described above does enhance the face validity of such models for prescriptive purposes.

## Utility Assessment

Standard methods suffice to assess the extrinsic-goal utility function we introduce here. It is worth pointing out, however, that it may be necessary to include hypothetical outcomes in the assessment process (see the discussion following Theorem 1, p. 6). It should also be emphasized that utility assessment should occur on the underlying goal attribute ("See my son graduate"), rather than on the survival-target proxy ("Live five years") we have been discussing above. The former can be uncoupled from survival duration (although some ingenuity may be required to frame this), whereas the latter confounds utility for survival duration with utility for goal achievement.

Notice first that for the full achievement level $g = g^*$, extrinsic-goal utility (4) reduces to the standard QALY form:

$$U(g^*, q, t) \sim U_Q(q)U_T(t).$$

Therefore, traditional techniques such as time trade-off or standard gamble can be used, albeit possibly on hypothetical combinations $(g^*, q, t)$ depending on the nature of goal achievement. Both time trade-off and standard gamble require that $U_T$ is known or already assessed. For time trade-off assessment, it is common to assume $U_T(t) = t$ without any assessment at all. Nevertheless, if necessary, $U_T$ can be assessed by standard probability-equivalent or certainty-equivalent procedures (e.g., Clemen and Reilly 2001) under the background assumptions $g = g^*$, $q = q^*$.

Assessing the goal achievement utility function $U_G(g)$ is unnecessary if the goal attribute is binary, because we already have the normalization $U_G(g^*) = 1$, $U_G(g_0) = 0$. If there are additional goal achievement levels, the usual probability-equivalent or certainty-equivalent methods could be applied. Again, it is important to use the original goal achievement attribute rather than any survival proxy to avoid confounding additional survival time and goal achievement.

Trade-off or swing-weighting techniques that have become standard (e.g, Keeney and Raiffa 1976) should also suffice for assessing the goal importance weight $k_G$. Again, hypothetical scenarios would probably be required using the underlying goal attribute rather than any survival proxy. A swing-weighting approach would ask directly how much more important goal achievement would be, compared, say, to subsequent survival of five years. A trade-off question would ask what proportion of subsequent survival time (or a standard gamble question would ask what risk of losing subsequent survival) a subject would be willing to sacrifice for goal achievement. This question is in principle no more difficult than time trade-off and standard gamble questions currently in use for assessing $U_Q$.

It might be helpful to devise *hypothetical* assessment questions that would never be given to actual subjects, but could usefully convey the meaning of the trade-off weight $k_G$ as part of an analysis. For instance, how much health quality should be given up to extend life duration from just below to just above a survival target $t_G$? An answer of $\Delta t$ would produce the indifference

$$(g_0, q^*, t_G - \Delta t) \sim (g^*, q, t_G).$$

Using the extrinsic-goal utility representation (4) with $U_T(\cdot)$ and $U_Q(\cdot)$ already known would yield the equation $U_T(t_G - \Delta t) - k_G = U_Q(q)U_T(t_G)$, from which $k_G$ could be determined. In the common case $U_T(t) = t$ of no time discounting, we obtain

$$k_G = (1 - U_Q(q))t_G - \Delta t. \tag{8}$$

If $\Delta t$ is negligibly small, it follows that $k_G/t_G$ is the most health quality the decision maker is willing to sacrifice to increase survival from just below $t_G$ to just above $t_G$.

In current medical applications, patients may be interviewed by medical staff, who use time trade-off or standard gamble techniques to elicit $U_Q(q)$ for anticipated health states $q$. Often this is all that time allows, and no elicitation of $U_T(t)$ is even attempted—the assumption being that for time trade-offs or standard gambles over small $t$, $U_T$ is linear in $t$. Adding extrinsic goals to this mix may be prohibitive, as this necessitates, at a minimum, the additional steps of determining the extrinsic goal and assessing $k_G$. Worse, if there are multiple extrinsic goals, the present paper gives no guidance as to what should be done. Therefore, at present, assessing extrinsic-goal utility likely falls short of practicality in typical settings.

However, further research on the types of extrinsic goals associated with particular interventions and patients may make it possible for medical interviewers to anticipate the identity of extrinsic goals just as they currently anticipate the identity of potential health states $q$. For such standardized goal scenarios, assessment for a binary extrinsic goal requires only one more assessment question to determine the goal weight $k_G$, certainly within the realm of practicality.

**Health Profiles and Extrinsic Goals**

A *health profile* $h$ is a function that assigns health state $q = h(s)$ to every time instant $s$ in its domain. The domain of $h$ is a closed interval $[0, t_h]$ whose upper bound $t_h > 0$ may vary across different health profiles $h$. Health profiles $h$ can represent potentially intricate sequences of health states unfolding over time. The combination $h = (q, t)$ represents a simple health profile with $t_h = t$. This profile is constant and equal to $q$ in the interval $[0, t]$.

In a health profile $h$, the convention is to think of the utility $U_Q(h(t))$ at time $t \in [0, t_h]$ as the *rate of health accrual* at time $t$—in the QALY model as applied to health profiles, overall health accumulates at time varying rate $U_Q(h(t))$. However, extrinsic goal achievement, being not time modulated, does not accrue at any particular rate, but instead is associated holistically with the entire *life profile* of an individual. For modeling purposes, then, it may be simplest to deal with pairs $(g, h)$, where $h$ is a health profile and $g$ is a level of extrinsic goal achievement that may or may not be directly associated with $h$.

Let $H$ be the set of feasible health profiles. We extend the marginality Assumption B3 to health profiles as follows.

Assumption B3 (Marginality Between $G$ and $H$). *Preference over gambles $(\tilde{g}, \tilde{h})$ on $G \times H$ depends only on the marginal distribution of $\tilde{g}$ and of $\tilde{h}$, and not on their joint distribution.*

Because of the additively separable form of $U(g, h)$ over $G \times H$ implied by B3′, we may speak of preferences over health profiles in $H$ without specifying the level $g$ of goal achievement. (That is, $H$ is *utility independent* of $G$ (Keeney and Raiffa 1976) which is really all we need below.) It follows if health profile $h$ over $[0, t_h]$ is indifferent to a simple health profile $(q, t_h)$, then using the form (3) of utility over goals and simple health profiles, we may conclude

$$U(g, h) = U(g, (q, t_h)) \sim U_Q(q)U_T(t_h) + k_G U_G(g). \quad (9)$$

We follow Pliskin et al. (1980) in assuming the existence of such a health state $q$. We suppose:

Assumption Q0. *For any health profile $h$ there is a level $q = \bar{Q}(h)$ of health such that $h \sim (q, t_h)$.*

In other words, if a sequence of health states represented by profile $h$ is indifferent to occupying a constant health state $q$ for the same duration $t_h$, then the health quality associated with $h$ should be equal to the health quality associated with $q$. Using (9), we obtain the extended representation

$$U(g, h) = U(g, \bar{Q}(h), t_h) \sim U_Q(\bar{Q}(h))U_T(t_h) + k_G U_G(g)$$

over $G \times H$. We note that if the utility $U_Q(\bar{Q}(h))$ of the health quality indicator $\bar{Q}(h)$ satisfies the time-weighted average equation

$$U_Q(\bar{Q}(h)) = \frac{1}{U_T(t_h)} \int_0^{t_h} U_Q(h(t))\, dU_T(t), \quad (10)$$

then we obtain an extension of the generalized QALY model

$$U(g, h) = \int_0^{t_h} U_Q(h(t))\, dU_T(t) + k_G U_G(g). \quad (11)$$

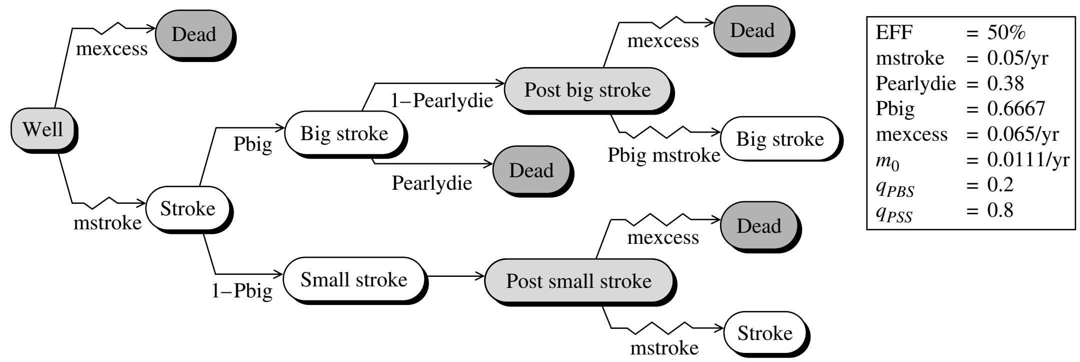
We note that survival-target proxy (5) for extrinsic goal achievement can be extended to health profiles in the obvious way: For health profile $h$ with domain $[0, t_h]$, take

$$s_G = [t_h \geq t_G] = \begin{cases} 1 & \text{if } t_h \geq t_G \\ 0 & \text{if } t_h < t_G. \end{cases} \quad (12)$$

The goal utility $\mathrm{E}[U_G(g) \mid h]$ induced by a health profile $h$ is given as in (6):

$$\mathrm{E}[U_G(g) \mid h] = F_G(t_h). \quad (13)$$

**Figure 6 Downstream Prognosis Following the Short-Term Sequelae of Accepting or Rejecting Carotid Endarterectomy**




*Notes.* Surgery reduces baseline stroke rate *mstroke* $= 0.05$/year by EFF $= 50\%$. However, surgery doubles short-term stroke risks (not shown). Short-term surgical or nonsurgical stroke results in initial state *Stroke* rather than *Well*. Health quality in the state *Post Big Stroke* is $q_{pbs} = 0.2$ and in *Post Small Stroke* is $q_{pss} = 0.8$.

**Healthy-Years Equivalents**

*Healthy-years equivalents* (HYEs) were introduced by Mehrez and Gafni (1989, 1991) as a method of health quality evaluation more general than the QALY model. The HYE $y_h$ of a health profile $h$ satisfies $h \sim (q^*, y_h)$, that is, $y_h$ is the number of years in full health that is preference equivalent to health profile $h$. In contrast to the QALY model, the HYE approach, suitably modified, can capture preferences for extrinsic goals. For instance, suppose we define a *goal-achieved* HYE $y_{g,h}$ by the holistic indifference

$$(g, h) \sim (g^*, q^*, y_{g,h}). \tag{14}$$

Then $y_{g,h}$ captures preference for extrinsic-goal achievement. Assuming that the level $g$ of extrinsic-goal achievement is functionally dependent on the health profile $h$, the holistic assessment (14) would require no more effort than assessing standard HYEs. However, as is well known (e.g., Dolan 2000), in most contexts there will be impracticably many possible health profiles $h$ requiring assessment. The extrinsic-goal utility model decomposes holistic preference into health quality, survival time, and extrinsic-goal achievement, requiring only the relatively simpler assessments of $U_Q$, $U_T$, $k_G$, and $U_G$.

If in (14) the pair $(g, h)$ is a (possibly uncertain) goal achievement and health profile, then under the utility model (4) with undiscounted time preference $U_T(t) = t$, the indifference (14) reduces to

$$\mathrm{E}U(g, h) = y_{g,h}. \tag{15}$$

In other words, expected extrinsic-goal utility computed via (4) is numerically equal to the goal-achieved HYE. Thus, although utility is unitless, it is intuitively useful to think of utility computed via (4) as having units of years. This would give units of years to the trade-off weight $k_G$ as well, as we do in the next example.

## A Decision Analysis Including Time Goals

We illustrate here the prescriptive use of a utility function capturing extrinsic goals in a small realistic decision analysis. The analysis is based on Matchar and Pauker (1986), in which a 55-year-old male patient, with partial blockage in the right carotid artery, must choose whether to undergo carotid endarterectomy (surgery) to clear the carotid artery. Surgery may prevent strokes in the near or distant future, but surgery itself may bring on a stroke. Figure 6 portrays this problem using a stochastic tree diagram (Hazen 1992, 1993).[1]

We assume an extrinsic goal with certain survival target $t_G = 6$ years as in (12), and a utility function (11) over health profiles in which life duration is not discounted. We use the quality coefficients assigned by

[1] A stochastic tree is a combination of a decision tree (the straight arrows) and a transition diagram for a continuous-time Markov chain (the wavy arrows). Chance branches (straight arrows) are labeled by probabilities, and stochastic transitions (wavy arrows) are labeled by transition rates.

**Table 1** **Outcome Measures and Trade-off Weight Thresholds $k_G$ for the Decision Problem in Figure 6, for Two Different Levels EFF of Surgical Efficacy**

| | Surgical efficacy | | | |
|---|---|---|---|---|
| | EFF = 50% | | EFF = 37% | |
| Intervention | Surgery | No surgery | Surgery | No surgery |
| E[QALY] | 8.588 year | 8.294 year | 8.369 year | 8.294 year |
| $E[U_G]$ (=P(Goal achieved)) | 0.558 | 0.577 | 0.552 | 0.577 |
| *HYE* when $k_G = 1.2$ year | 8.057 year | 7.786 year | 7.768 year | 7.786 year |
| Threshold value for the trade-off weight $k_G$ | $k_G = 15.2$ year | | $k_G = 0.492$ year | |

*Note*. *Surgery* has a higher mean QALY, but lower probability of achieving the six-year survival goal. A patient just willing to decrease health quality from *well* to *post small stroke* in order to increase survival from just below six years to the six-year goal would have $k_G = 1.2$ year, which we take as baseline. In the case EFF = 50%, *surgery* remains optimal for $k_G$ as high as 15.2 year. However, in case EFF = 37%, *no surgery* becomes overall optimal even though *surgery* is better considering only QALYs. It remains optimal for $k_G$ exceeding 0.492 year.

Matchar and Pauker, namely, $U_Q = 1.0$ to the state *Well*, $U_Q = 0.8$ to the state *Post Small Stroke*, and $U_Q = 0.2$ to the state *Post Big Stroke*.

To quantify the goal trade-off weight $k_G$, we assume that the patient is just willing to decrease health quality from *Well* to *Post Small Stroke* in order to increase survival duration from just below the six-year survival goal to just above it. Using (8) with $\Delta t$ negligibly small, we obtain $k_G = (1 - q_{PSS}) \cdot t_G = 1.2$ year at baseline. In short, goal achievement is worth 1.2 years of life.

Outcome measures and trade-off weight thresholds $k_G$ for this decision problem are listed in Table 1, for two different levels efficacy EFF. *Surgery* is preferred if only mean quality-adjusted lifetime is considered (mean QALY 8.588 year versus 8.294 year). However, the expected goal achievement utility $E[U_G]$, equal to the probability of achieving the six-year survival goal, is higher for *No Surgery*. This makes intuitive sense, because *Surgery* carries a risk of short-term death due to operative mortality or operative stroke.

Overall expected utility is slightly higher for *Surgery* as $E[U \mid \textit{Surgery}] = 8.057$ versus $E[U \mid \textit{No Surgery}] = 7.786$. According to (15), the overall expected utilities in Table 1 can be interpreted as goal-achieved HYEs, equal to 8.057 years for *Surgery* versus 7.786 years for *No Surgery*.

As indicated in the table, in the case EFF = 50% (the base case analyzed by Matchar and Pauker), *No Surgery* does not become optimal until the trade-off weight $k_G$ rises to $k_G = 15.2$ years, a relatively high level. However, when EFF = 37%, *No Surgery* becomes optimal, and the threshold for $k_G$ is only 0.492 year. In this case, the optimal solution is very sensitive to the goal trade-off weight $k_G$.

## General Survival-Duration Proxies for Degree of Goal Achievement

We consider the general case in which there is a proxy attribute for extrinsic-goal achievement that depends only on survival duration. It turns out that under such proxy attributes, the extrinsic-goal utility model collapses to an equivalent QALY model if health status is suitably augmented. We illustrate why this is so, and comment on why it is nevertheless conceptually valuable to retain the extrinsic-goal utility perspective.

Given a health profile $h$ over $[0, t_h]$, we suppose that induced expected goal utility depends only on $t_h$, that is, there is a function $F_G(t)$ with $F_G(0) = 0$ such that

$$E[U_G(g) \mid h] = F_G(t_h). \tag{16}$$

This is identical to (13), although here we make no assumption that $F_G$ is a distribution function of any survival target $t_G$. It follows from (11) that overall utility $E[U \mid h]$ induced by $h$ is given by

$$E[U \mid h] = \int_0^{t_h} U_Q(h(t))\, dU_T(t) + k_G F_G(t_h).$$

Assume that $F_G$ is absolutely continuous. Then (see, e.g., Rudin 1966, §8.17) $F_G$ is almost everywhere differentiable, and

$$F_G(t_h) = \int_0^{t_h} F'_G(t)\, dt,$$

where $F'_G(t)$ is the derivative of $F_G$ at $t$. Supposing that $U_T$ is differentiable, overall induced utility becomes

$$\begin{aligned}\mathrm{E}[U \mid h] &= \int_0^{t_h} U_Q(h(t))U'_T(t)\,dt + k_G \int_0^{t_h} F'_G(t)\,dt \\ &= \int_0^{t_h} \left(U_Q(h(t)) + k_G \frac{F'_G(t)}{U'_T(t)}\right) U'_T(t)\,dt.\end{aligned}$$

Notice this is *the same* as the discounted QALYs associated with a modified health profile obtained by adding $k_G(F'_G(t)/U'_T(t))$ to health state quality at time $t$. We summarize as follows.

Theorem 2. *Suppose there is a proxy attribute for extrinsic-goal achievement under which expected goal utility depends only on survival duration, as in* (16). *Under the extrinsic-goal utility model* (11) *and the assumptions above, the expected utility* $\mathrm{E}[U \mid h]$ *induced by health profile $h$ is equal to the discounted QALY of a new health profile obtained from $h$ by augmenting the health quality $U_Q(q)$ of each health state $q$ in $h$ at time $t$ by the amount $k_G(F'_G(t)/U'_T(t))$.*

The absolute continuity assumption on $F_G$ in this theorem prevents it from applying to the case of a survival target proxy (5) with certain target $t_G$ (because then $F_G$ is discontinuous). However, it does apply if $t_G$ is uncertain with continuous distribution. It also applies in the following situation.

### A Proportionate-Duration Proxy for Extrinsic Goal Achievement

Consider the situation in which there are intermediate extrinsic-goal achievement levels $g$ in addition to full achievement $g^*$ and no achievement $g_0$. A *proportionate-duration proxy* for degree of goal achievement is given by *survival as a proportion of a target duration* $t_G$. For instance, the degree of achievement of the goal of raising a six-year-old to adulthood might be represented by the percentage of the 12 remaining childhood years that the parent survives.

Formally, the proportionate-duration proxy $p_G$ associated with health profile $h$ over $[0, t_h]$ consists of survival time as a percentage up to 100% of a critical duration $t_G$, that is,

$$p_G = \min\{t_h/t_G, 1\}. \tag{17}$$

Suppose the induced goal-achievement utility given health profile $h$ is in fact equal to $p_G$:

$$\mathrm{E}[U_G(g) \mid h] = \min\{t_h/t_G, 1\}.$$

In this case $F_G(t) = \min\{t/t_G, 1\}$ is an absolutely continuous function of $t$, and Theorem 2 applies. Assuming no discounting ($U_T(t) = t$), we obtain the following result by invoking Theorem 2.

Corollary. *Suppose degree of extrinsic-goal achievement is measured by the proportionate-duration proxy* (17). *Under the extrinsic-goal utility model* (11) *with no discounting, the utility of a health profile $h$ is equal to the QALY of a modified health profile in which all health states $q$ occupied before time $t_G$ are replaced by states $q^+$ having health quality $U_Q(q^+) = U_Q(q) + k_G/t_G$.*

This result is useful for incorporating extrinsic goals into decision analyses, as it enables one to retain the widespread and efficient procedures available for computing QALYs. It does not, however, imply that the standard QALY model is adequate conceptually to handle extrinsic goals, because the associated utility assessment approaches might differ substantially. For instance, we suspect that subjects would likely find it easier to directly consider extrinsic goals and the value of achieving them—as would be required when a goal attribute is explicitly introduced as we have done here—rather than attempting to trade off time in goal-achieved versus goal-unachieved versions of every relevant health state $q$, as implied in this situation by the QALY approach.

## Conclusion

We have presented an axiomatically justified augmentation of the QALY model that accounts for extrinsic goals, and showed that this model can accommodate plausible observed preference behavior not allowed by the standard QALY model. It remains to determine empirically if accounting for extrinsic goals can help explain this plausible behavior, or improve the correlation of measured preferences with current health. The model can be reasonably applied to medical decision analyses, although the impact of adding extrinsic goals to published medical decision analyses has yet to be investigated.

Despite its limitations, the QALY model has become the standard for modeling patient preferences in medical decision analyses. The extrinsic-goal model presented here constitutes a fundamental augmentation of this standard, and has, we believe, the potential to benefit society by substantially broadening the types of patient and community preferences included in these analyses.

## Acknowledgments
This research was supported by a grant from the National Science Foundation (SES–0451672). The author thanks all three referees for their constructive comments. As a result, the author believes the paper is much improved from its original version. In response to comments from two of the referees, the author has significantly improved the examples presented. All three made him think more carefully about utility assessment, and an excellent suggestion from one referee yielded a more parsimonious version of the primary theorem.

## Appendix. Proof of Theorem 1
In the following, we take as given the utility function $U(g, q, t)$ over goal-quality-duration triples $(g, q, t)$. If we write $U$ without all of its arguments, then we intend that the remaining arguments are at their $^*$-level. For example:

$$U(t) = U(g^*, q^*, t) \qquad U(g, q) = U(g, q, t^*)$$
$$U(q) = U(g^*, q, t^*)$$

In the following lemma, we have in mind $X = G \times Q$, although the result holds for any attribute $X$.

Lemma A.1. *Suppose $t^* \succ t_0$ given $X = x^*$. If $T$ GUI $X$, then*

$$U(x, t) = U(x)U_T(t) + U(x, t_0)(1 - U_T(t))$$

*where*

$$U_T(t) = \frac{U(t) - U(t_0)}{U(t^*) - U(t_0)}.$$

Proof. The GUI assumption yields

$$U(x, t) = a(x) + b(x)U(x^*, t) = a(x) + b(x)U(t) \tag{A1}$$

for some functions $a(x)$ and $b(x)$. Set $t = t^*$ and $t = t_0$ to get

$$U(x) = a(x) + b(x)U(t^*)$$
$$U(x, t_0) = a(x) + b(x)U(t_0)$$

Solving for $a(x)$ and $b(x)$ yields

$$b(x) = \frac{U(x) - U(x, t_0)}{U(t^*) - U(t_0)}$$
$$a(x) = \frac{-U(x)U(t_0) + U(x, t_0)U(t^*)}{U(t^*) - U(t_0)}.$$

Substitute back into Equation (A1) to get

$$\begin{aligned} U(x, t) &= \frac{-U(x)U(t_0) + U(x, t_0)U(t^*)}{U(t^*) - U(t_0)} + \frac{U(x) - U(x, t_0)}{U(t^*) - U(t_0)}U(t) \\ &= \frac{U(x)(U(t) - U(t_0)) + U(x, t_0)(U(t^*) - U(t))}{U(t^*) - U(t_0)} \\ &= U(x)U_T(t) + U(x, t_0)(1 - U_T(t)). \quad \square \end{aligned}$$

We begin with sufficiency in Theorem 1. Invoking the lemma, the assumption $TGUI(G, Q)$ yields

$$U(g, q, t) = U(g, q)U_T(t) + U(g, q, t_0)(1 - U_T(t)).$$

Note that $U_T(t_0) = 0$ and $U_T(t^*) = 1$. Then the conditional zero condition implies

$$U(g, q, t_0) = U(g, q^*, t_0) = U(g, t_0).$$

Combine to get

$$\begin{aligned} U(g, q, t) &= U(g, q)U_T(t) + U(g, t_0)(1 - U_T(t)) \\ &= U(g, t_0) + (U(g, q) - U(g, t_0))U_T(t). \end{aligned}$$

We now apply marginality between $G$ and $Q, T$. Consider the difference multiplying $U_T(t)$ in the last equation. We have

$$\begin{aligned} &U(g, q) - U(g, t_0) \\ &\quad = U(g, q, t^*) - U(g, q^*, t_0) \\ &\quad = U(g^*, q, t^*) - U(g^*, q^*, t_0) \quad \text{by marginality} \\ &\quad = U(q) - U(t_0). \end{aligned}$$

This establishes the following result.

Lemma A.2. *If $t^* \succ t_0$ given $Q = q^*$, $G = g^*$, and $TGUI(G, Q)$, marginality holds between $G$ and $Q$, $T$, and the conditional zero condition holds, then there is a function $U_T(t)$ over $t \in T$ such that*

$$U(g, q, t) = U(g, t_0) + (U(q) - U(t_0))U_T(t)$$

*where $U_T(t_0) = 0$ and $U_T(t^*) = 1$.*

This is essentially the decomposition stated in Theorem 1 once things are renormalized. To accomplish this, define $U_Q(q)$ by

$$U(q) - U(t_0) = (U(t^*) - U(t_0))U_Q(q).$$

This is well defined because $U(t^*) > U(t_0)$, by the assumption $t^* \succ t_0$ given $g^*, q^*$. Note that $U_Q(q^*) = 1$. Also define $U_G(g)$ by

$$\begin{aligned} U(g, t_0) &= U(g_0, t_0) + (U(g^*, t_0) - U(g_0, t_0))U_G(g) \\ &= U(g_0, t_0) + (U(g^*) - U(g_0))U_G(g) \quad \text{by marginality} \end{aligned}$$

where $U(g^*) - U(g_0) > 0$ by the assumption that $g^* \succ g_0$ given $Q = q^*$, $T = t^*$, so $U_G(g)$ is well defined. Note that $U_G(g^*) = 1$ and $U_G(g_0) = 0$. Substitute the expressions above for $U(q) - U(t_0)$ and $U(g, t_0)$ to obtain

$$\begin{aligned} U(g, q, t) = U(g_0, t_0) &+ (U(g^*) - U(g_0))U_G(g) \\ &+ (U(t^*) - U(t_0))U_Q(q)U_T(t). \end{aligned}$$

Drop the leading constant and divide through by $U(t^*) - U(t_0) > 0$ to obtain

$$U(g, q, t) \sim k_G U_G(g) + U_Q(q) U_T(t),$$

where

$$k_G = \frac{U(g^*) - U(g_0)}{U(t^*) - U(t_0)}.$$

This establishes the representation in Theorem 1. We have therefore shown sufficiency. Necessity is easy to verify. □

## References


Clemen, R. T., T. Reilly. 2001. *Making Hard Decisions with Decision Tools*. Duxbury, Pacific Groves, CA.

Dolan, P. 1996. Modelling valuations for health states: The effect of duration. *Health Policy* **38**(3) 189–203.

Dolan, P. 1997. Modeling valuations for EuroQol health states. *Medical Care* **35**(11) 1095–1108.

Dolan, P. 2000. A note on QALYs versus HYEs: Health states versus health profiles. *Internat. J. Tech. Assessment Health Care* **16**(4) 1220–1224.

Dolan, P., P. Stalmeier. 2003. The validity of time trade-off values in calculating QALYs: Constant proportional time trade-off versus the proportional heuristic. *J. Health Econom.* **22**(3) 445–458.

Feeny, D., W. Furlong, M. Boyle, G. W. Torrance. 1995. Multi-attribute health status classification systems: Health utilities index. *Pharmacoeconomics* **7**(6) 490–502.

Fishburn, P. C. 1965. Independence in utility theory with whole product sets. *Oper. Res.* **13**(1) 28–45.

Fishburn, P. C., R. L. Keeney. 1975. Generalized utility independence and some implications. *Oper. Res.* **23**(5) 928–940.

Gold, M. R., D. L. Patrick. G. W. Torrence, D. G. Fryback, D. C. Hadorn, M. S. Kamlet, N. Daniels, M. C. Weinstein. 1996. Identifying and valuing outcomes. M. R. Gold, J. E. Siegel, L. B. Russell, M. C. Weinstein, eds. *Cost-Effectiveness in Health and Medicine*. Oxford University Press, New York.

Hazen, G. B. 1992. Stochastic trees: A new technique for temporal medical decision modeling. *Medical Decision Making* **12** 163–178.

Hazen, G. B. 1993. Factored stochastic trees: A tool for solving complex temporal medical decision models. *Medical Decision Making* **13** 227–236.

Hunink, M., P. Glasziou, J. Siegel, J. Weeks, J. Pliskin, A. Elstein, M. Weinstein. 2001. *Decision Making in Health and Medicine: Integrating Evidence and Values*. Cambridge University Press, New York.

Kahneman, D., A. Tversky. 1979. Prospect theory: An analysis of decision under risk. *Econometrica* **47**(2) 263–291.

Keeney, R. L., H. Raiffa. 1976. *Decisions with Multiple Objectives: Preferences and Value Trade-Offs*. John Wiley and Sons and Cambridge University Press, New York. Reprint 1993.

Matchar, D. B., S. G. Pauker. 1986. Transient ischemic attacks in a man with coronary artery disease: Two strategies neck and neck. *Medical Decision Making* **6** 239–249.

Mehrez, A., A. Gafni. 1989. Quality adjusted life years, utility theory, and healthy-years equivalents. *Medical Decision Making* **9** 142–149.

Mehrez, A., A. Gafni. 1991. The healthy-years equivalents: How to measure them using the standard gamble approach. *Medical Decision Making* **11** 140–146.

Miyamoto, J. M. 1999. Quality-adjusted life years (QALY) utility models under expected utility and rank dependent utility assumptions. *J. Math. Psych.* **43** 201–237.

Miyamoto, J. M., S. A. Eraker. 1988. A multiplicative model of the utility of survival duration and health quality. *J. Experiment. Psych.: General* **117** 3–20.

Miyamoto, J. M., P. P. Wakker, H. Bleichrodt, H. J. M. Peters. 1998. The zero-condition: A simplifying assumption in QALY measurement and multiattribute utility. *Management Sci.* **44**(6) 839–849.

Nease, R. F. 1994. Commentary. Risk attitudes in gambles involving length of life: Aspirations, variations and ruminations. *Medical Decision Making* **14** 201–203.

Pliskin, J. S., D. S. Shepard, M. C. Weinstein. 1980. Utility functions for life years and health status. *Oper. Res.* **28** 206–224.

Rudin, W. 1966. *Real and Complex Analysis*. McGraw-Hill, New York.

Schwartz, A., G. B. Hazen, A. Leifer. 2007. Life goals and health decisions. *Medical Decision Making*. Forthcoming.

Stalmeier, P. F. M., J. J. V. Busschbach, L. M. Lamers, P. F. M. Krabbe. 2005. The gap effect: Discontinuities of preference around dead. *J. Health Econom.* **14**(7) 679–685.

Stalmeier, P. F. M., G. B. Chapman, A. G. E. M. de Boer, J. J. B. van Lanschot. 2001. A fallacy of the multiplicative QALY model for low-quality weights in students and patients judging hypothetical health states. *Internat. J. Tech. Assessment Health Care* **17**(4) 488–496.

Sutherland, H. J., H. Llewellyn-Thomas, N. F. Boyd, J. E. Till. 1982. Attitudes towards quality of survival: The concept of maximal endurable time. *Medical Decision Making* **2**(3) 299–309.

Tsevat, J. 2000. What do utilities measure? *Medical Care* **38**(9, Supplement II) II-160–II-164.

Tversky, A., D. Kahneman. 1981. The framing of decisions and the psychology of choice. *Science* **211**(4481) 453–458.

Verhoef, L. C. G., A. F. J. de Haan, W. A. J. van Daal. 1994. Risk attitude in gambles with years of life: Empirical support for prospect theory. *Medical Decision Making* **14**(2) 194–200.