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# Lottery Acquisition versus Information Acquisition: Prices and Preference Reversals

GORDON B. HAZEN
*IE/MS Department, Northwestern University, Evanston, IL 60208*

JAYAVEL SOUNDERPANDIAN
*Department of Business, University of Wisconsin-Parkside, Kenosha, WI 53141*

*Abstract*

Suppose you must choose between two pieces of information A and B. In the absence of cost, you would prefer to obtain A rather than B, and in fact would be willing to take more risk to obtain A than B. Nevertheless, you would pay more money for B than for A. Are your preferences consistent with expected utility? The answer is yes; they may very well be. We give an example to illustrate how this may happen, and relate this reversal phenomenon to the well-known discrepancy between buying and selling prices for lotteries. Along the way, we demonstrate that even though *selling* an information source is strictly analogous to selling a lottery, *buying* an information source is not strictly analogous to buying a lottery. However, for any collection of lotteries there is a decision problem with corresponding information sources, each source having both buying price and selling price equal to the buying and selling prices of the corresponding lottery. The existence of preference reversals for mode of information acquisition dispels any notion that the relative value of competing information acquisitions should not depend on the nature of the acquisition. Among expected utility maximizers, only those with constant risk attitude avoid these reversals.

**Key words:** information value, preference reversals

**JEL Classification:** D80 Information and Uncertainty: General

## 1. Introduction

Suppose you must choose between acquiring information about A and acquiring information about B. In the absence of cost, you would prefer to learn about A rather than B, and in fact would be willing to take more risk to learn about A. Nevertheless, you would pay more money to learn about B than A. Are your preferences consistent with expected utility? The answer is yes; they may very well be. A closely related question involves certainty equivalents (cash equivalents, selling prices) and buying prices for lotteries. As is well known, buying and selling prices need not coincide unless the risk attitude is constant. However, consider an expected utility maximizer who would pay more for lottery $X$ than lottery $Y$. Might she demand a higher selling price for $Y$ if she owned it than she would for $X$ if she owned it? Once again the answer is yes.

In what follows, we give examples to illustrate how these reversals may happen. We also show that among expected utility maximizers, only those with constant risk attitude avoid these phenomena, either for lottery payoffs or for information. While the discrepancies between buying and selling prices of lotteries are well known, the question of whether preference reversals are possible and which utility functions allow them seems not to have been treated adequately in the literature. Bell (1988) establishes the type of utility functions that allow only one preference reversal over the entire range of wealth levels for the case of lotteries, without reference to buying or selling. Our emphasis here is on buying and selling information which involves changes in decision strategy as well.

In Section 2 we begin by briefly reviewing the literature on information value, and discussing measures of information value. Our approach involves reducing questions about valuing information to the analogous questions about valuing payoff lotteries, and this approach is presented in Section 3, where we present our main results. As the reader will see, even though *selling* an information source is strictly analogous to selling a lottery, *buying* an information source is not strictly analogous to buying a lottery. In Section 4 we provide some concluding remarks.

## 2. Information value

### *Background*

Early work in the area of information value as it pertains to decision analysis is attributable to Howard (1966, 1967) and Matheson (1968). Their consideration of the value of clairvoyance led to the concept of perfect information and a methodology for calculating the expected value of perfect information. General discussions of information value may be found in Raiffa (1968), Gould (1974), and Howard (1988). Rothkopf (1971) advocates EVPI as a measure of venture risk. Hazen and Felli (1997) propose information value on input parameters as the proper way to measure problem sensitivity. In recently developed normative expert systems (*e.g.*, Heckerman 1991), information value is used to determine what question to ask the user next.

Information value is notorious for its lack of convenient mathematical properties. For example, it is well known that information value is not additive across sources (see Howard (1988) for a discussion). LaValle (1968), Gould (1974), and Hilton (1981) show the lack of any general relationship between information value and the level of wealth, the degree of absolute or relative risk aversion, or the Rothschild-Stiglitz degree of uncertainty in the prior. Miller (1975) examines situations where it is possible to obtain information sequentially throughout the decision process and determines that the value of any particular piece of information is a function of the prices of all other obtainable pieces of information. In a production model with uncertain demand, Merkhofer (1977) shows that the value a decision maker places on a given piece of information depends on the flexibility of his decisions. In the non-expected utility framework, it is well known that nonlinearity in probability can give to rise to negative value of information (*e.g.*, Wakker 1988). More details about information evaluation under non-additive expected utility theory can be found in LaValle and Xu (1990).

*Measures of information value*

Suppose a decision maker will receive uncertain payoff $V_a$ when choosing action $a$, and $V_a$ depends directly or indirectly on an uncertainty $X$. We assume that the decision maker acts to maximize expected utility under some utility function $u$ defined, continuous, and increasing over payoffs. Let $a^*$ be an optimal action in the absence of further information, the event which we denoted $I_\varnothing$. Letting $V$ be the overall payoff, we have

$$E\left[u(V) \mid I_\varnothing\right] = \max_a E\left[u(V_a)\right] = E\left[u(V_{a^*})\right].$$

Let $I_X$ denote the event that the value of the uncertain quantity $X$ will be available prior to choosing, and let $a^*(x)$ be an action maximizing $E[u(V_a) \mid X = x]$. Then

$$E\left[u(V) \mid I_X\right] = E_X\left[\max_a E\left[u(V_a) \mid X\right]\right] = E_X\left[E\left[u\left(V_{a^*(X)}\right) \mid X\right]\right]$$
$$= E\left[u\left(V_{a^*(X)}\right)\right].$$

The standard approach to quantifying information value is to ask what the decision maker would give up to acquire the information. The *buying price* $BPI_X$ is defined as the maximum payoff the decision maker would forgo to learn $X$ before choosing. It satisfies:

$$E\left[u(V - BPI_X) \mid I_X\right] = E\left[u(V) \mid I_\varnothing\right].$$

If we let $CE[V \mid I_X] = u^{-1}(E[u(V) \mid I_X])$ be the *certainty equivalent* of $V$ given $I_X$, then we may write the last equality as

$$CE\left[(V - BPI_X) \mid I_X\right] = CE\left[V \mid I_\varnothing\right].$$

The *utility increase* $EUI_X$ is defined as the increase in utility obtained by being able to observe $X$ before choosing:

$$EUI_X = E\left[u(V) \mid I_X\right] - E\left[u(V) \mid I_\varnothing\right].$$

*EUI* is more mathematically tractable than *BPI*, and has therefore seen some use in theoretical contexts (*e.g.*, Bernardo and Smith 1994). A related measure of information value is the *certainty equivalent increase CEI*, defined as

$$CEI_X = CE[V \mid I_X] - CE[V \mid I_\varnothing].$$

Another measure is the *selling price* of an information. The selling price of $I_X$, denoted by $SPI_X$, is the minimum price a seller who already possesses $I_X$ (but has not used it) would ask for giving up $I_X$, and it satisfies

$$E[u(V) \mid I_X] = E[u(V + SPI_X) \mid I_\varnothing].$$

Yet another measure of information value is the *Probability Price* which is the maximum probability of a large, designated loss that the decision maker is willing to bear in order to acquire the information. To define it formally, suppose there is a payoff $v_0$ which bounds below the payoff variables $V_a$ in the decision problem at hand. Imagine that in exchange for $I_X$, the decision maker can take on some chance $p$ of obtaining $v_0$ instead of $V_a$. The probability price of $I_X$, denoted by $PPI_X$, is the probability that satisfies

$$PPI_X u(v_0) + (1 - PPI_X)E[u(V) \mid I_X] = E[u(V) \mid I_\varnothing].$$

Now let us consider the question of how different measures rank order different information sources. For any $I_X$ the three measures, namely, $EUI_X, CEI_X$ and $SPI_X$ are readily seen to be increasing transformations of one another. Coming to $PPI_X$, we can rewrite its defining equation as

$$\frac{PPI_X}{1 - PPI_X} = \frac{EUI_X}{E[u(V) \mid I_\varnothing] - u(v_0)}$$

revealing that it is also an increasing transformation of $EUI_X$ and therefore of $CEI_X$ and $SPI_X$. Thus all four measures will always rank different information sources in the same order.

Let us say that two measures of information value are *ordinally equivalent* if they rank information sources identically. We then have the following formal assertion.

**Proposition 1.** *CEI, EUI, SPI and PPI are ordinally equivalent measures of information value, that is, for any two uncertainties X and W,*

$$CEI_X \geq CEI_W \Leftrightarrow EUI_X \geq EUI_W \Leftrightarrow SPI_X \geq SPI_W \Leftrightarrow PPI_X \geq PPI_W.$$

In what follows we will use any one or another of these four measures for comparison with the fifth measure, *BPI*. The reason we keep track of all four measures is that each one has its own advantage. *CEI* is relevant for the delta property discussed below, $EUI$ is tractable, $SPI$ has a practical significance, and $PPI$ has the flavor of buying the information.

When two measures of information value are not ordinally equivalent, there will exist reversals, pairs $X$, $W$ of uncertainties such that the information value of $X$ is higher than that of $W$ according to one measure but lower according to the other. The primary question we address in this paper is whether the measure $BPI$ is ordinally equivalent to $SPI$ (and therefore to $CEI$, $EUI$ and $PPI$), or in other words, whether there exist what we term *BP-SP reversals* for information.

The more specific question of whether $CEI$ and $BPI$ are *equal* (not merely ordinally equivalent) can be addressed by invoking familiar results on buying and selling prices. If the utility function $u$ has constant risk attitude (*i.e.*, $u$, is linear or exponential), then certainty equivalents obey the *delta property* (Howard, 1967).

$$CE[V + \Delta] = CE[V] + \Delta.$$

Invoking the delta property on the defining equation for $BPI$ yields $BPI_X = CEI_X$. This is an extension of the familiar result for lotteries that certainty equivalent and buying price are equal if (and only if) utility is linear or exponential (Raiffa, 1968; Howard, 1970).

At first glance it may appear that there is no oddity if $SPI$ and $BPI$ are not ordinally equivalent. But if they are not, then $PPI$ and $BPI$ will also not be ordinally equivalent. This implies that one might rationally reverse the preference for information depending upon how one pays for the information, paying money or accepting risk. Accepting risk to acquire information is not as rare as one might think. For example, medical patients accept the mortality risks of exploratory surgery and the cancer hazard of X-ray examination, and police detectives interrogate dangerous criminals to pursue investigative leads.

In view of the above it is tempting to hope that $SPI$ and $BPI$ are ordinally equivalent. In the next section we show that in fact this equivalence fails in general; that is, $BP$-$SP$ reversals can happen for information sources. Such reversals are basically the analogies to $BP$-$SP$ reversals for simple payoff lotteries.

## 3. Valuing Information and valuing payoff lotteries

### *Examples of reversals*

We first illustrate a $BP$-$SP$ reversal for payoff lotteries. Suppose the utility function $u$ is given by $u(w) = w^{1/2}$ for nonnegative $w$. Let the initial wealth $L$ be \$100 and the payoff of the lotteries $Y_1$ and $Y_2$ be as shown in Figure 1. Because $E[u(L + Y_1)] = 15$ and $E[u(L + Y_2)] = 15.11$, the decision maker will prefer receiving $Y_2$ as gift. Since $E[u(L + Y_1)] = 15 = u(\$225)$, it follows that the certainty equivalent of $L + Y_1$ is \$225, an increase of \$125 over the initial wealth \$100. Similarly, the certainty equivalent of $L + Y_2$ is \$228.31, and increase of \$128.31 over initial wealth.


*Figure 1.* Payoff lotteries $Y_1, Y_2$ exhibiting *BP-SP* reversal with wealth = \$100 under square-root utility.

The buying price $b_i$ for $Y_i$ satisfies the equation

$$E\left[u(L + Y_i - b_i)\right] = u(L).$$

Calculation shows that $b_1$ = \$93.75 and $b_2$ = \$83.79, in contrast to the respective certainty equivalent, increases \$125 and \$128.31. These respective quantities would be equal if risk attitude were constant.

An analogous reversal can be generated for information acquisition by appropriately embedding $Y_1$ and $Y_2$ into a decision problem. Consider the decision problem in Figure 2. In this problem, the decision maker must choose to play game 1 or game 2 or not to play. If game $i$ is chosen, then the decision maker must guess the value of a discrete uncertainty $X_i$. A correct guess increases wealth by $Y_i$, whereas an incorrect guess yields a penalty $k$.

Take $k$ = \$75 and let $X_1, X_2$ be independent ternary lotteries with $\mathrm{P}(X_1 = x) = \mathrm{P}(X_2 = x) = 1/3$ for each possible x. Extend the utility function $u(w) = w^{1/2}$ to $w < 0$ in any way that leaves $u$ increasing. Then with wealth = $L$ = \$100 as above, *Don't Play* is optimal in the absence of information about $X_1$ or $X_2$. It is not difficult to show that the certainty equivalent increases and buying prices for information about the independent uncertainties $X_1$ and $X_2$ are identical to the

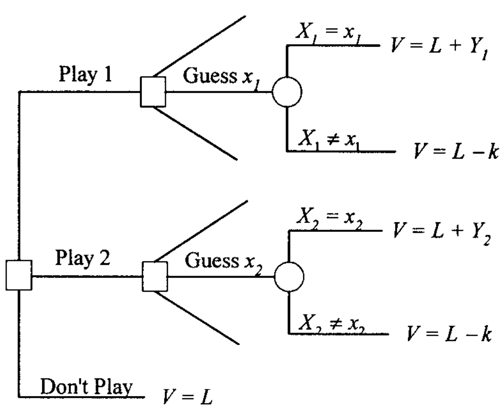


*Figure 2.* Embedding the payoff lotteries $Y_1, Y_2$ into a decision problem so that buying and selling prices for $Y_i$ are identical to buying and selling prices for the information source $X_i$.

corresponding quantities for the payoff lotteries $Y_1$ and $Y_2$ above, that is,

$$\begin{aligned} CEI_{X_1} &= \$125 \quad & CEI_{X_2} &= \$128.31 \\ BPI_{X_1} &= \$93.75 \quad & BPI_{X_2} &= \$83.79. \end{aligned}$$

We have therefore exhibited a *BP-CE* and therefore a *BP-SP* reversal for information sources.

*Payoff lotteries induced by information acquisition choices*

There is, of course, an intimate relationship between the operators *CE* and *BP* over random variables and operators *CEI* and *BPI* over information sources. For certainty equivalents, the relationship is the obvious one: $CEI_X$ is the amount an owner of $V_{a^*(X)}$ would demand to give it up in exchange for $V_{a^*}$, that is, $CEI_X$ is the difference between the certainty equivalents of $V_{a^*(X)}$ and $V_{a^*}$. In detail, because $E[u(V) \mid I_X] = E[u(V_{a^*(X)})]$, we have

$$\begin{aligned} CEI_X &= u^{-1}\big(E\big[u(V) \mid I_X\big]\big) - u^{-1}\big(E\big[u(V) \mid I_\varnothing\big]\big) \\ &= u^{-1}\big(E\big[u\big(V_{a^*(X)}\big)\big]\big) - u^{-1}\big(E\big[u\big(V_{a^*}\big)\big]\big) \\ &= CE\big[V_{a^*(X)}\big] - CE\big[V_{a^*}\big] \end{aligned}$$

where $CE[Z] = u^{-1}(E[u(Z)]$ denotes the certainty equivalent of a lottery $Z$. Because of this relationship, properties of certainty equivalents and selling prices extend naturally from lotteries to information sources.

One might naively expect that the buying price for $I_X$ is the price an owner of the lottery $V_{a^*}$ would pay to exchange it for the lottery $V_{a^*(X)}$. However, in general this statement is *false*, and one cannot invoke this kind of reasoning to extend properties of buying prices from lotteries to information sources. To see why, let $BP[Z \mid Wealth = L]$ be the buying price of the lottery $Z$ given (possibly uncertain) wealth $L$. If $BP[Z \mid Wealth = L] = b$, then $b$ satisfies the equation

$$E\big[u(L + Z - b)\big] = E\big[u(L)\big].$$

The buying price $BPI_X$ is the quantity $b$ satisfying

$$E\big[u(V - b) \mid I_X\big] = E\big[u(V) \mid I_\varnothing\big]$$

or equivalently

$$E_X\Big[\max_a E\big[u(V_a - b) \mid X\big]\Big] = E\big[u(V_{a^*})\big].$$

Let $a^*_{V-b}(X)$ be the maximizer on the left side of this equation. The act $a^*_{V-b}(X)$ is optimal after paying $b$ for $I_X$ and observing $X$. When risk attitude is not constant, $a^*_{V-b}(X)$ need not equal $a^*(X) = a^*_V(X)$, which is the optimal act after observing $X$ for free. When $a^*_{V-BPI_X}(X) = a^*(X)$ with probability 1, then paying the buying price for $I_X$ does not affect the optimal act when $X$ is observed. In this case we say that $a^*(X)$ is *purchase invariant*. When $a^*(X)$ is purchase invariant, the last displayed equation becomes

$$E\left[u\left(V_{a^*(X)} - BPI_X\right)\right] = E\left[u\left(V_{a^*}\right)\right].$$

We have established the following result.

**Proposition 2.** *Consider a decision problem with uncertainty $X$.*

(a) *$CEI_X$ is the amount an owner of the lottery $V_{a^*(X)}$ would demand to give it up in exchange for the lottery $V_{a^*}$, that is*

$$CEI_X = CE\left[V_{a^*(X)}\right] - CE\left[V_{a^*}\right].$$

(b) *Suppose $a^*(X)$ is purchase invariant. Then $BPI_X$ is the price an owner of the lottery $V_{a^*}$ would pay to exchange it for the lottery $V_{a^*(X)}$, that is,*

$$BPI_X = BP\left[V_{a^*(X)} - V_{a^*} \mid Wealth = V_{a^*}\right].$$

To summarize, while *selling* an information source is strictly analogous to selling a lottery, *buying* an information source is not analogous to buying a lottery unless purchase invariance holds.

### *Information acquisition choices induced by payoff lotteries*

Because Proposition 2(b) fails in the absence of purchase invariance, one cannot in general invoke it to infer properties of BPI from those of BP. However, it would be useful in light of Proposition 2 if for any payoff lottery $Y$, we could find a decision problem containing an uncertainty $X$ with $a^*(X)$ purchase invariant such that $V_{a^*(X)} - V_{a^*}$ was equal to $Y$. In fact, we have essentially accomplished this already in the construction of Figure 2.

**Proposition 3.** *Let $u$ be any strictly increasing utility function $u$. Then for any wealth level $L$ and any random variables $Y_1, \ldots, Y_n$ with $E[u(L + Y_i)] > u(L)$ for each $i$, there is a decision problem with independent uncertainties $X_1, \ldots, X_n$ such that $BPI_{X_i} = BP[Y_i \mid Wealth = L]$ and $CEI_{X_i} = CE[Y_i + L] - CE[L]$.*

Figure 2 illustrates this construction for $n = 2$. As before, the decision maker must first choose to play or not, and if s/he chooses to play $i$, s/he must guess the value of the uncertainty $X_i$. A correct guess yields independent incremental payoff $Y_i$, but an incorrect guess yields incremental payoff $-k < 0$. Without further information, the expected utility of *Play i* is $p_i(x_i)u(L + Y_i) + (1 - p_i(x_i))u(L - k)$, where $p_i(x_i) = P(X_i = x_i)$. Because $u(L - k) < u(L)$, we can, for each possible value $x_i$ of $X_i$, choose $p_i(x_i)$ sufficiently small so that for each $x_i$, $p_i(x_i)u(L + Y_i) + (1 - p_i(x_i))u(L - k), < u(L)$; that is, the decision maker would prefer to not play. So the optimal action $a^*$ without information is $a^* =$ "Don't Play," and $V_{a^*} = L$. But if $X_i = x_i$ can be observed prior to deciding, then because $E[u(L + Y_i)] > u(L)$, it is optimal to play $i$ and guess $x_i$. Then $a^*(x_i) =$ "Play $i$ and guess $x_i$," and $V_{a^*(X_i)} = L + Y_i$. Therefore $V_{a^*(X_i)} - V_{a^*} = Y_i$.

If we let $BP(Y_i \mid Wealth = L) = b_i$, then by definition of $BP$

$$E\big[u(L + Y_i - b_i)\big] = u(L),$$

so $b_i \geq 0$ because $E[u(L + Y_i)] > u(L)$. Therefore because $u$ is increasing, we have

$$E\big[u(L + Y_i - b_i)\big] \geq u(L - b_i).$$

This inequality implies $a^*_{V-b_i}(x_i) =$ "Play and guess $x_i$" $= a^*(x_i)$. Therefore $a^*(X)$ is purchase invariant. Hence by Proposition 2, $BPI_{X_i} = BP[Y_i \mid Wealth = L]$. Finally it is easy to check that $CEI_{X_i} = CE[Y_i + L] - CE[L]$. This establishes Proposition 3.

*Equality of certainty equivalent and buying price*

Proposition 3 allows us to extend known properties of certainty equivalents and buying prices for payoff lotteries to information sources. The following well known result on payoff lotteries receives general mention with partial or no proof by Howard (1970) and by Raiffa (1968). Because we did not locate a specific proof of necessity in the literature, we provide our own.

**Proposition 4.** *In order that $BP[Z \mid Wealth = L] = CE[Z + L] - CE[L]$ for all nonnegative payoff lotteries Z it is necessary and sufficient that the utility function $u(x)$ has constant risk attitude (i.e., is linear or exponential).*

*Proof:* Sufficiency is shown by LaValle (1968) and Howard (1970). To demonstrate necessity, note that buying price has the so-called $\Delta$-*property*:

$$BP\big[Z + \Delta \mid Wealth = L\big] = BP\big[Z \mid Wealth = L\big] + \Delta \qquad \Delta \geq 0.$$

Therefore $CE[Z + L]$ also possesses the $\Delta$-property, *i.e.*, $CE[Z + L + \Delta] = CE[Z + L] + \Delta$ for all $\Delta \geq 0$. Then the risk premium, as defined in Pratt (1964), is constant in wealth, and it follows from results there that $u$ must have constant risk attitude. □

**Corollary.** *$BPI_X = CEI_X$ for all decision problem uncertainties X if and only if the utility function $u(x)$ has constant risk attitude (i.e., is linear or exponential).*

*Reversals for certainty equivalents and buying prices*

Although the question of *BP-CE equality* is treated in the literature, we are aware of no examination of *BP-CE reversals* for payoff lotteries. Say that a utility function *u allows no BP-CE reversals* if for every pair of payoff lotteries $X, Y$

$$BP[X] \geq BP[Y] \Leftrightarrow CE[X] \geq CE[Y].$$

Here we omit reference to wealth, for expositional simplicity. Of course, linear or exponential utility functions allow no reversals because for them, $BP = CE$. Are there any other utility functions that do not allow reversals? We answer that question as follows.

**Proposition 5.** *A utility function u allows no BP-SP reversals for payoff lotteries if and only if its risk attitude is constant (i.e., u is linear or exponential).*

*Proof:* Only necessity remains to be established. Switching SP to CE, the assumption of no *BP-SP* reversals implies no *BP-CE* reversals. By simple logical manipulation then

$$CE[X] = CE[Y] \Leftrightarrow BP[X] = BP[Y].$$

From this it follows that *CE* is a *function* of *BP*, that is, there is some function $g$ such that $CE[X] = g(BP[X])$ for all $X$. But for *constant* payoff lotteries $X, CE[X] = BP[X]$, so it must be that $g$ is the identity function. Therefore, $CE[X] = BP[X]$ for all payoff lotteries $X$. □

Once again, invoking Proposition 3 extends this result to information sources.

**Corollary.** *BPI and SPI are ordinally equivalent measures of information value for a given utility function u if and only if u has constant risk attitude (i.e., u is linear or exponential).*

## 4. Conclusion

We have shown that although utility increase, certainty equivalent increase, selling price and probability price are ordinally equivalent measures of information value, none of these is ordinally equivalent to buying price. In particular, probability price and buying price may rank information sources differently. Only utility functions with constant risk attitude avoid this phenomenon. It is interesting that this lack of equivalence between buying and selling prices for information sources is *not* a naïve consequence of their lack of equivalence for lotteries. As we have shown, the analogy fails because while selling an information source $I_X$ is equivalent to selling the lottery resulting from an optimal response to learning $X$, purchasing an information source $I_X$ is not equivalent to purchasing that lottery, because due to wealth effects, the act of purchasing may change the optimal response to learning $X$.

We suspect that for many, information is a fundamental entity, in that the relative values of competing information acquisitions should not depend on the nature of the acquisition. This viewpoint must be abandoned in light of the fact that an expected utility maximizer can be willing to take more risk to learn about $X$ (*i.e.*, $X$ has a greater probability price), but pay more to learn about $Y$ (*i.e.*, $Y$ has a higher buying price). The point is not a trivial one: Utility increase is often preferred over buying price as a measure of information value due to its analytical tractability. Neither measure is more correct than the other—they are merely different measures. Analysts should be aware of the potential conflict between these measures due to their lack of ordinal equivalence.

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