## IEMS 326, Homework 3, Due 4/27/2011

- 1. Consider the oil well example we discussed in class. Now assume that the test for oil is not perfect but instead has a false positive probability of p and a false negative probability of q. That is, the probability of a positive test result if the oil field is poor is p. Similarly, the probability of a negative test result if the oil field is rich is q. Draw and solve the resulting decision tree assuming p=25% and q=10%. For compactness you may replace the subtree for the alternative "don't test" with a single outcome node (whose value we calculated in class). Write a sentence describing the optimal strategy. What is the most you would pay for the test (i.e., how much value is it creating)?
- 2a. Luenberger Chapter 5, #9 (p132-133). The problem mentions a trinomial lattice; you can think of it as a tree. Page 485 gives the solution.
- 2b. Now suppose that the price of oil can fluctuate: that it is equally likely to go up or down \$1.5/barrel each year. Note that you already know the initial price (\$10/barrel) and that costs and revenues are the beginning of the year.
- 3. (From Kellogg Finance 440) This is an involved problem involving taxes which we do not cover in lecture (and which will not be tested on). You can find information on taxes in chapter 12 of Newnan et al. Create and turn in a spreadsheet for this problem. Use a separate worksheet for each part. Provide well-written answers when it asks for an explanation.

One day, as you drive around in your BMW (after you get your MBA from Kellogg), you discover a cottage in the country that's for sale for \$380,000 (it's a nice cottage...). You want to buy it; and you've managed to save more than enough to cover the \$80,000 for a down payment. You shop around for mortgage loans and narrow your search down to two alternatives to finance the \$300,000 difference; both are 15-year loans.

Points	Quoted annual rate
2.5%	8%
0%	10%

Points are paid up-front out of savings (in addition to the down payment) and are a percentage of the mortgage principal amount (\$300,000 here). The mortgage principal is the same, \$300,000, in both alternatives. Note that points and mortgage interest payments are tax deductible (i.e., they reduce your taxable income). Assume that payments are end-of-period, interest is compounded monthly, taxes are also paid monthly (for simplicity), and that your tax rate is 30%, both for ordinary income and capital gains.

- a) What are the monthly payments associated with each of these loans?
- b) Use a spreadsheet to break down the first three years of payments of each loan into their principal and interest components. (The mortgage.xls spreadsheet on the course website may be helpful here.) Also break out the dollar amount of the tax savings each month due to the deductibility of interest and/or points.
- c) Assume that you plan to sell the cottage at the end of three years, and will use part of the

proceeds to pay off the remaining balance of the mortgage. The anticipated sales price is \$440,000. You pay capital gains tax on the difference between the sales and purchase prices. Do a present value calculation to determine which mortgage to take, discounting after-tax cashflows by an after-tax discount rate. Assume that the appropriate after-tax discount rate to use is 7% (quoted annual percentage rate, with monthly compounding). Intuitively, how would you expect your answer to change if you only intended to own the cottage for one year?

- d) Sensing your indecision, the current owners offer to rent you the cottage for \$2400 month (fixed over the three years) instead of selling it to you; rental payments are made at the beginning of each month. If you rent you avoid the mortgage payments and also save \$300 per month in maintenance costs and property taxes. The rental payments, however, are not tax deductible and you forego the expected capital gain. Should you rent or buy (assuming all else is the same as in parts (a) to (c))? Explain.
- e) With all else as in part (d), at what rental rate are you indifferent between renting and buying?