1 Time Value of Money

1.1 Future Value

The future value of \( x \) after \( n \) periods of growth at (annual) interest rate \( a \) compounded \( m \) times per year is

\[
x(1 + r)^n
\]

where \( r = a/m \) is the per-period interest rate.

The effective annual interest rate is

\[
i = (1 + a/m)^m - 1.
\]

The future value of \( x \) after \( t \) years of growth at annual growth rate \( d \) is

\[
x(1 + d)^t.
\]

1.2 Present Value

In the following, \( r \) is the per-period discount rate, \( d \) is the annual discount rate, and there are \( m \) periods per year.

The present value of \( y \) to be received \( n \) periods later is

\[
y(1 + r)^{-n} = \frac{y}{(1 + r)^n}.
\]

The present value of \( y \) to be received \( t \) years later is

\[
y(1 + d)^{-t} = \frac{y}{(1 + d)^t}.
\]

The relationship between \( r \) and \( d \) is

\[
d = (1 + r)^m - 1 \quad \text{and} \quad r = (1 + d)^{1/m} - 1.
\]
1.3 Present Value: Perpetuities and Annuities

When the discount rate is \( r \) per period, an annuity making \( n \) payments of \( C \), each one period apart, starting in one period:

\[
C \frac{1}{r} (1 - (1 + r)^{-n}).
\]

Present value of a perpetuity of \( C \) per period, starting in one period:

\[
\frac{C}{r}.
\]

2 Bonds

A coupon payment of a bond with face value \( F \), coupon rate \( c \) and \( m \) coupon payments per year is

\[
\frac{Fc}{m}.
\]

If the yield (quoted annually) is \( y \) for a bond making \( m \) coupon payments per year, the corresponding per-period discount rate is (because of the yield quotation convention)

\[
r = \frac{y}{m}.
\]

The price of a bond with face value \( F \), coupon rate \( c \), \( m \) coupon payments per year, next coupon payment in 1 period, \( n \) coupon payments remaining, and yield \( y \) is

\[
F(1 + r)^{-n} + \frac{Fc}{y} (1 - (1 + r)^{-n}).
\]

3 Inflation

When \( p \) is a nominal cost that grows at rate \( h \) per year, the nominal cost after \( t \) years is

\[
p(1 + h)^t.
\]

When \( i \) is an inflation rate and \( p \) is a nominal cost occurring at time \( u \), the real cost as measured in time \( s \) dollars is

\[
p(1 + i)^{s-u}.
\]

The real cost, as measured in base-\( b \) dollars, of an actual cost \( A \) at time \( t \), is

\[
R = A(1 + f)^{b-t},
\]

where \( f \) is the annual rate of inflation. If the actual cost of something at time \( t \) is \( A_t \), and its actual cost changes at an annual rate \( g \), then its actual cost at time \( u \) is

\[
A_u = A_t (1 + g)^{u-t}.
\]

The relationship between the inflation rate \( f \), the actual discount rate \( d_A \), and the real discount rate \( d_R \) is

\[
(1 + f)(1 + d_R) = 1 + d_A.
\]