1 Time Value of Money

1.1 Future Value

The future value of \(x\) after \(n\) periods of growth at (annual) interest rate \(a\) compounded \(m\) times per year is

\[x(1 + r)^n\]

where \(r = a/m\) is the per-period interest rate.

The effective annual interest rate is

\[i = (1 + a/m)^m - 1.\]

The future value of \(x\) after \(t\) years of growth at annual growth rate \(d\) is

\[x(1 + d)^t.\]

1.2 Present Value

In the following, \(r\) is the per-period discount rate, \(d\) is the annual discount rate, and there are \(m\) periods per year.

The present value of \(y\) to be received \(n\) periods later is

\[y(1 + r)^{-n} = \frac{y}{(1 + r)^n}.\]

The present value of \(y\) to be received \(t\) years later is

\[y(1 + d)^{-t} = \frac{y}{(1 + d)^t}.\]

The relationship between \(r\) and \(d\) is

\[d = (1 + r)^m - 1 \quad \text{and} \quad r = (1 + d)^{1/m} - 1.\]
1.3 Present Value: Perpetuities and Annuities

When the discount rate is $r$ per period, an annuity making $n$ payments of $C$, each one period apart, starting in one period:

$$ \frac{C}{r}(1 - (1 + r)^{-n}). $$

Present value of a perpetuity of $C$ per period, starting in one period:

$$ \frac{C}{r}. $$

2 Bonds

A coupon payment of a bond with face value $F$, coupon rate $c$ and $m$ coupon payments per year is

$$ \frac{Fc}{m}. $$

If the yield (quoted annually) is $y$ for a bond making $m$ coupon payments per year, the corresponding per-period discount rate is (because of the yield quotation convention)

$$ r = \frac{y}{m}. $$

The price of a bond with face value $F$, coupon rate $c$, $m$ coupon payments per year, next coupon payment in 1 period, $n$ coupon payments remaining, and yield $y$ is

$$ F(1 + r)^{-n} + \frac{Fc}{y(1 - (1 + r)^{-n})}. $$

3 Inflation

When $p$ is a nominal cost that grows at rate $h$ per year, the nominal cost after $t$ years is

$$ p(1 + h)^t. $$

When $i$ is an inflation rate and $p$ is a nominal cost occurring at time $u$, the real cost as measured in time $s$ dollars is

$$ p(1 + i)^{s-u}. $$

The real cost, as measured in base-$b$ dollars, of an actual cost $A$ at time $t$, is

$$ R = A(1 + f)^{b-t}, $$

where $f$ is the annual rate of inflation. If the actual cost of something at time $t$ is $A_t$, and its actual cost changes at an annual rate $g$, then its actual cost at time $u$ is

$$ A_u = A_t(1 + g)^{u-t}. $$

The relationship between the inflation rate $f$, the actual discount rate $d_A$, and the real discount rate $d_R$ is

$$ (1 + f)(1 + d_R) = 1 + d_A. $$