Midterm 1

1. (21 points) For all the parts to this problem, let the annual discount rate be 5%.
   a) Find the present value of the following cashflow: receive $10 every year for 30 years with the first payment being 10 years from now.
      
      Answer: $99.09

   b) Find the present value of the following cashflow: receive $10m now and the same amount a year from today and pay $3m a year forever with the first payment being a year from today.
      
      Answer: $-40.48m

   c) Consider the following two cashflows. For cashflow A, you receive $10 every year for 5 years with the first payment being today. For cashflow B, you receive x dollars every year forever with the first payment being today. What is the value of x in order for cashflow B to have the same present value as cashflow A?
      
      Answer: $2.16

2. (15 points) Today, you’re in charge of the nation’s finances. Suppose that projected 2015 shortfall is $418 billion and projected 2030 shortfall is $1,345 billion. In present value terms, how large is the difference of the two budget shortfalls? Assume a 3% discount rate.
      
      Answer: $396 billion

3. (18 points) Suppose that you borrowed $20k for 36 months to buy a car last year at an annual interest rate of 5% compounded monthly.
   a) What is the amount of monthly payment?
      
      Answer: $599.42

   b) Calculate the effective annual interest rate for both the car loan and for a rate of 6% compounded quarterly. Which is larger?
      
      Answer: for car loan: 5.12%  
                          For 6% compounded quarterly: 6.14%

   c) You made monthly payments for the last 12 months. But you still have to make 24 more payments. What is the present value of the remaining payments?
      
      Answer: $12,998 (or $13,663 for PV at year 12)
4. (15 points) Suppose that you consider some mortgage options. The price of home is $200k. Calculate your monthly payments for each option:
- Option A: 20% down payment at 15-year fixed annual rate of 4%
- Option B: 15% down payment at 30-year fixed annual rate of 4.5%
- Option C: 10% down payment at 30-year fixed annual rate of 6%

Answer: Option A: \(160000 \times 0.04/12/(1-(1+0.04/12)^{-180}) = $1183.50\)
Option B: \(170000 \times 0.045/12/(1-(1+0.045/12)^{-360}) = $861.37\)
Option C: \(180000 \times 0.06/12/(1-(1+0.06/12)^{-360}) = $1079\)

5. (15 points) Suppose that an account has $6m now. The money is invested and obtains a return of 2%. Your business projections are that in year one you take out $2m, in year two you take out $0.7m, in year three you add $1m to the account, and in year four you add $4m to the account. Calculate the amount of money in the account a year from now, two years from now, three years from now, and four years from now.

Answer:
Just after the interest payment and before the external transaction
Year 1 $6.12m
Year 2 $4.2024m
Year 3 $3.5724m
Year 4 $4.6639m

Just after the external transaction
Year 1 $4.12m
Year 2 $3.5024m
Year 3 $4.5724m
Year 4 $8.6639m

6. (16 points) Consider a 30-year mortgage with a 5% interest rate and a 20% down payment. If you can afford a $1000 monthly payment, how expensive a house can you buy?

Answer: \(1000 = x \times 0.05/12/(1-(1+.05/12)^{-360}) \Rightarrow x = 186280\)
You can afford at most \(x/0.8 = $232,850\)