Midterm 1 Practice Problems

1. Calculate the present value of each cashflow using a discount rate of 7%. Which do you most prefer most? Show and explain all supporting calculations!

Cashflow A: receive $60 today and then receive $60 in four years.
Cashflow B: receive $12 every year, forever, starting today.
Cashflow C: pay $50 every year for five years, with the first payment being next year, and then subsequently receive $30 every year for 20 years.
Cashflow D: receive $9 every other year, forever, with the first payment being next year.

2. Consider a project costing $1m each year from year 1 to year T. Then starting in year T+1, the project will generate a profit of $700k each year, forever.
     a) Write a formula for the present value of this project with a discount rate of r.
     b) Write a formula in terms of r for the value of T at which you break even (ignoring the issue of whether T is an integer).

3. Suppose you had $10,000 to invest for one year. You are deciding between a savings account with a 2% annual interest rate compounded daily (alternative A) and one with a 2% annual interest rate compounded monthly (alternative B). You are about to invest in the alternative A, but then you realize that since that bank is in downtown Chicago, you'll need to spend an extra $1 for parking when opening the account. Alternative B does not have this cost (it's a bank in Evanston). Should you change your decision or stick with alternative A? Show and explain all supporting calculations!

4. What is the effective annual interest rate in each situation?
a. A savings account with 4% annual interest rate compounded daily (assume a year consists of 365 days)?

b. A savings account with 4% annual interest rate compounded monthly?

5. Consider the following cashflow stream and a bank account paying 3% annual interest. What is the present value? Is the account value ever negative?

Year	Cashflow
0	8
1	2
2	4
3	-15
4	16

6. Which of the following cashflows do you most prefer using a discount rate of 10%? Using a discount rate of 1%? Show and explain all supporting calculations!

Cashflow A: receive $10 every year, forever, with the first payment next year
Cashflow B: receive $19 every other year, forever, with the first payment being next year

Cashflow C: pay $5 every year for 20 years, with the first payment being today, and then subsequently receive $30 every year for 20 years.
Cashflow D: receive $70 today and then receive $50 in five years.

7. Irene Engels recently graduated with an MBA. In August 2007, she borrowed $50,000, and she borrowed another $50,000 in August 2008. Her student loan has an annual interest rate of 2% compounded monthly. Irene doesn't make any payments on her student debt until she starts a lucrative Wall St. job. Then starting in September 2009 she makes a payment of $1000 every month. Now bonus time is coming near. For January 2010 she plans to make another $1000 payment (her 5th) and also apply her bonus to the debt. How big must her bonus be so that she will have completely paid-off the debt at the end of this January?

8. You are analyzing the value of the company Twitter using a 15% discount rate. You expect its cashflows over the next 4 years to be as shown below and you estimate its NPV as $1B. Explain.

Year	Cashflow
0	-20M
1	-10M
2	0
3	12M
4	40M

9. A bank offers a savings account with a 3% annual interest rate, compounded monthly.
9.1 What is the effective annual interest rate?
9.2 Stu wants to open a savings account and make one deposit now that will enable him to withdraw $700 to go on vacation 5 months from now and $2000 for a deposit on a rental apartment when he starts working in 20 months from now. How much money does Stu need to deposit now?

10. If the discount rate is 12%, what is the present value of receiving $1000 per year at the end of each of the next 8 years?

11. Using a discount rate of 5%, what is the net present value of the following cashflow stream?

Year	Cashflow
0	-20
1	2
2	4
3	8
5	16

12. You bought a $200k condo. You got a 15-year fixed-rate mortgage and made a 20% down payment.

a) What is your monthly payment?

b) Would the monthly payment be bigger or smaller with a 30-year mortgage at the same interest rate?

13. Consider the following cashflow stream and a bank account paying 10% annual interest. Today the account has $9.

Year	Cashflow
1	-4
2	+3
3	-1

What is the largest amount ever in the account?

14. Calculate the PV of the following cashflows using a 7% discount rate.

a) 30 payments of 100 starting 5 years from today

b) you pay 10/yr for 3 years with the first payment being today, and then starting a year from today you will receive $6/yr for 6 years.

15. Suppose that a construction project costs $10m (in present value) if you start it today. What are the savings (in present value) of delaying it by 3 years. Assume a 10% discount rate and that the price remains the same.

16. A 1% monthly rate of return is equivalent to what annual rate (compounded yearly)?

17. You've taken out a 30-year mortgage for 120k with a 4.2% rate.

a) What's your minimum monthly payment?

b) You've paid $700/mo for one and a half years. You're now trying to refinance. What's the principal remaining on your mortgage?

18. You and two friends are considering buying a house in Chicagoland to live here together after you graduate. You can get a 15-year fixed-rate mortgage with a mortgage rate of 5% if you make a 20% down payment on the house. You will split the monthly mortgage payment equally among the three of you. Each of the three of you can afford to contribute up to $1,000 per month towards the mortgage payment. You each have $10,000 available towards the down payment. How expensive a house can you afford to buy?

Solutions

1. PV of A = 60+60*1.07^-4 = $105.77.
PV of B = 12+12/0.07= $183.43
PV of C = -50/0.07*(1-1.07^-5)+30/0.07*(1-1.07^-20)*1.07^-5 = $21.59
PV of D = 9/(1.07^2-1)*1.07 = $66.46
The PV of cashflow B is largest and thus most preferred.

2. $PV = \$-1m*(1-(1+r)^{-T})/r + \$700k*(1+r)^{-T}/r$
$PV=0$ implies $(\$700k+\$1m)(1+r)^{-T} = \$1m$
so $T = \log 1.7 / \log (1+r)$

3. FV of alternative A: $\$9999*(1+0.02/365)^{365}=\$10,200.99$. FV of alternative B: $10,000*(1+0.02/12)^{12}=\$10,201.84$. Since the FV of B is greater than the FV of A, you should change your decision and go with alternative B.

4a. $(1+0.04/365)^{365} - 1 = 0.0408 = \textbf{4.08\%}$

4b. $(1+0.04/12)^{12} - 1 = 0.0407 = \textbf{4.07\%}$

5. Present value equals $8+2*1.03^{-1} + 4*1.03^{-2} - 15*1.03^{-3} + 16*1.03^{-4} = \textbf{14.20}$. If the account value is ever negative, then it will be at the end of year 3. The present value up cashflows through year 3 is $8+2*1.03^{-1} + 4*1.03^{-2} - 15*1.03^{-3}=-0.01$. Since this is negative, **the account will be negative at the end of year 3**.

6. The present value of cashflow A is $10/r$, or 100 when $r=10\%$ and 1000 when $r=1\%$. The two period interest rate is $s=(1+r)^2-1$, or 21% when $r=10\%$ and 2.01% when $r=1\%$. The present value of cashflow B is $(1+r)*19/s$ where the $1+r$ factor accounts for the fact that the first payment is in one year (half of a two year period). Thus the present value is 99.52 when $r=10\%$ and 955 when $r=1\%$. The present value of cashflow C is $-5-5/r*(1-(1+r)^{-19})+(1+r)^{-19}*(30/r*(1-(1+r)^{-20})$, or -5.06 when $r=10\%$ and 357 when $r=1\%$. The present value of cashflow D is $70+50*(1+r)^{-5}$, or 101 when $r=10\%$ and 118 when $r=1\%$. **Thus when $r=10\%$ then cashflow D is preferred and when $r=1\%$ then cashflow A is preferred.**

7. Let $r=0.02/12$ be the monthly interest rate. The future value of the debt at the end of August 2009 is $50000*(1+r)^{24}+50000*(1+r)^{12} = 103,048$. The present value at the end of August 2009 of the future payments is $1000/r*(1-(1+r)^{-5}) = 4975$. Thus the value of the debt at the end of August 2009 is $103,048-4975=98,073$. Thus the future value of the debt at the end of January 2010 is $98,073*(1+r)^5=\textbf{\$98,893}$. A bonus this big would allow her to pay off the debt.

8. Clearly the present value of the cashflows over the next 4 years is less than \$1B. So to have a present value of \$1B the cashflows after year 4 must be pretty big. Another way of saying the same thing is that the value of Twitter, X, at the end of year 4 must be quite high. We can actually calculate X. The future value X at year 4 is $X=(1B-20M)*1.15^4 - 10M*1.15^3+12M*1.15^1+40M = 1.753B$.

9.1 Answer: Since the annual interest rate a = 3%, compounded in m =12 periods, then the effective annual interest rate i is

$$(1 + a /m)^m -1 = (1 + 3\% /12)^{12} -1 = 3.04\%.$$

9.2 Answer: For future value y1 = \$700 received in n1 = 5 months later, the present value is

$$y1 *(1 + a/n1)^{-n1} = 700 *(1 + 3\%/12)^{-5} = 691.32.$$

For future value y2 = \$2000 received in n2 = 20 months later, the present value is

$$y2 *(1 + a/n2)^{-n2} = 2000 *(1 + 3\%/12)^{-20} = 1902.58.$$

Thus the money Stu needs to deposit now is

$$691.315 + 1902.578 = 2593.89.$$

10. Answer: PV=1000 * (1/0.12) * (1 − 1/(1.12^8)) = **\$4967.64**.

11. Answer: NPV = -20 + $2(1.05)^{-1}$ + $4(1.05)^{-2}$ + $8(1.05)^{-3}$ + $16(1.05)^{-5}$ = 4.98.

12a. principal = 80% 200k = 160k. Payment = 160k r/(1-(1+r)^-180). Here r is the monthly interest rate, the annual rate divided by 12.

12b. smaller.

13. 9.49

14a. 946.68

14b. 0.52

15. 2.49m

16. 12.7%

17a. 586.82

17b. 114,807.22

18. **The maximum monthly payment is \$3000. Suppose you pay 180 months to pay off the debt. PV = A[1-1/(1+r)^n]/r = \$379,365.73. This is the maximum principal you can pay. So the total cost of the house is \$379,365.73/(1-0.2) = \$474207.16 and the down payment is \$474207.16 * 20% = \$94,841.43 > \$30,000. So the most expensive house you can afford is \$30,000 / 20% = \$150,000.**