IEMS 326, Homework 3, Due 10/21/2011

1. The pension fund for Evanston city employees is worried about the coming wave of retirements. It expects to pay out $\$ 10 \mathrm{~m}$ a year, indefinitely, starting 10 years from today. The rate of inflation is $\mathrm{f}=2 \%$. Assuming a real rate of return $\mathrm{d}_{\mathrm{R}}=3 \%$. How much would the fund need to set aside today to cover these expenditures (the $\$ 10 \mathrm{~m} /$ year)?
2. Suppose the city invested $\$ 1 \mathrm{~m}$ ten years ago (October 2000). The annual returns over the following ten years were $4 \%$ (year 1 ), $9 \%, 2 \%, 7 \%, 6 \%, 12 \%, 9 \%,-15 \%,-5 \%$, and $1 \%$. Use the CPI-U to determine the cumulative real annual return from October 2000 to October 2010.
3. (Decision tree with imperfect test) Consider the oil well example we discussed in class. Now assume that the test for oil is not perfect but instead has a false positive probability of $p$ and a false negative probability of $q$. That is, the probability of a positive test result if the oil field is poor is $p$. Similarly, the probability of a negative test result if the oil field is rich is $q$. Draw and solve the resulting decision tree assuming $p=25 \%$ and $q=10 \%$. For compactness you may replace the subtree for the alternative "don't test" with a single outcome node (whose value we calculated in class). Suppose initially that the test costs $\$ 1 \mathrm{~m}$. Write a sentence describing the optimal strategy. Now suppose that the test cost is negotiable. What is the most you would pay for the test (i.e., how much value is it creating)?
4. (Put option) Assume that you own $\$ 10,000$ worth of Facebook shares. Suppose each month the value of Facebook shares either increase by $3 \%$ with probability 0.7 or decreases by 1 $1 / 1.03=2.9 \%$ with probability 0.3 . You friend on Wall Street makes you a standing offer until one year from now, to buy your shares for $\$ 10,000$. What is the expected value of your friend's offer? Use a $1 \%$ annual discount rate compounded monthly.
