Formula Sheet

1 Time Value of Money

1.1 Future Value
The future value of $x$ after $n$ periods of growth at (annual) interest rate $a$ compounded $m$ times per year is

$$x(1 + r)^n$$

where $r = a/m$ is the per-period interest rate.

The effective annual interest rate is

$$i = (1 + a/m)^m - 1.$$ 

The future value of $x$ after $t$ years of growth at annual growth rate $d$ is

$$x(1 + d)^t.$$ 

1.2 Present Value
In the following, $r$ is the per-period discount rate, $d$ is the annual discount rate, and there are $m$ periods per year.

The present value of $y$ to be received $n$ periods later is

$$y(1 + r)^{-n} = \frac{y}{(1 + r)^n}.$$ 

The present value of $y$ to be received $t$ years later is

$$y(1 + d)^{-t} = \frac{y}{(1 + d)^t}.$$ 

The relationship between $r$ and $d$ is

$$d = (1 + r)^m - 1 \quad \text{and} \quad r = (1 + d)^{1/m} - 1.$$
1.3 Present Value: Perpetuities and Annuities

When the discount rate is $r$ per period, the present value $P$ of an annuity making $n$ payments of $C$, each one period apart, starting in one period:

$$P = \frac{C}{r} (1 - (1 + r)^{-n}), \quad C = \frac{Pr}{1 - (1 + r)^{-n}}$$

Present value of a perpetuity of $C$ per period, starting in one period:

$$\frac{C}{r}.$$

2 Inflation

When $p$ is a nominal cost that grows at rate $h$ per year, the nominal cost after $t$ years is

$$p(1 + h)^t.$$

When $i$ is an inflation rate and $p$ is a nominal cost occurring at time $u$, the real cost as measured in time $s$ dollars is

$$p(1 + i)^{s-u}.$$

The real cost, as measured in base-$b$ dollars, of an actual cost $A$ at time $t$, is

$$R = A(1 + f)^{b-t},$$

where $f$ is the annual rate of inflation. If the actual cost of something at time $t$ is $A_t$, and its actual cost changes at an annual rate $g$, then its actual cost at time $u$ is

$$A_u = A_t(1 + g)^{u-t}.$$

The relationship between the inflation rate $f$, the actual discount rate $d_A$, and the real discount rate $d_R$ is

$$(1 + f)(1 + d_R) = 1 + d_A.$$
3 Probability

If $A$ and $B$ are two evens, then Bayes’ rule is

$$
\Pr[A|B] = \frac{\Pr[A \text{ and } B]}{\Pr[B]} = \frac{\Pr[B|A] \Pr[A]}{\Pr[B]}
$$

Let $X$ be a random variable. If there are $n$ total scenarios with probabilities $p_1, \ldots, p_n$, and $X_i$ is the value of $X$ in scenario $i$, then the mean of $X$ is

$$
E[X] = \sum_{i=1}^n p_i X_i.
$$

Regardless of how many scenarios there are, the variance

$$
\text{Var}[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2
$$

and the standard deviation $\sigma[X] = \sqrt{\text{Var}[X]}$.

Let $Y$ be another random variable. The covariance between $X$ and $Y$ is

$$
$$

and the correlation between $X$ and $Y$ is

$$
\rho(X,Y) = \frac{\text{Cov}[X,Y]}{\sigma[X]\sigma[Y]}.
$$

A linear combination of $X$ and $Y$, where $v$ and $w$ are constants, has mean and variance

$$
E[vX + wY] = vE[X] + wE[Y] \quad \text{and} \quad \text{Var}[vX + wY] = v^2\text{Var}[X] + 2vw\text{Cov}[X,Y] + w^2\text{Var}[Y].
$$

If $w_1, \ldots, w_m$ are constants and $X_1, \ldots, X_m$ are random variables, then the linear combination $\sum_{j=1}^m w_j X_j$ has mean and variance

$$
E \left[ \sum_{j=1}^m w_j X_j \right] = \sum_{j=1}^m w_j E[X_j] \quad \text{and} \quad \text{Var} \left[ \sum_{j=1}^m w_j X_j \right] = \sum_{j=1}^m w_j^2 \text{Var}[X_j] + 2 \sum_{j=1}^m \sum_{k<j} \text{Cov}[X_j,X_k].
$$

4 Bonds

A coupon payment of a bond with face value $F$, coupon rate $c$ and $m$ coupon payments per year is

$$
Fc/m.
$$

If the yield (quoted annually) is $y$ for a bond making $m$ coupon payments per year, the corresponding per-period discount rate is (because of the yield quotation convention)

$$
r = y/m.
$$

The price of a bond with face value $F$, coupon rate $c$, $m$ coupon payments per year, next coupon payment in 1 period, $n$ coupon payments remaining, and yield $y$ is

$$
F(1+r)^{-n} + \frac{F(c/m)}{r} \left(1 - (1+r)^{-n}\right).
$$
5 Capital Asset Pricing Model

Let \( r_f \) be the risk-free rate and \( R_M \) be the return of the market portfolio. Also let \( r_m = E[R_M] \) be the expected return of the market portfolio and \( \sigma_M = \sigma[R_M] \) be the standard deviation of the market portfolio’s return.

The beta of an asset whose return is \( R \) equals the covariance of its returns with the market portfolio’s return \( R_M \), divided by the variance of the market portfolio’s return:

\[
\beta = \frac{\text{Cov}[R_M, X]}{\sigma_M^2}.
\]

Capital Asset Pricing Model: the expected return of this asset is

\[
r = r_f + \beta (r_m - r_f).
\]

This equation is also known as the Security Market Line.

Capital Market Line:

\[
r = r_f + \frac{r_m - r_f}{\sigma_M} \sigma
\]

where \( r \) is the expected return and \( \sigma \) is the standard deviation of a portfolio that lies on this line.

6 Weighted Average Cost of Capital

A company’s (before-tax) WACC is

\[
r_d \frac{D}{V} + r_e \frac{E}{V}
\]

where

- \( r_d \) is the required return on debt,
- \( D \) is the value of the company’s debt,
- \( r_e \) is the required return on equity,
- \( E \) is the company’s market capitalization, and
- \( V = D + E \) is the company’s total market value.