1 Time Value of Money

1.1 Future Value

The future value of $x$ after $n$ periods of growth at (annual) interest rate $a$ compounded $m$ times per year is

$$x(1 + r)^n$$

where $r = a/m$ is the per-period interest rate.

The effective annual interest rate is

$$i = (1 + a/m)^m - 1.$$  

The future value of $x$ after $t$ years of growth at annual growth rate $d$ is

$$x(1 + d)^t.$$  

1.2 Present Value

In the following, $r$ is the per-period discount rate, $d$ is the annual discount rate, and there are $m$ periods per year.

The present value of $y$ to be received $n$ periods later is

$$y(1 + r)^{-n} = \frac{y}{(1 + r)^n}.$$  

The present value of $y$ to be received $t$ years later is

$$y(1 + d)^{-t} = \frac{y}{(1 + d)^t}.$$  

The relationship between $r$ and $d$ is

$$d = (1 + r)^m - 1 \quad \text{and} \quad r = (1 + d)^{1/m} - 1.$$  

1
1.3 Present Value: Perpetuities and Annuities

When the discount rate is \( r \) per period, the present value \( P \) of an annuity making \( n \) payments of \( C \), each one period apart, starting in one period:

\[
P = \frac{C}{r} (1 - (1 + r)^{-n}), \quad C = \frac{Pr}{1 - (1 + r)^{-n}}
\]

Present value of a perpetuity of \( C \) per period, starting in one period:

\[
\frac{C}{r}.
\]

2 Inflation

When \( p \) is a nominal cost that grows at rate \( h \) per year, the nominal cost after \( t \) years is

\[
p(1 + h)^t.
\]

When \( i \) is an inflation rate and \( p \) is a nominal cost occurring at time \( u \), the real cost as measured in time \( s \) dollars is

\[
p(1 + i)^{s-u}.
\]

The real cost, as measured in base-\( b \) dollars, of an actual cost \( A \) at time \( t \), is

\[
R = A(1 + f)^{b-t},
\]

where \( f \) is the annual rate of inflation. If the actual cost of something at time \( t \) is \( A_t \), and its actual cost changes at an annual rate \( g \), then its actual cost at time \( u \) is

\[
A_u = A_t(1 + g)^{u-t}.
\]

The relationship between the inflation rate \( f \), the actual discount rate \( d_A \), and the real discount rate \( d_R \) is

\[
(1 + f)(1 + d_R) = 1 + d_A.
\]