

IEMS 326  
Homework 2

Show and explain all supporting calculations! For the mortgage problems, the mortgage.xls spreadsheet on the course website may be helpful.

1. In August 2003, someone got a 30-year fixed-rate mortgage with a rate of 5.25% and principal of \$310,000 (first scheduled payment: September 2003).

1a. Suppose he makes a \$2000 payment every month until his mortgage is paid off (i.e. reaches a principal balance of zero: the payment in the last month may be less than \$2,000 in order to reach a principal balance of exactly zero). In what month will the mortgage be paid off?

1b. Same as part a, but now calculate the remaining balance after making his September 2009 payment.

1c. Same as part a but assume the monthly payment is \$1000.

2. Continuing the mortgage problem: after making his payment in September 2009, his principal balance was \$190,000 (less than indicated by the initial amortization schedule, due to extra principal payments he made during the first 6 years). He is considering two choices for refinancing his mortgage now: a new 30-year fixed-rate mortgage with a rate of 5.16%, or a new 15-year fixed-rate mortgage with a rate of 4.66%. Overall, he has three choices:

1. Do not refinance; keep the old mortgage.
2. Refinance and get the new 30-year fixed-rate mortgage.
3. Refinance and get the new 15-year fixed-rate mortgage.

If he refinances, he must pay fees of \$2,500 now. These fees are rolled into the refinanced mortgage by increasing the principal by \$2500. Regardless of which mortgage he has, he intends to make payments of \$2,000 per month every month, until his mortgage is paid off (i.e. reaches a principal balance of zero: the payment in the last month may be less than \$2,000 in order to reach a principal balance of exactly zero). For each of the three choices, calculate in what month the mortgage will be paid off.

3 (extra credit). According to the current tax code, homeowners get a tax deduction for the interest they pay. If  $I_t$  is the amount of interest accrued in month  $t$ , then suppose that the value of the tax deduction is  $0.28 \cdot I_t$  (for simplicity assume that the homeowner receives the value of the deduction at the same time he makes his monthly payment). He uses a discount rate of 4% (because he believes this is the rate at which his safe investments will grow in the long run). What is the present value of this tax deduction for the three options described in problem 2? Which of the three options is now the best?

4. In January 2006, three bond investors, Bill Isaacs, Beth Ingersoll, and Bo Ip, each bought \$1 million worth of bonds. All bonds in this question have an annual coupon period. All three bond investors bought only newly issued bonds, and the bonds were all sold for face value and hence had coupon rates equal to their yields when issued. They neither added money to their bond investing accounts nor took any out: they reinvested any coupon or principal repayments according to the strategies below, until January 2009, when they compared their wealth.

- Every January, Bill invested everything in bonds with a 1-year maturity.
- Beth always bought bonds with maturity January 2009.
- Bo sold all his bonds every January and then invested all the proceeds in those bonds with the highest yield.

Bill and Beth never sold any bonds before maturity. The table below shows yield curves each January.

**4a.** How much money did Bill, Beth, and Bo each have in January 2009?

**4b.** Who took on the least interest-rate and reinvestment risk? Who took on the most?

at time	maturity (in years)			
	1	2	3	10
Jan. 2006	4.5%	4.4%	4.3%	4.3%
Jan. 2007	5.0%	4.8%	4.7%	6.7%
Jan. 2008	2.9%	2.6%	2.6%	4.1%
Jan. 2009	0.4%	0.7%	1.0%	3.4%