6.1 Solution: The total return of the stock \( R = \frac{x_1}{x_0} \)

The total return on the short position equal to \( \frac{x_0 + (x_0 - x_1)}{x_0} = \frac{2x_0 - x_1}{x_0} = 2 - \frac{x_1}{x_0} = 2 - R \)

6.3 Solution: portfolio return \( R = \alpha R_A + (1 - \alpha) R_B \)

\[
\text{Var}(R) = \alpha^2 \text{Var}(R_A) + (1 - \alpha)^2 \text{Var}(R_B) + 2\alpha(1 - \alpha) \text{Cov}(R_A, R_B)
\]

\[
= \alpha^2 \sigma_A^2 + (1 - \alpha)^2 \sigma_B^2 + 2\alpha(1 - \alpha) \rho \sigma_A \sigma_B
\]

\[
= \alpha^2 \sigma_A^2 + (1 - 2\alpha + \alpha^2) \sigma_B^2 + 2(\rho \sigma_A \sigma_B)\alpha - 2(\rho \sigma_A \sigma_B) \alpha^2
\]

\[
= [\sigma_A^2 + \sigma_B^2 - 2\rho \sigma_A \sigma_B] \alpha^2 - 2(\rho \sigma_A \sigma_B) \alpha + \sigma_B^2
\]

The minimum is obtained at \( \alpha^* = \frac{\sigma_B^2 - \rho \sigma_A \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2\rho \sigma_A \sigma_B} = \frac{19}{23} \)

(fix derivative and set it to 0)

\[
\alpha^* = \alpha^* R_A + (1 - \alpha^*) R_B
\]

\[
= 13.7\%
\]

\[
\sigma^* = \sqrt{\text{Var}(R^*)} = 11.4\%
\]

6.5 Solution: (a) pay 6 + 0.5U

If rain, receive 6

If not rain, receive 3 \times 10^6

Expected return \( \frac{0.5 \times 3 \times 10^6 + 0.5 \times U}{10^6 + 0.5 U} - 1 = \frac{0.5 \times 10^6}{10^6 + 0.5 U} \)
(b) If $u = 3 \times 10^6$, then he has the same return in both scenarios, so the variance is 0. The expected return is 20%.

7.1 Solution:
(a) \[ \bar{r} = 7\% + \frac{\sigma}{32\%} (23\% - 7\%) \]
\[ = 7\% + 0.5\sigma \]
(b) \[ 39\% = 7\% + 0.5\sigma \Rightarrow \sigma = 64\% \]

The proportion invested in the market should be \( \frac{64\%}{32\%} = 2 \)
and the proportion invested in the T-bills should be \( 1 - 2 = -1 \)
i.e. borrow $1000 and invest $2000 in market

(c) In this case, \( \sigma = 0.7 \times 32\% \)
\[ \Rightarrow \bar{r} = 7\% + 0.5 \times 0.7 \times 32\% = 18.2\% \]

You should expect $1118.2 at the end of the year.

5. Solution: This is an open question. There is no one right answer. Almost all analysis in the submissions makes sense.