6.1 Solution: The total return of the stock $R = \frac{X_1}{X_0}$

The total return on the short position $= \frac{X_0 + (X_0 - X_1)}{X_0}$

$$= \frac{2X_0 - X_1}{X_0}$$

$$= 2 - \frac{X_1}{X_0}$$

$$= 2 - R$$

6.3 Solution: portfolio return $r = \alpha r_A + (1-\alpha) r_B$

$$Var(r) = \alpha^2 Var(r_A) + (1-\alpha)^2 Var(r_B) + 2\alpha(1-\alpha) cov(r_A, r_B)$$

$$= \alpha^2 \sigma_A^2 + (1-\alpha)^2 \sigma_B^2 + 2\alpha(1-\alpha)\rho\sigma_A\sigma_B$$

$$= \alpha^2\sigma_A^2 + (1-2\alpha+\alpha^2)\sigma_B^2 + 2(\rho\sigma_A\sigma_B)\alpha - 2(\rho\sigma_A\sigma_B)\alpha^2$$

$$= [\sigma_A^2 + \sigma_B^2 - 2\rho\sigma_A\sigma_B]\alpha^2 - 2(\sigma_B^2 - \rho\sigma_A\sigma_B)\alpha + \sigma_B^2$$

The minimum is obtained at $\alpha^* = \frac{\sigma_B^2 - \rho\sigma_A\sigma_B}{\sigma_A^2 + \sigma_B^2 - 2\rho\sigma_A\sigma_B} = \frac{19}{23}$

(take derivative and set it to 0)

$$r^* = \alpha^* r_A + (1-\alpha^*) r_B$$

$$= 13.7\%$$

$$\sigma^* = \sqrt{Var(r)^*} = 11.4\%$$

6.5 Solution: (a) ~~payoff~~ $10^6 + 0.5u$

If rain, receive u

If not rain, receive 3×10^6

Expected return $\frac{0.5 \times 3\times10^6 + 0.5 \times u}{10^6 + 0.5u} - 1 = \frac{0.5 \times 10^6}{10^6 + 0.5u}$

(b) If $u = 3\times 10^6$, then he has the same return in both scenarios, so the variance is 0. The expected return is 20%

7.1 Solution: (a) $r = 7\% + \frac{\sigma}{32\%}(23\% - 7\%)$

$= 7\% + 0.5\sigma$

(b) $39\% = 7\% + 0.5\sigma \Rightarrow \sigma = 64\%$

The proportion invested in the market should be $\frac{64\%}{32\%} = 2$ and the proportion invested in the T-bills should be $1 - 2 = -1$ i.e. borrow \$1000 and invest \$2000 in market

(c) In this case, $\sigma = 0.7 \times 32\%$

$\Rightarrow r = 7\% + 0.5 \times 0.7 \times 32\% = 18.2\%$

You should expect \$1182 at the end of the year.

5. Solution: This is an open question. There is no one right answer. Almost all analysis in the submissions makes sense.