Formula Sheet

1 Time Value of Money

1.1 Future Value

The future value of \( x \) after \( n \) periods of growth at (annual) interest rate \( a \) compounded \( m \) times per year is

\[
x(1 + r)^n
\]

where \( r = a/m \) is the per-period interest rate.

The effective annual interest rate is

\[
i = (1 + a/m)^m - 1.
\]

The future value of \( x \) after \( t \) years of growth at annual growth rate \( d \) is

\[
x(1 + d)^t.
\]

1.2 Present Value

In the following, \( r \) is the per-period discount rate, \( d \) is the annual discount rate, and there are \( m \) periods per year.

The present value of \( y \) to be received \( n \) periods later is

\[
y(1 + r)^{-n} = \frac{y}{(1 + r)^n}.
\]

The present value of \( y \) to be received \( t \) years later is

\[
y(1 + d)^{-t} = \frac{y}{(1 + d)^t}.
\]

The relationship between \( r \) and \( d \) is

\[
d = (1 + r)^m - 1 \quad \text{and} \quad r = (1 + d)^{1/m} - 1.
\]
1.3 Present Value: Perpetuities and Annuities

When the discount rate is $r$ per period, an annuity making $n$ payments of $C$, each one period apart, starting in one period:

$$\frac{C}{r}(1 - (1 + r)^{-n}).$$

Present value of a perpetuity of $C$ per period, starting in one period:

$$\frac{C}{r}.$$

2 Bonds

A coupon payment of a bond with face value $F$, coupon rate $c$ and $m$ coupon payments per year is

$$Fc/m.$$ 

If the yield (quoted annually) is $y$ for a bond making $m$ coupon payments per year, the corresponding per-period discount rate is (because of the yield quotation convention)

$$r = y/m.$$ 

The price of a bond with face value $F$, coupon rate $c$, $m$ coupon payments per year, next coupon payment in 1 period, $n$ coupon payments remaining, and yield $y$ is

$$F(1 + r)^{-n} + \frac{Fc}{y}(1 - (1 + r)^{-n}).$$

3 Inflation

When $p$ is a nominal cost that grows at rate $h$ per year, the nominal cost after $t$ years is

$$p(1 + h)^t.$$ 

When $i$ is an inflation rate and $p$ is a nominal cost occurring at time $u$, the real cost as measured in time $s$ dollars is

$$p(1 + i)^{s-u}.$$ 

The real cost, as measured in base-$b$ dollars, of an actual cost $A$ at time $t$, is

$$R = A(1 + f)^{b-t},$$

where $f$ is the annual rate of inflation. If the actual cost of something at time $t$ is $A_t$, and its actual cost changes at an annual rate $g$, then its actual cost at time $u$ is

$$A_u = A_t(1 + g)^{u-t}.$$ 

The relationship between the inflation rate $f$, the actual discount rate $d_A$, and the real discount rate $d_R$ is

$$(1 + f)(1 + d_R) = 1 + d_A.$$
4 Probability

Let $X$ be a random variable. If there are $n$ total scenarios with probabilities $p_1, \ldots, p_n$, and $X_i$ is the value of $X$ in scenario $i$, then the mean of $X$ is

$$E[X] = \sum_{i=1}^{n} p_i X_i.$$ 

Regardless of how many scenarios there are, the variance

$$\text{Var}[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

and the standard deviation $\sigma[X] = \sqrt{\text{Var}[X]}$.

Let $Y$ be another random variable. The covariance between $X$ and $Y$ is


and the correlation between $X$ and $Y$ is

$$\rho(X,Y) = \frac{\text{Cov}[X,Y]}{\sigma[X] \sigma[Y]}.$$ 

A linear combination of $X$ and $Y$, where $v$ and $w$ are constants, has mean and variance

$$E[vX + wY] = vE[X] + wE[Y] \quad \text{and} \quad \text{Var}[vX + wY] = v^2 \text{Var}[X] + 2vw \text{Cov}[X,Y] + w^2 \text{Var}[Y].$$

If $w_1, \ldots, w_m$ are constants and $X_1, \ldots, X_m$ are random variables, then the linear combination $\sum_{j=1}^{m} w_j X_j$ has mean and variance

$$E \left[ \sum_{j=1}^{m} w_j X_j \right] = \sum_{j=1}^{m} w_j E[X_j] \quad \text{and} \quad \text{Var} \left[ \sum_{j=1}^{m} w_j X_j \right] = \sum_{j=1}^{m} w_j^2 \text{Var}[X_j] + 2 \sum_{j=1}^{m} \sum_{k \neq j} \text{Cov}[X_j, X_k].$$

5 Capital Asset Pricing Model

Let $r_f$ be the risk-free rate and $R_M$ be the return of the market portfolio. Also let $r_m = E[R_M]$ be the expected return of the market portfolio and $\sigma_M = \sigma[R_M]$ be the standard deviation of the market portfolio’s return.

The beta of an asset whose return is $R$ equals the covariance of its returns with the market portfolio’s return divided by the variance of the market portfolio’s return:

$$\beta = \frac{\text{Cov}[R_M, X]}{\sigma_M^2}.$$ 

Capital Asset Pricing Model: the expected return of this asset is

$$r = r_f + \beta (r_m - r_f).$$

This equation is also known as the Security Market Line.

Capital Market Line:

$$r = r_f + \frac{r_m - r_f}{\sigma_M} \sigma$$

where $r$ is the expected return and $\sigma$ is the standard deviation of a portfolio that lies on this line.
6 Weighted Average Cost of Capital

A company’s (before-tax) WACC is

\[ \frac{r_d D}{V} + \frac{r_e E}{V} \]

where

- \( r_d \) is the required return on debt,
- \( D \) is the value of the company’s debt,
- \( r_e \) is the required return on equity,
- \( E \) is the company’s market capitalization, and
- \( V = D + E \) is the company’s total market value.